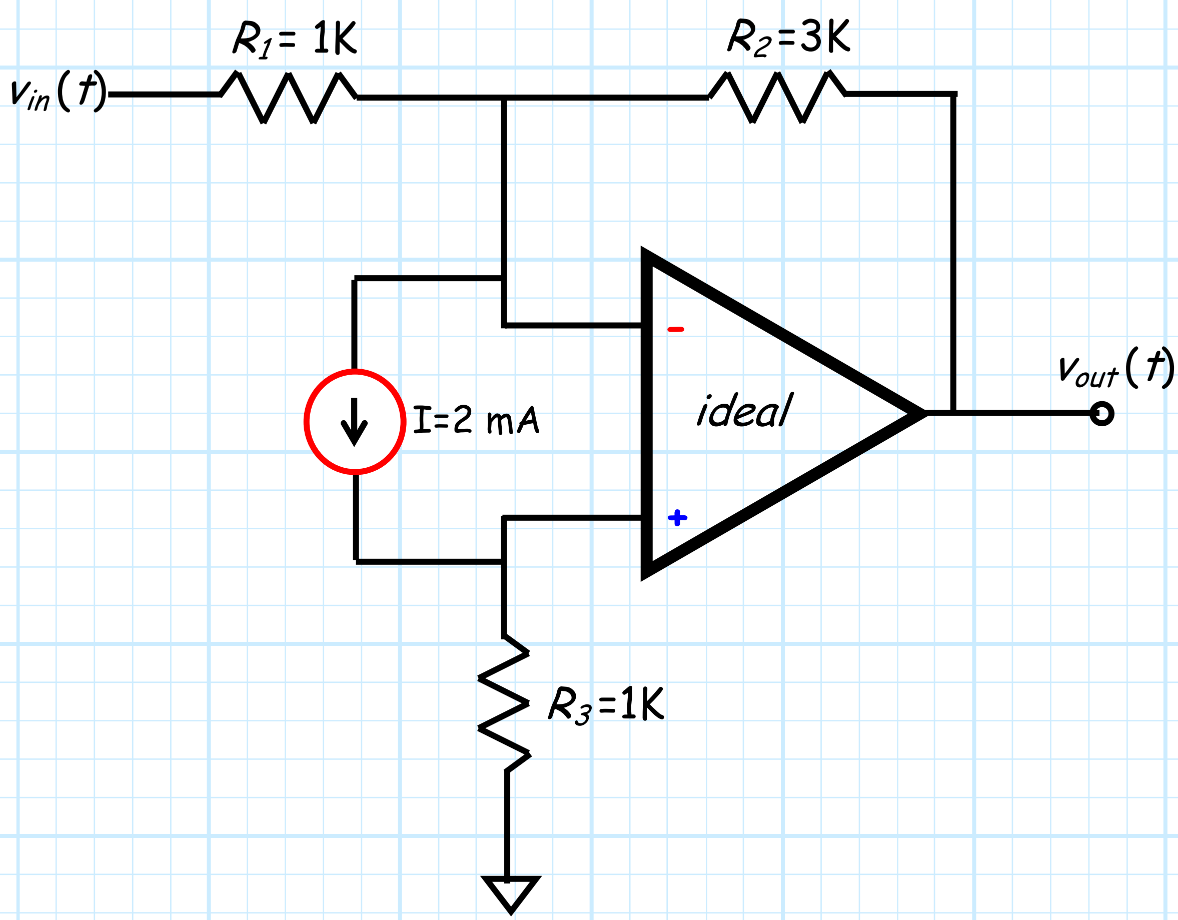
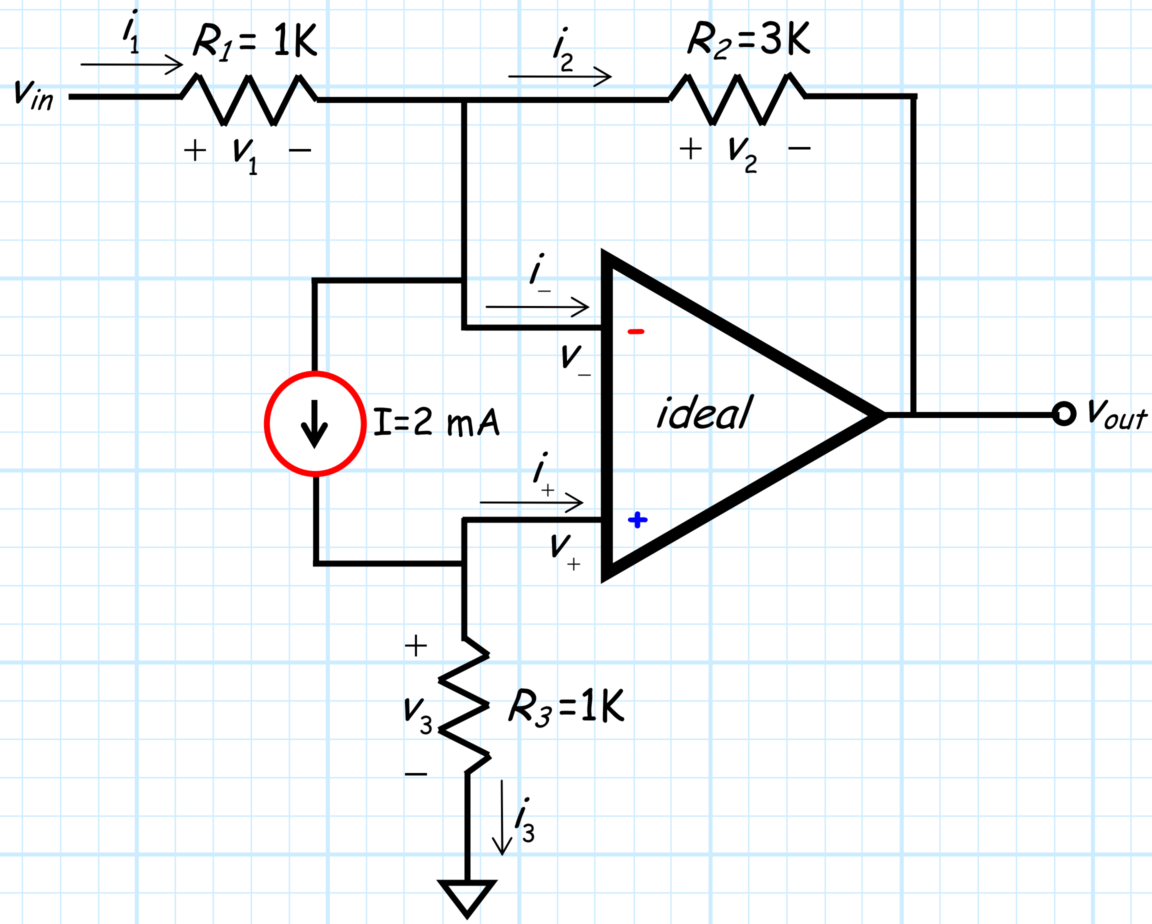
Example: An op-amp circuit analysis

