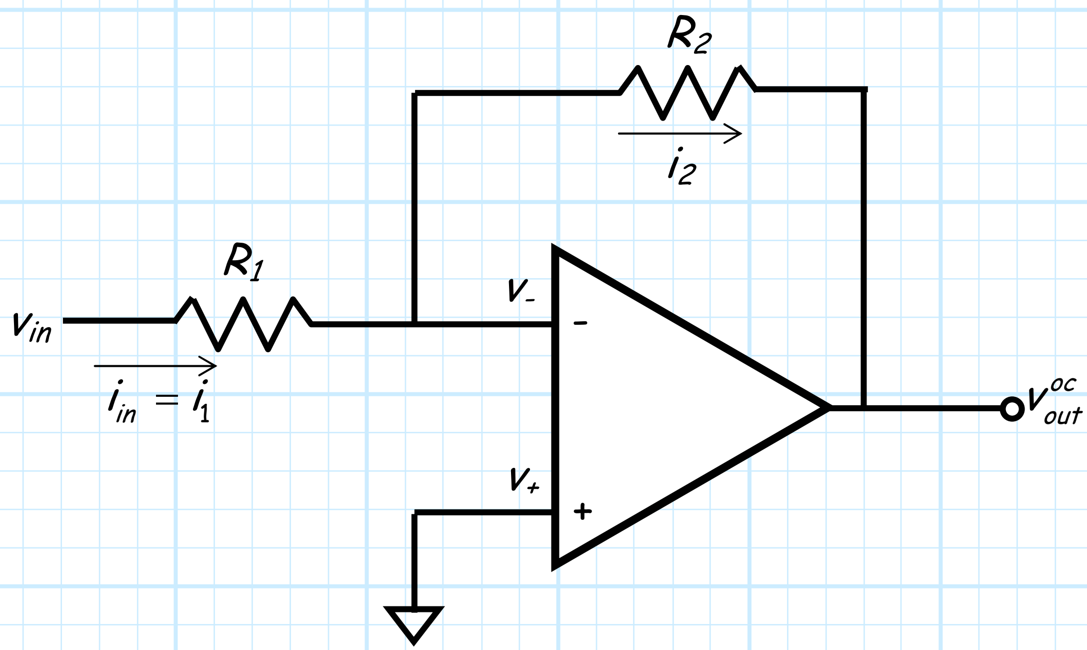
R_{in} and R_{out} of the Inverting Amplifier

Recall that the input resistance of an amplifier is:

$$R_{in} = \frac{v_{in}}{i_{in}}$$

For the **inverting** amplifier, it is evident that the input current i_{in} is equal to i_1:



Its input resistance

From **Ohm's Law**, we know that this current is:

