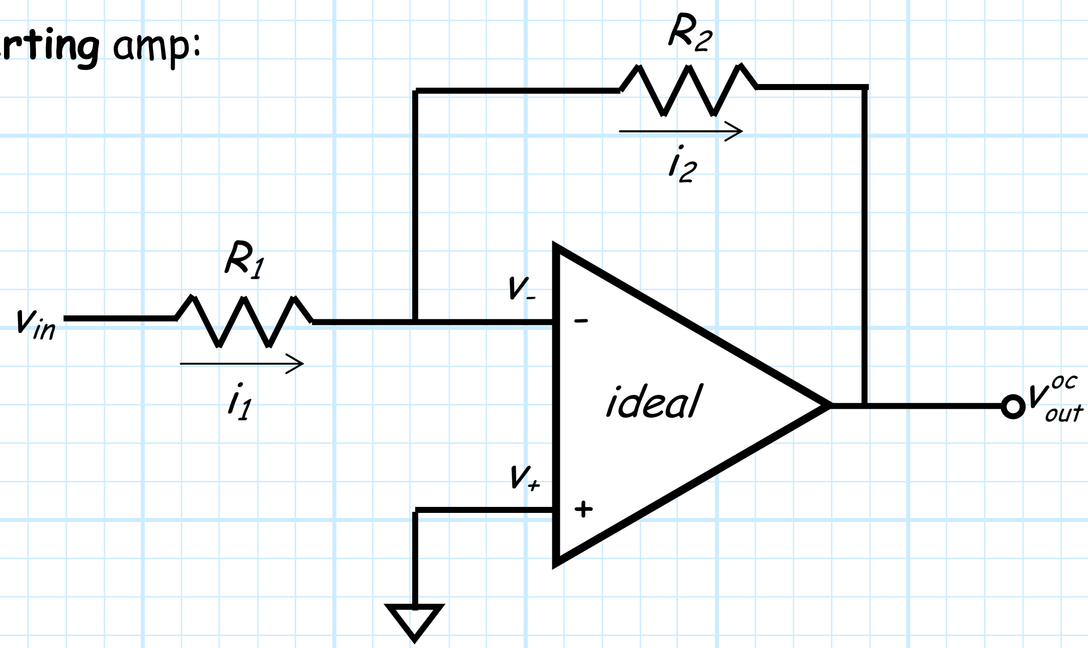
Feedback Stability

Recall that for the **inverting** amp:



we have the **feed-forward** equation:

$$v_{out}^{oc} = -A_{op}\, v_-$$

and the **feed-back** equation:

$$v_- = \frac{R_2\, v_{in} + R_1\, v_{out}^{oc}}{R_1 + R_2}$$

Nothing more fun than calculus!

Taking **derivatives** of these equations, we find that:

$$\frac{\partial v_{out}^{oc}}{\partial v_-} = \frac{\partial\left(-A_{op}\, v_-\right)}{\partial v_-} = -A_{op}$$

and:

$$\frac{\partial v_-}{\partial v_{out}^{oc}} = \frac{\partial}{\partial v_{out}^{oc}}\left(\frac{R_2\, v_{in} + R_1\, v_{out}^{oc}}{R_1 + R_2}\right) = \frac{R_1}{R_1 + R_2}$$

These derivatives are **very** important in determining the **stability** of the feedback amplifier.

Feed-forward

To see this, consider what happens when, for some reason, v_- **changes** some small value Δv_- from its nominal value of $v_-=0$.

The **output** voltage will then likewise change by a value Δv_{out}^{oc}:

$$\Delta v_{out}^{oc} \approx \left(\frac{\partial v_{out}^{oc}}{\partial v_-}\right) \Delta v_- = -A_{op}\,\Delta v_1$$

Note if Δv_- is **positive**, then Δv_{out}^{oc} will be **negative**—an **increase** in v_- leads to a **decrease** in v_{out}^{oc}.

This describes the feed-**forward** portion of the "loop."