Let's determine the **output voltage** $v_{out}(t)$ of the circuit below:



Without this step, your answer (and thus your grade) mean nothing

The first step in **EVERY** circuit analysis problem is to **label all currents and voltages**:

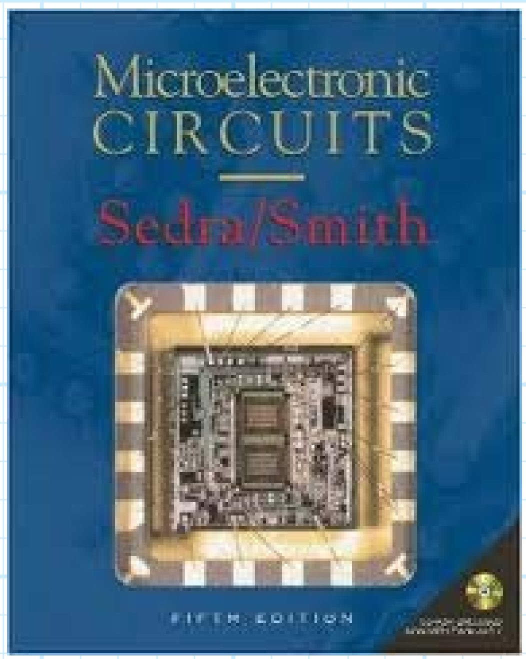


The search for a template…

Q: *I looked and looked at the notes, and I* ***even*** *looked at the* ***book****, but I can't seem to find the* ***right equation*** *for this configuration!*

A: That's because the **"right equation"** for this circuit does **not** exist—at least **yet**.

It's up to **you** to use **your** knowledge and **your** skills to **determine** the "right equation" for the output voltage v_{out}!



You have the tools to determine this yourself—no need to find a template!

Q: *OK, let's see; the output voltage is:*

$$v_{out} = ????$$

*I'm stuck. Just **how** do I determine the output voltage?*

A: Open up your circuit analysis **tool box**. Note it consists of **three** tools and three tools **only**:

Tool 1: KCL

Tool 2: KVL

Tool 3: Device equations (e.g., Ohm's Law and the virtual short).