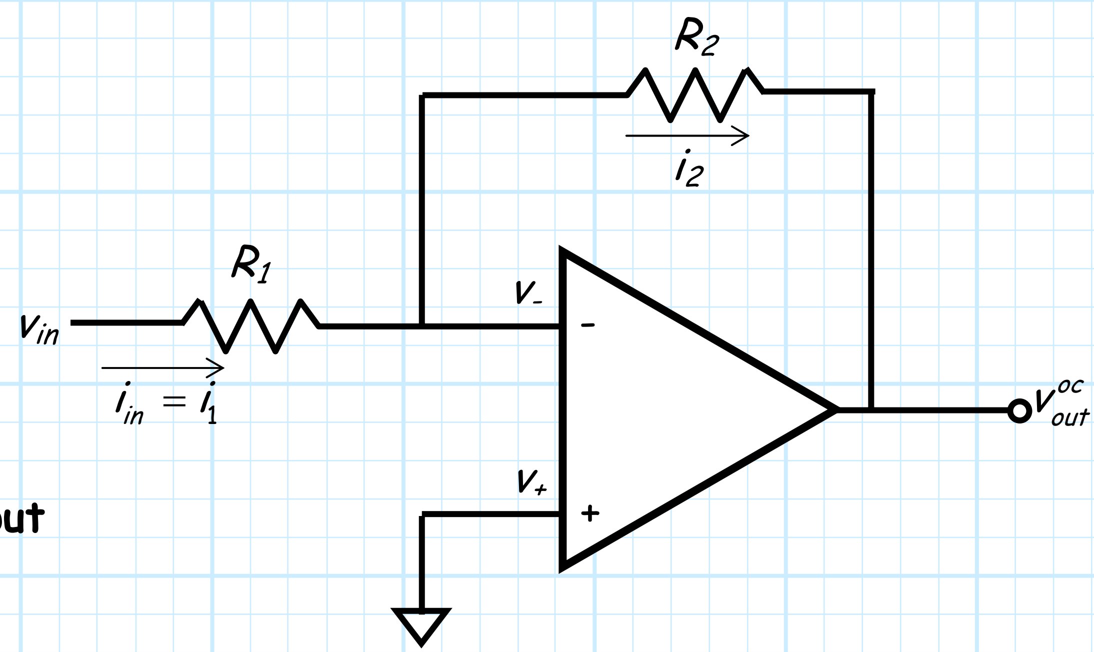
$$i_{in} = i_1 = \frac{v_{in} - v_1}{R_1}$$

The non-inverting terminal is "connected" to **virtual ground**:

$$v_- = 0$$

and thus the **input current** is:

$$i_{in} = i_1 = \frac{v_{in}}{R_1}$$



We now can determine the **input** resistance:

$$R_{in} = \frac{v_{in}}{i_{in}} = v_{in}\left(\frac{R_1}{v_{in}}\right) = R_1$$

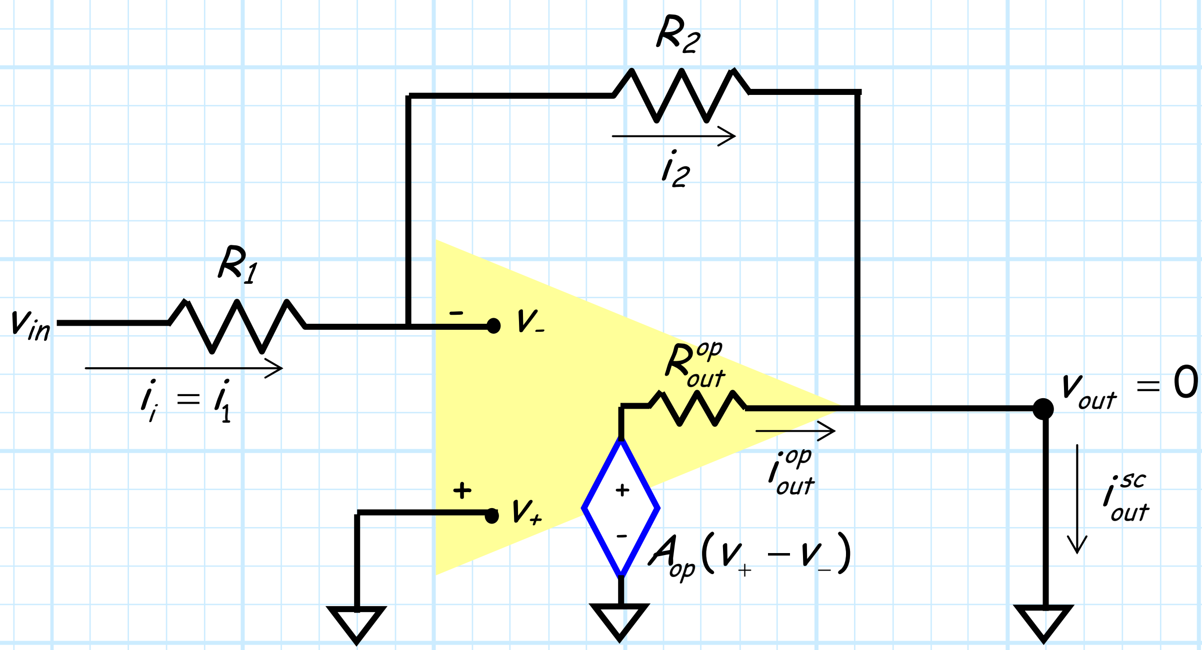
The **input resistance** of this inverting amplifier is therefore $R_{in} = R_1$!