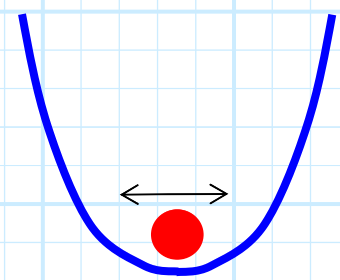
Feed-back

The feed-**back** equation states that a small change in the **output** voltage (i.e., Δv_{out}^{oc}), will likewise result in a small change in v_-:

$$\Delta v_- \approx \left(\frac{\partial v_-}{\partial v_{out}^{oc}} \right) \Delta v_{out}^{oc} = \left(\frac{R_1}{R_1 + R_2} \right) \Delta v_{out}^{oc}$$

Note in this case, a **decreasing** output voltage will result in a **decreasing** inverting terminal voltage v_-.

Thus, if the inverting terminal voltage tries to **increase** from its correct value of v_1=0, the control loop will react by **decreasing** the voltage v_1—essentially **counteracting** the initial change!



Negative feedback-in this case it's a good thing!

Note that the **loop product**:

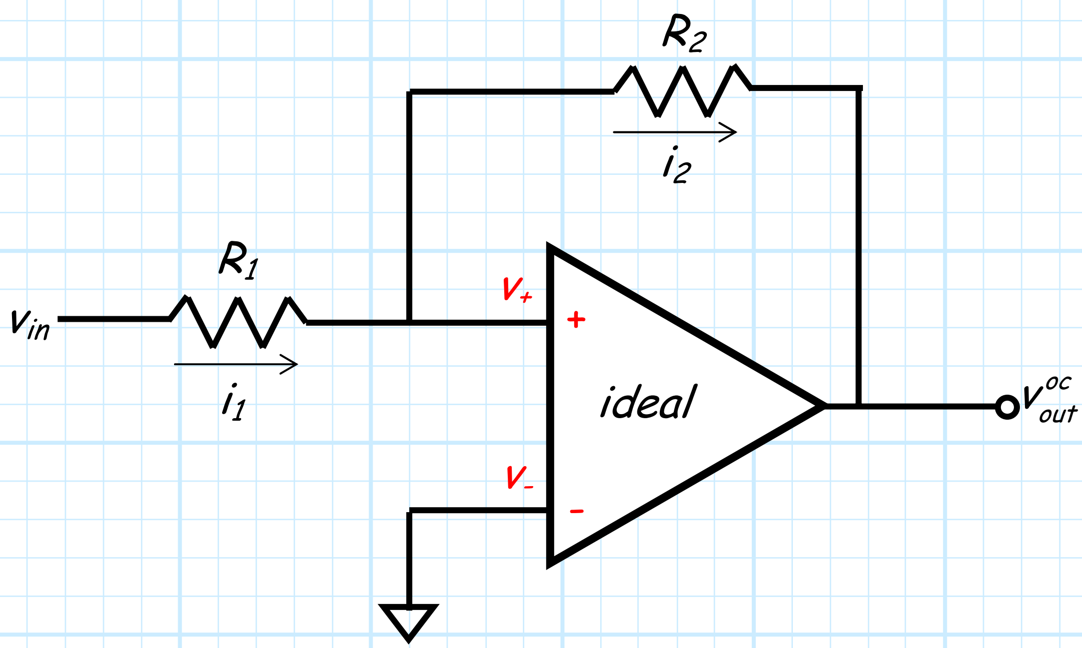
$$\frac{\partial v_{out}^{oc}}{\partial v_-}\,\frac{\partial v_-}{\partial v_{out}^{oc}} = -A_{op}\left(\frac{R_1}{R_1+R_2}\right)$$

is a **negative** value; we refer to this case as **negative feedback**.

Negative feedback keeps the inverting voltage **in place** (i.e., $v_-=0$)—it "enforces" the concept of the virtual ground!