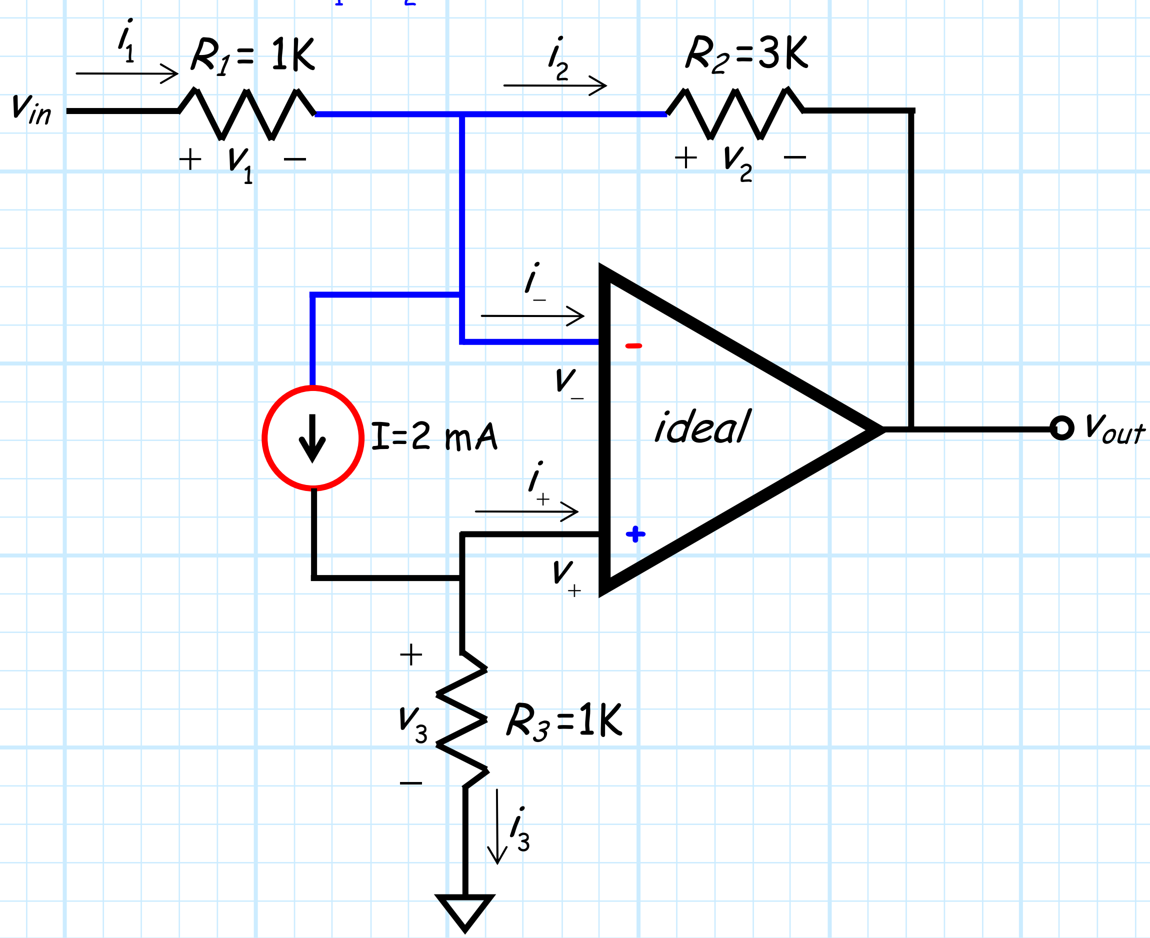
Let's use **these** tools to determine the "right" equation!

First, let's apply **KCL** (I'm quite partial to KCL).

The first KCL

Note there are **two nodes** in this circuit. The KCL for the **first** node is:

$$i_1 = i_2 + i_- + I$$

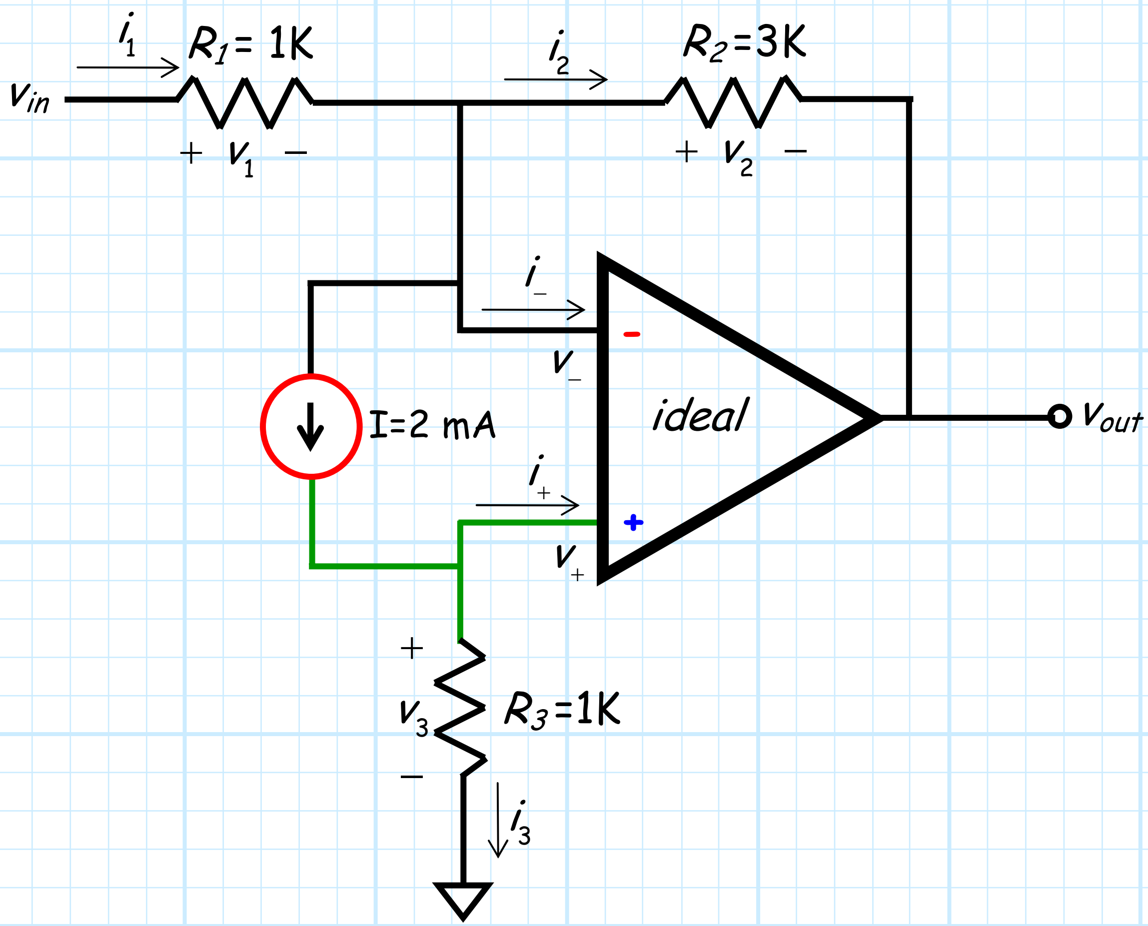


Note the **potential** of this node (with respect to ground) is that of the **inverting** op-amp terminal (i.e., v_-).

The second KCL

The KCL of the **second** node is:

$$I = i_+ + i_3$$



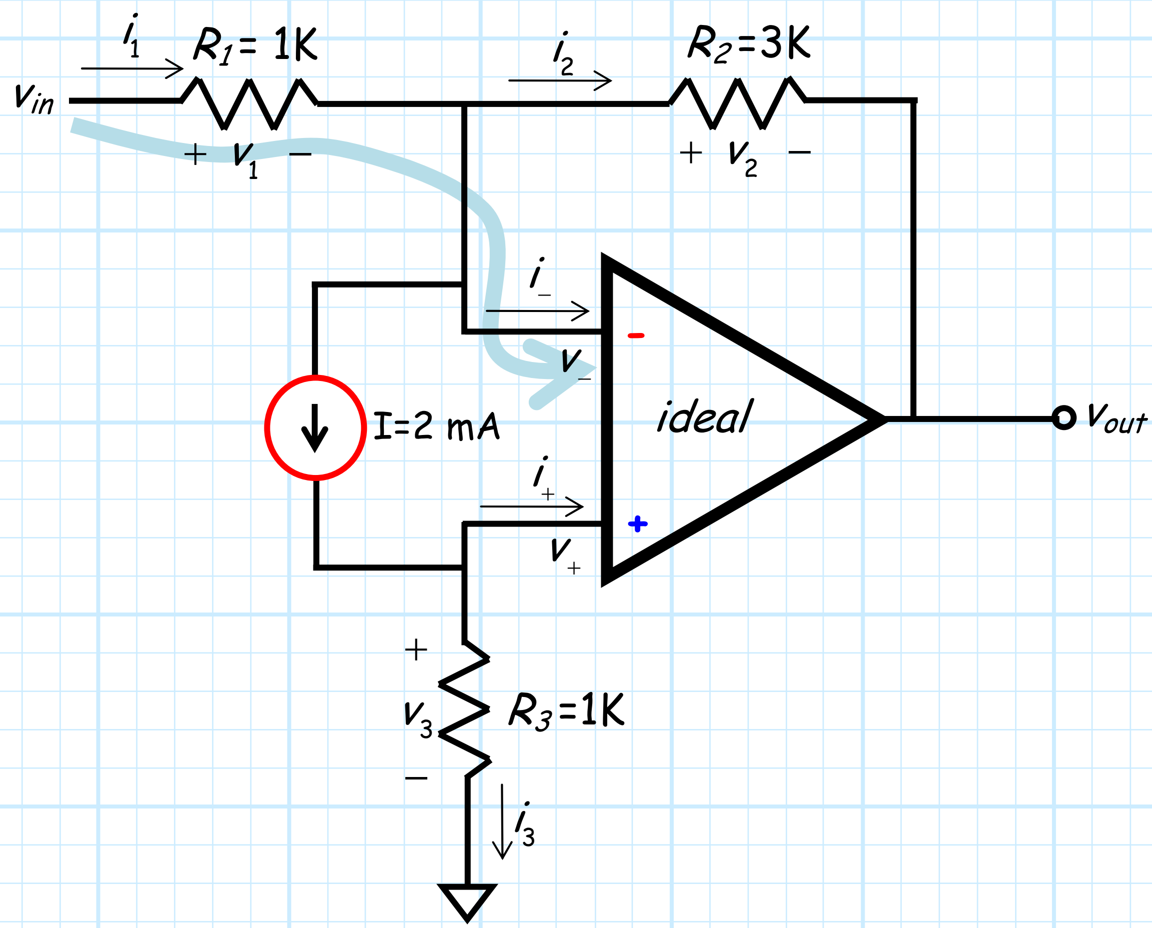
Note the **potential** of this node (with respect to ground) is that of the **non-inverting** op-amp terminal (i.e., v_+).

The first KVL

Now for our second tool—**KVL**.