Output resistance is harder

Now, let's attempt to determine the **output resistance** R_{out}.

Recall that we need to determine **two** values: the **short-circuit output current** $\left(i_{out}^{sc}\right)$ and the **open-circuit output voltage** $\left(v_{out}^{oc}\right)$.

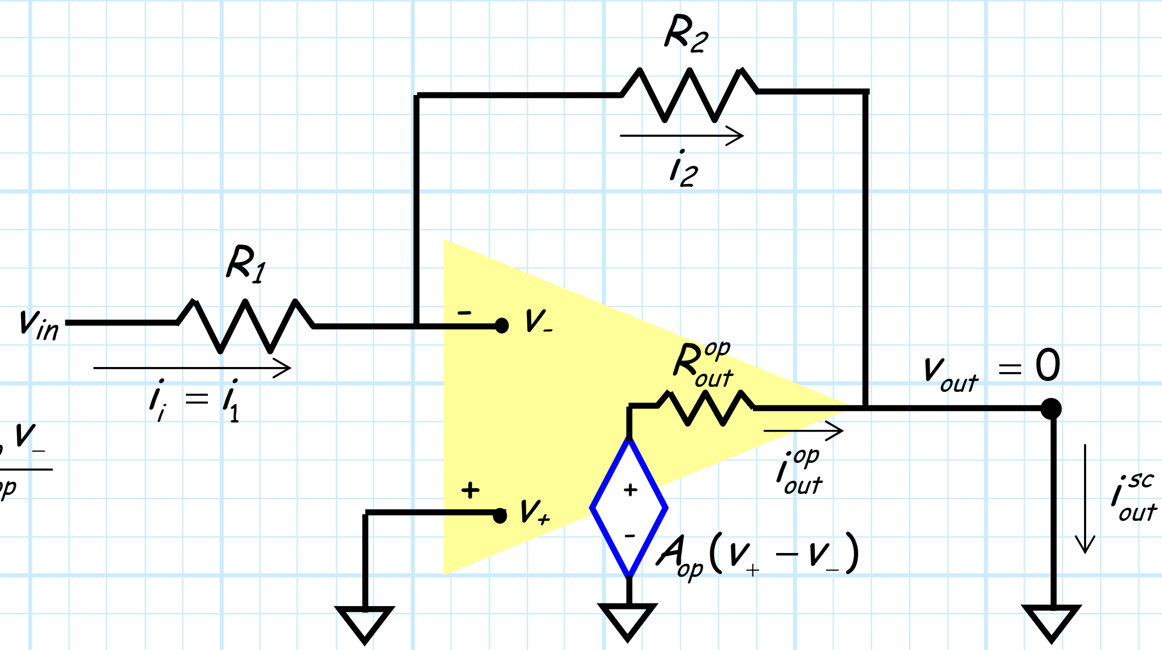
To accomplish this, we must replace the op-amp in the circuit with its **linear circuit model**:



First, the short circuit output current

From KCL, we find that:

$$i_{out}^{sc} = i_2 + i_{out}^{op}$$



where:

$$i_{out}^{op} = \frac{-A_{op}\,v_- - v_{out}^{oc}}{R_o^{op}} = \frac{-A_{op}\,v_-}{R_o^{op}}$$

and:

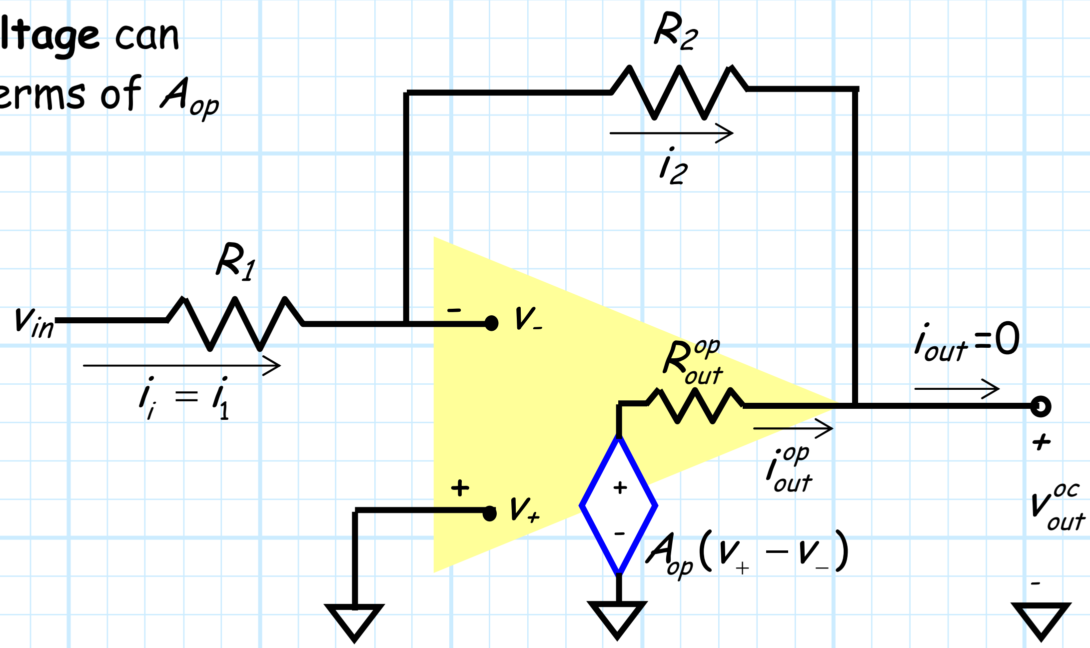
$$i_2 = \frac{v_- - v_{out}^{oc}}{R_2} = \frac{v_-}{R_2}$$

Therefore, the **short-circuit output current** is:

$$i_{out}^{sc} = \frac{v_-}{R_2} - \frac{A_{op}\,v_-}{R_{out}^{op}} = \left(\frac{R_{out}^{op} - R_2\,A_{op}}{R_2\,R_{out}^{op}}\right) v_- \cong -\frac{A_{op}}{R_{out}^{op}}\,v_-$$

Now, the open circuit output voltage

The **open-circuit output voltage** can likewise be determined in terms of A_{op} and v_- .



Here, it is evident that since $i_{out} = 0$:

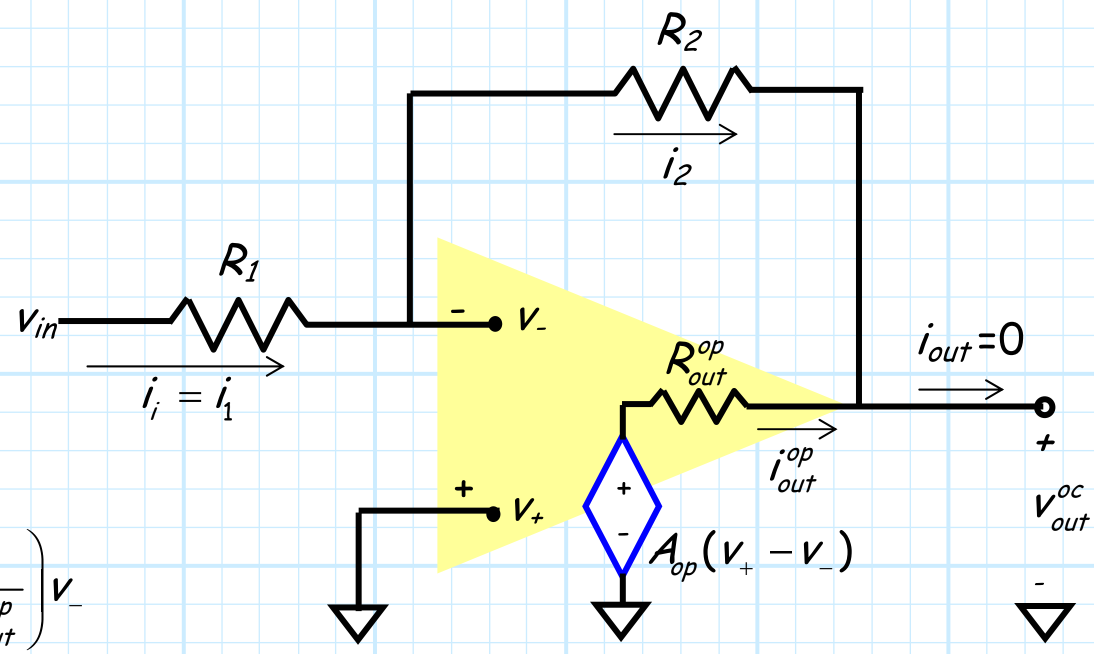
$$i_2 = -i_{out}^{op}$$

where we find from Ohm's Law:

$$i_2 = \frac{v_- - (-A_{op} v_-)}{R_2 + R_{out}^{op}} = \left(\frac{1 + A_{op}}{R_2 + R_{out}^{op}} \right) v_-$$

The open-circuit output voltage

Now from KVL:



$$v_{out}^{oc} = v_- - R_2\, i_2$$

Inserting the expression for i_2:

$$\begin{aligned} v_{out}^{oc} &= v_- - R_2 \left(\frac{1 + A_{op}}{R_2 + R_{out}^{op}} \right) v_- \\ &= \left(\frac{R_2 + R_{out}^{op}}{R_2 + R_{out}^{op}} - \frac{R_2 \left(1 + A_{op}\right)}{R_2 + R_{out}^{op}} \right) v_- \\ &= \left(\frac{R_o^{op} - R_2\, A_{op}}{R_2 + R_{out}^{op}} \right) v_- \\ &\cong - \frac{R_2\, A_{op}}{R_2 + R_{out}^{op}}\, v_- \end{aligned}$$

Now we find the output resistance

Now, we can find the **output resistance** of this amplifier:

$$
\begin{aligned}
R_{out} &= \frac{v_{out}^{oc}}{i_{out}^{sc}} \\
&= \left(\frac{-R_2 A_{op}}{R_2 + R_o^{op}}\right)\left(\frac{-A_{op}}{R_o^{op}}\right)^{-1} \\
&= \frac{R_2 R_o^{op}}{R_2 + R_o^{op}} \\
&= R_2 \| R_o^{op}
\end{aligned}
$$