Let's try some positive feedback

Contrast this behavior with that of the following circuit:



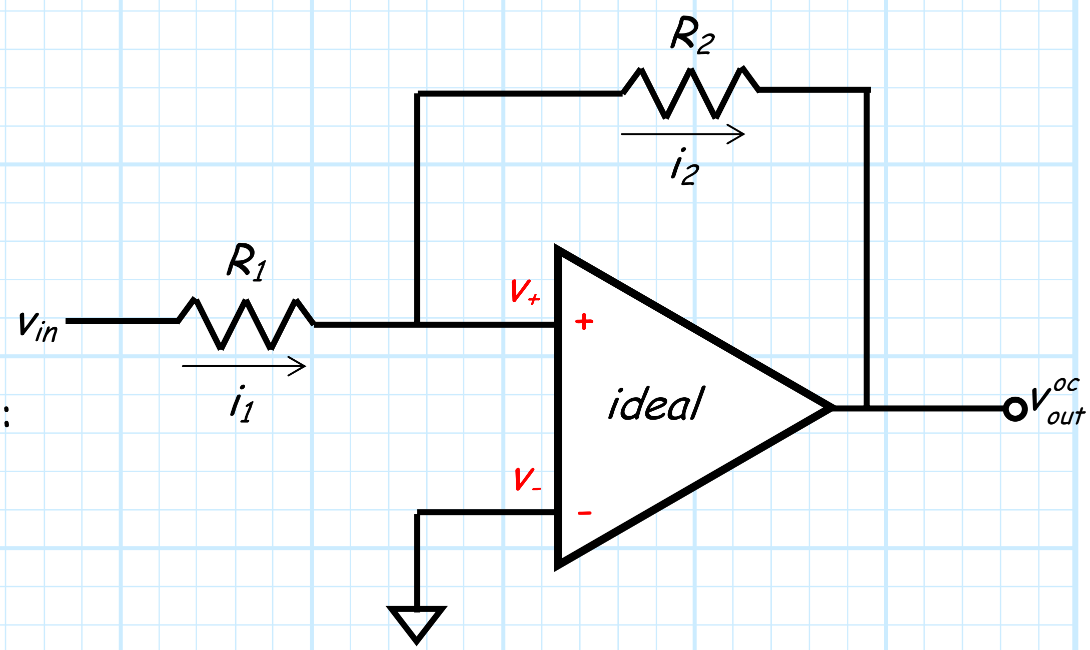
Q: *Isn't this precisely the **same** circuit as before?*

A: NO!

Note that the feedback resistor is now connected to the **non**-inverting terminal, and the **inverting** terminal is now grounded.

Positive derivatives!

The feed-**forward** equations for **this** circuit are thus:



$$v_{out}^{oc} = A_{op}\, v_+$$

And so:

$$\frac{\partial v_{out}^{oc}}{\partial v_+} = \frac{\partial\left(A_{op}\, v_+\right)}{\partial v_+} = A_{op}$$

while the feed-**back** equations are:

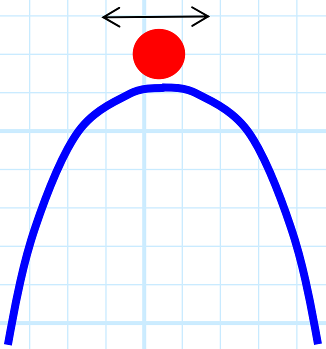
$$v_+ = \frac{R_2\, v_{in} + R_1\, v_{out}^{oc}}{R_1 + R_2}$$

and:

$$\frac{\partial v_+}{\partial v_{out}^{oc}} = \frac{\partial}{\partial v_{out}^{oc}}\left(\frac{R_2\, v_{in} + R_1\, v_{out}^{oc}}{R_1 + R_2}\right) = \frac{R_1}{R_1 + R_2}$$

Note in this case, **both** derivatives are **positive**.

Positive feedback-in this case it's a bad thing!



This means that an **increase** in v_+ will lead to an **increase** in v_{out}^{oc}. The problem is that the feedback will react by **increasing** v_+ even more—the error is **not** corrected, it is instead **reinforced**!

The result is that the output voltage will be sent to $v_{out}^{oc} = \infty$ or $v_{out}^{oc} = -\infty$ (i.e., the amplifier will **saturate**).

Note that the **loop product** for this case is **positive**:

$$\frac{\partial v_{out}^{oc}}{\partial v_+} \frac{\partial v_+}{\partial v_{out}^{oc}} = A_{op}\left(\frac{R_1}{R_1 + R_2}\right)$$

Thus, we refer to this case as **positive** feedback.

Positive feedback typically leads to amplifier **instability**!

As a result, we find that the **feed-back** portion of an op-amp circuit almost always is connected to its **inverting** (-) terminal!