We can conclude:

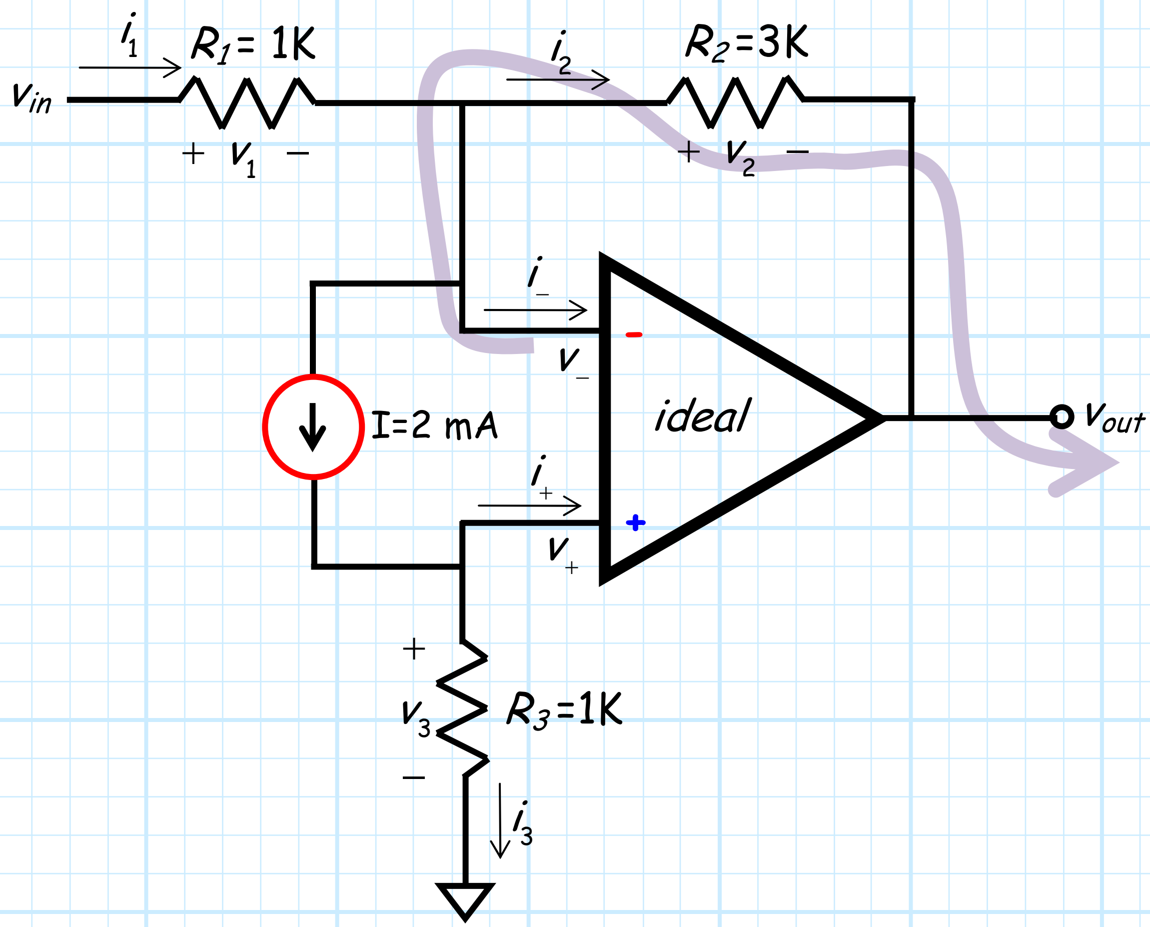
$$v_{in} - v_1 = v_- \quad \Rightarrow \quad v_1 = v_{in} - v_-$$



The second KVL

And also:

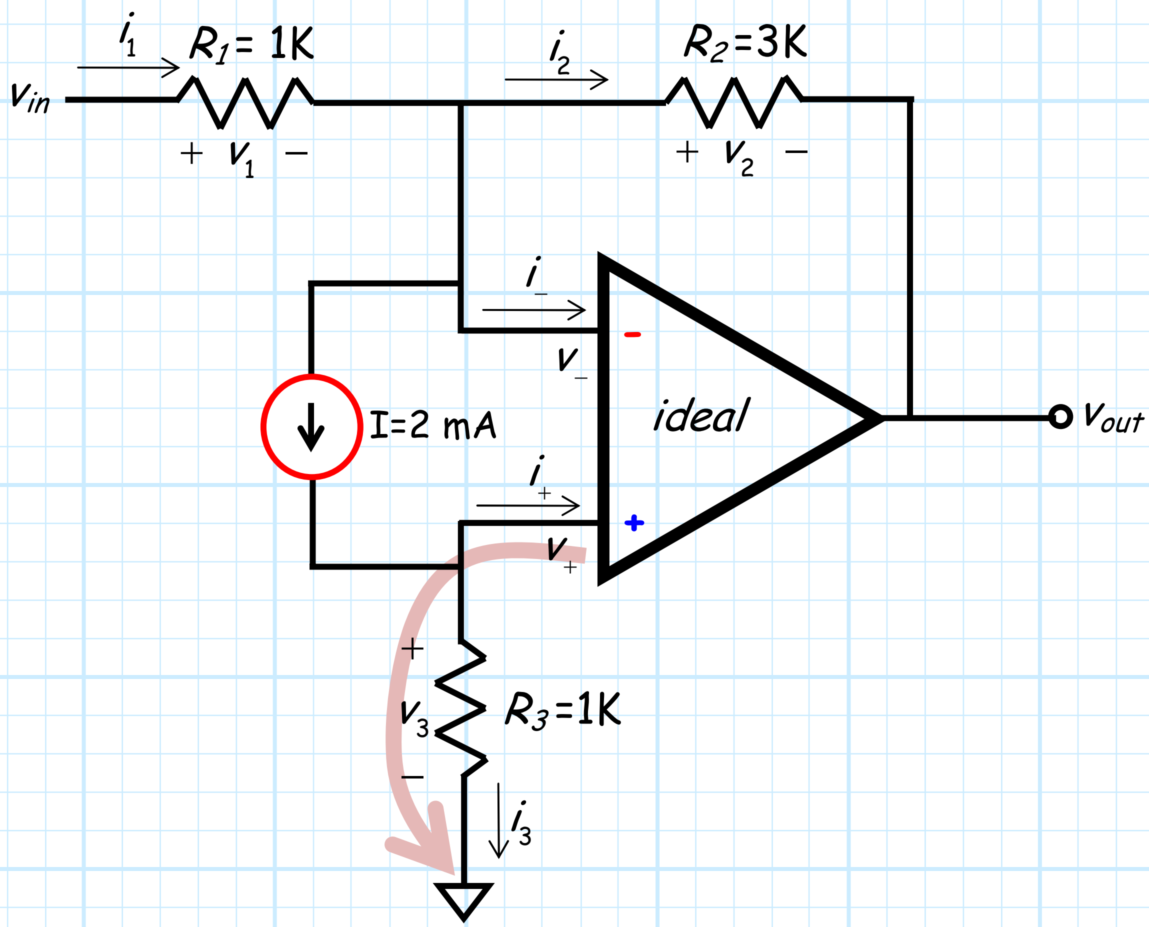
$$v_- - v_2 = v_{out} \quad \Rightarrow \quad v_2 = v_- - v_{out}$$



The third KVL

And likewise:

$$v_+ - v_3 = 0 \qquad \Rightarrow \qquad v_3 = v_+$$



There are seven device equations

Finally, we add in the **device equations**.

Note in this circuit there are **three resistors**, a **current source**, and an **op-amp**

From **Ohm's Law** we know:

$$i_1 = \frac{v_1}{R_1} \qquad i_2 = \frac{v_2}{R_2} \qquad i_3 = \frac{v_3}{R_3}$$

And from the **current source**:

$$I = 2$$

And from the **op-amp**, three equations!

$$i_- = 0 \qquad i_+ = 0 \qquad v_- = v_+$$

12 equations and 12 unknowns!

Q: *Yikes! **Two** KCL equations, **three** KVL equations, and **seven** device equations—together we have **twelve equations**. Do we really need all these?*

A: Absolutely! These 12 equations **completely describe** the circuit. There are each **independent**; without **any one** of them, we could **not** determine v_{out}!

To prove this, just count up the number of variables in these equations:

We have **six** currents:

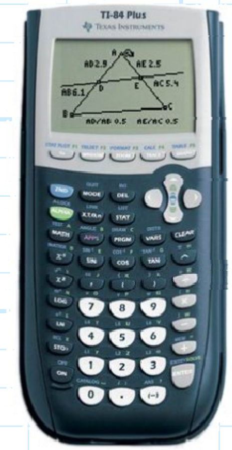
$$i_1, i_2, i_3, i_+, i_-, I$$

And **six** voltages:

$$v_1, v_2, v_3, v_+, v_-, v_{out}$$

Together we have **12 unknowns**—which works out well, since we have **12 equations**!

Thus, the only task remaining is to solve this **algebra** problem!



Don't ask the calculator to figure this out!

Q: *OK, here's where I take out my trusty programmable calculator, type in the equations, and let **it** tell me the answer!*

A: Nope. I will **not** be at all impressed with such results (and your **grade** will reflect this!).

Instead, put together the equations in a way that makes complete **physical sense**—just **one step** at a time.