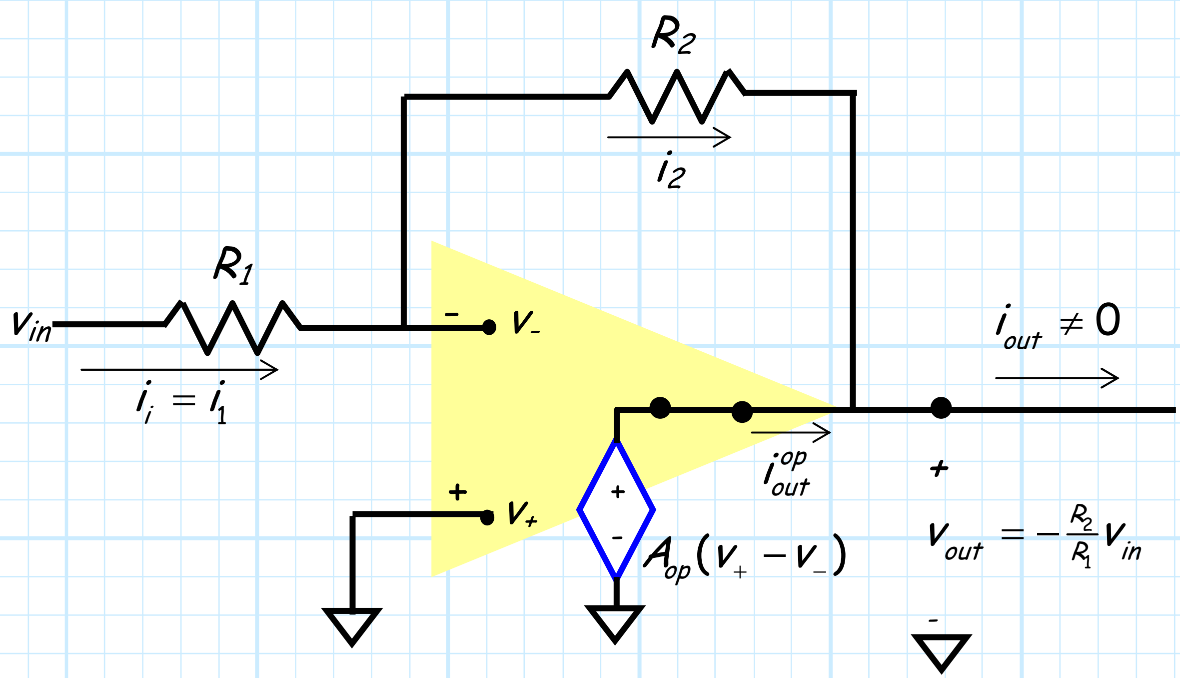
In other words, the **inverting amplifier output resistance** is simply equal to the value of the **feedback** resistor R_2 in **parallel** with **op-amp** output resistance R_{out}^{op}.

This is zero if the op-amp is ideal

Ideally, of course, the op-amp output resistance is **zero**, so that the output resistance of the inverting amplifier is **likewise zero**:

$$\begin{aligned} R_{out} &= R_2 \| R_{out}^{op} \\ &= R_2 \| 0 \\ &= 0 \end{aligned}$$

Note for this case—where the output resistance is **zero**—the output voltage will be the **same**, regardless of what **load** is attached at the output (e.g., **regardless** of i_{out})!



For real op-amps the output resistance is small

Thus, if $R_{out} = 0$, then the output voltage is equal to the **open-circuit** output voltage—even when the output is **not** open circuited:

$$v_{out} = -\frac{R_2}{R_1} v_{in} \qquad \text{for } \mathbf{all} \ \ i_{out} \ \ !!$$

Recall that it is this property that made $R_{out} = 0$ an "ideal" amplifier characteristic.

We will find that real (i.e., non-ideal!) op-amps typically have an output resistance that is **very small** ($R_{out}^{op} \ll R_2$), so that the **inverting amplifier** output resistance is **approximately equal** to the op-amp output resistance:

$$\begin{aligned} R_{out} &= R_2 \parallel R_{out}^{op} \\ &\approx R_{out}^{op} \end{aligned}$$

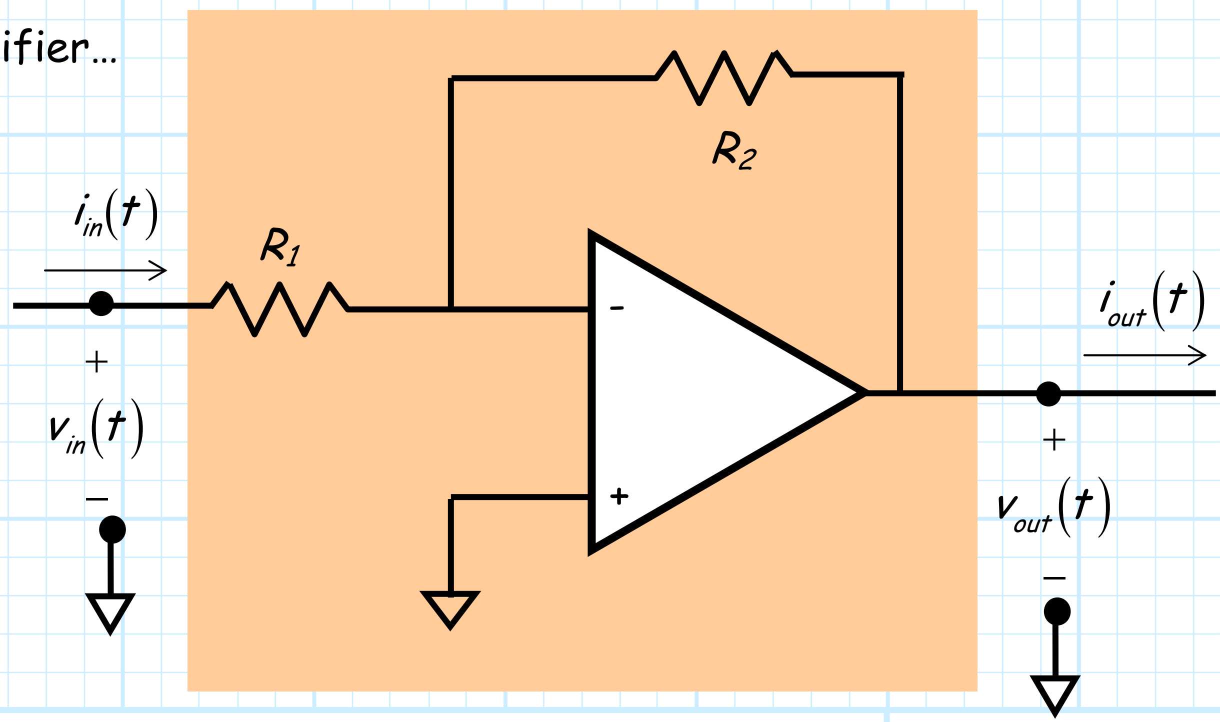
A summary

Summarizing, we have found that for the inverting amplifier:

$$R_{in} = R_1$$

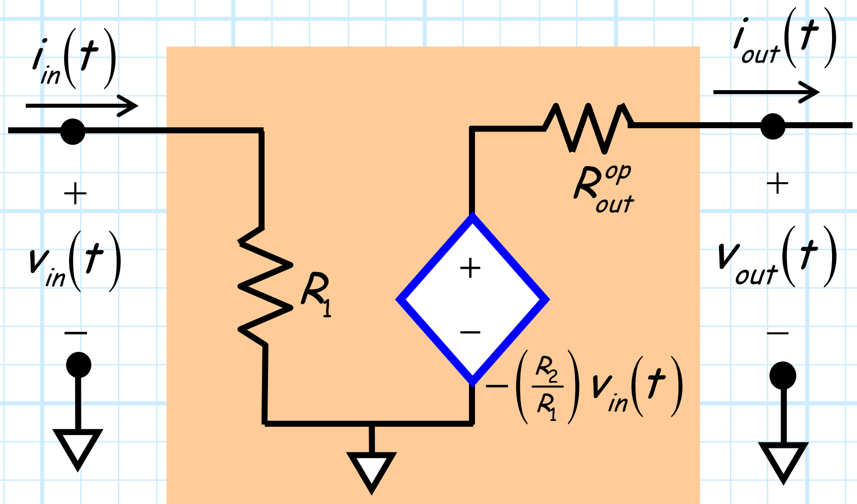
$$R_{out} \approx R_{out}^{op} \qquad \text{(ideally zero)}$$

Thus, **this** inverting amplifier…



The inverting amp equivalent circuit

...has the **equivalent circuit**:



Note the input resistance and open-circuit voltage gain of the **inverting amplifier** is **VERY different** from that of the **op-amp** itself!