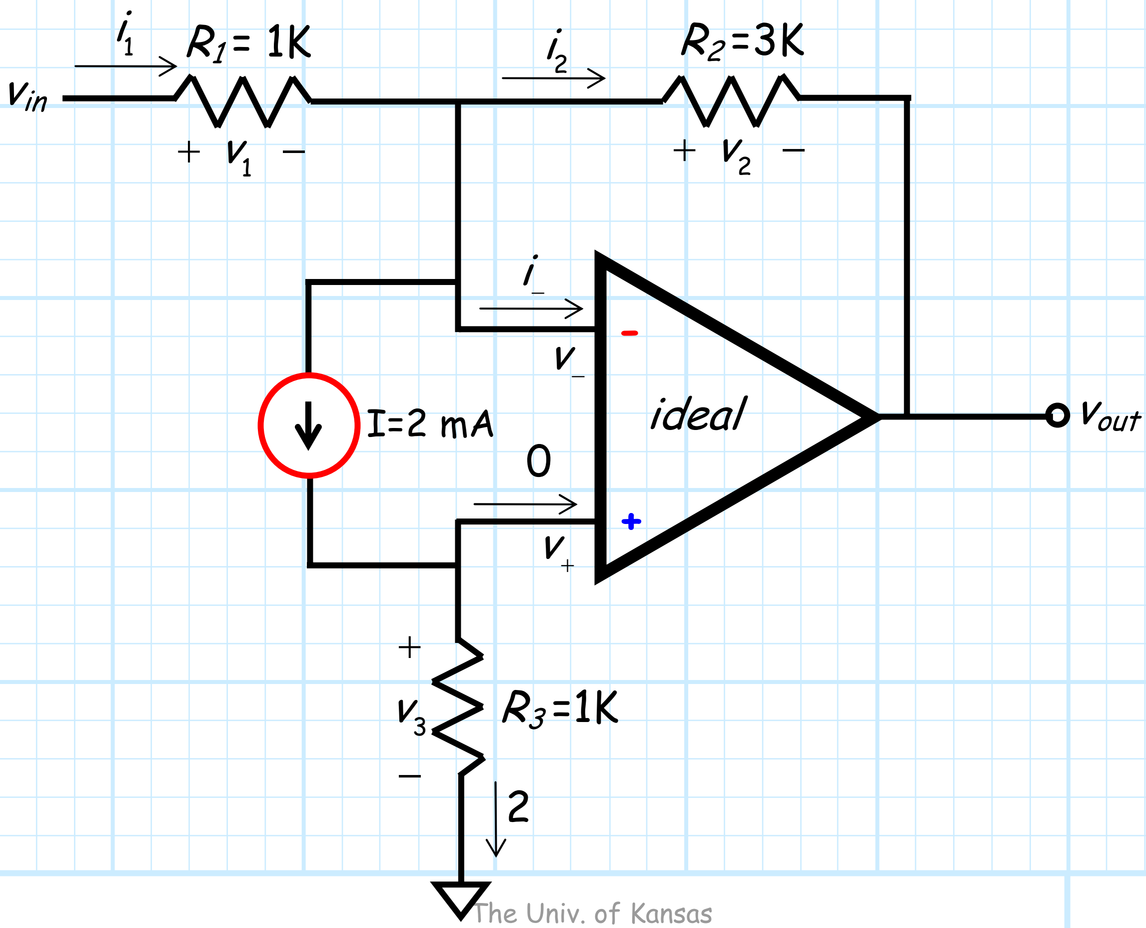
i_3 = 2 mA

First, we take the **two device equations**:

$$i_+ = 0 \qquad \text{and} \qquad I = 2$$

And from the **second KCL** equation:

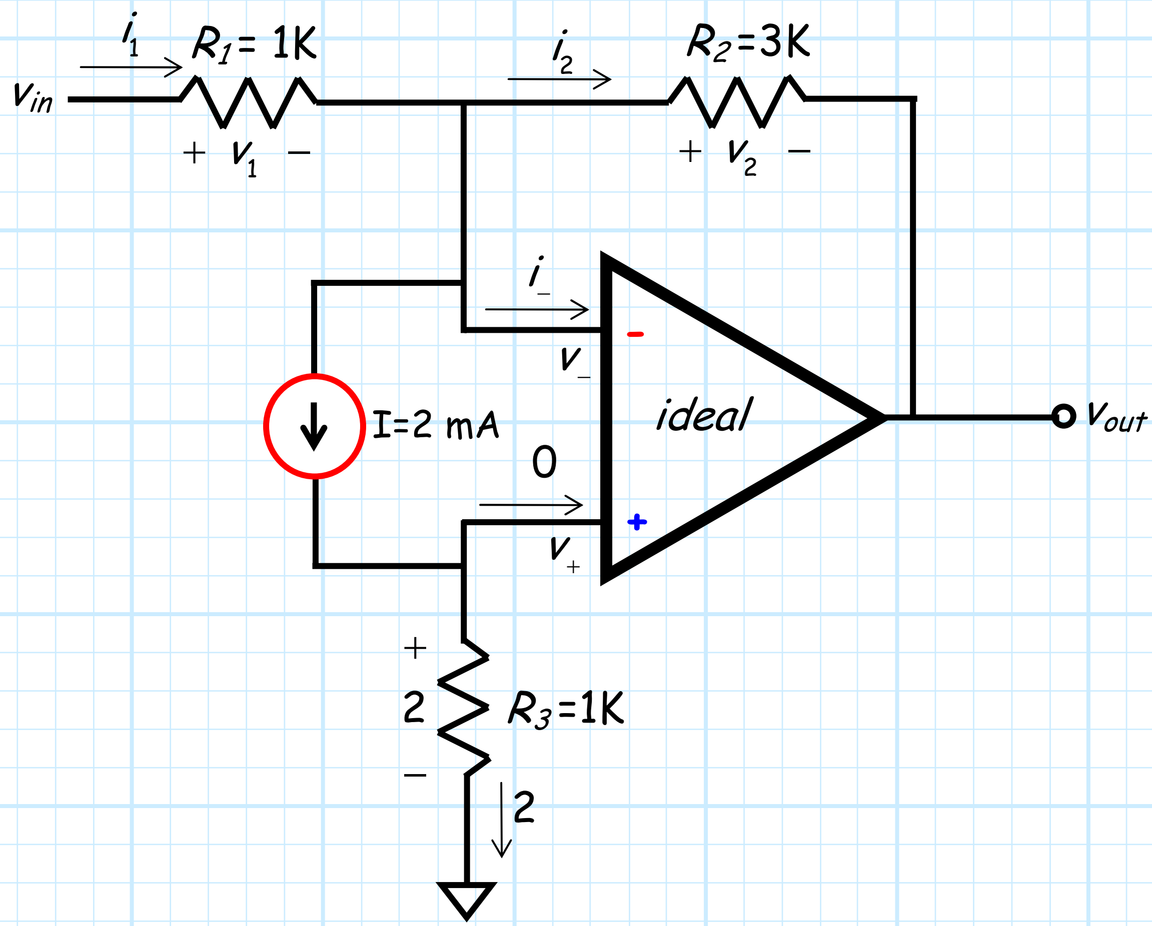
$$I = i_+ + i_3 \quad \Rightarrow \quad 2 = 0 + i_3 \quad \Rightarrow \quad i_3 = 2$$



So v_3 = 2.0 V

Now that we know the current through R_3, we can determine the voltage across it (um, using **Ohm's law**…).

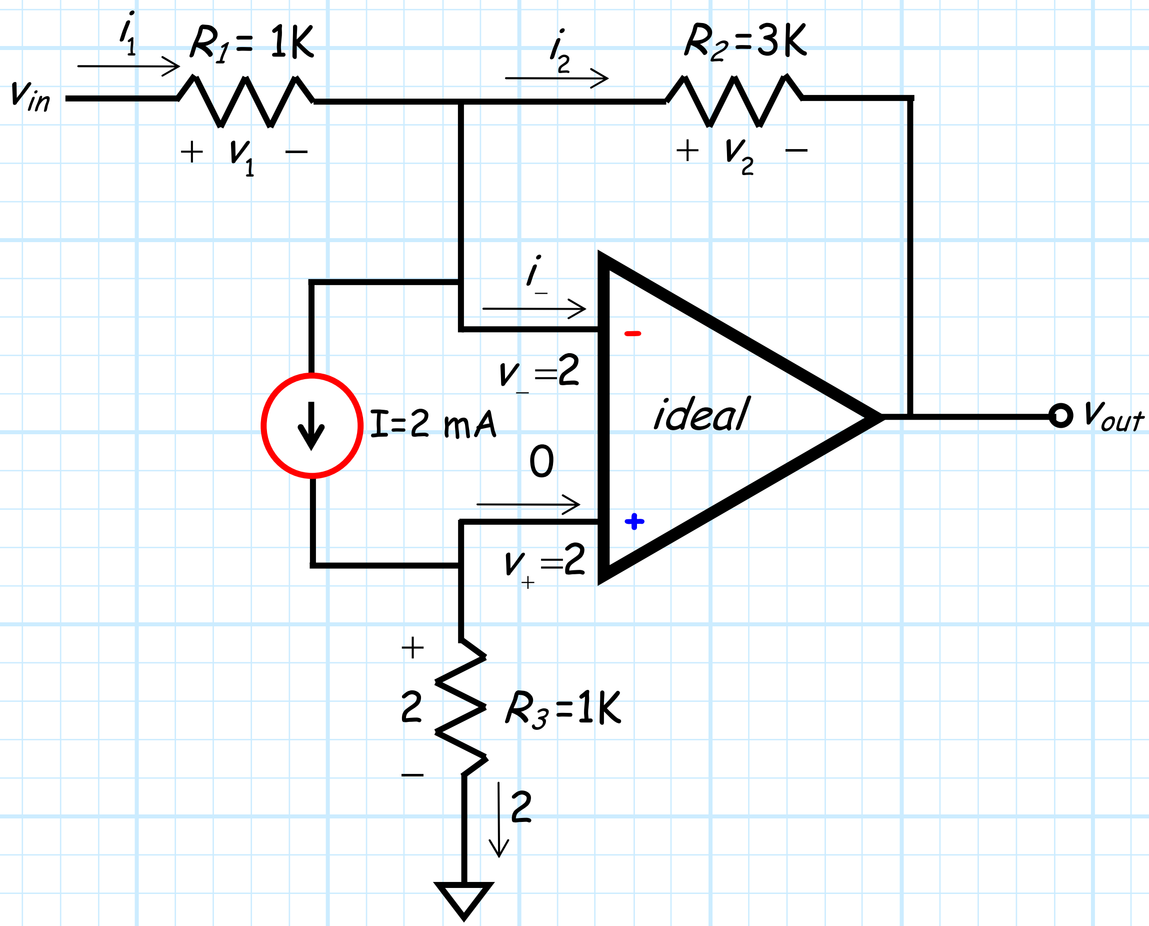
$$v_3 = i_3 R_3 = 2(1) = 2$$



Thus v_- = 2.0 V

Thus, we can now determine both v_+ (from a **KVL** equation) and v_- (from a **device equation**):

$$v_+ = v_3 = 2 \qquad \text{and} \qquad v_- = v_+ = 2$$



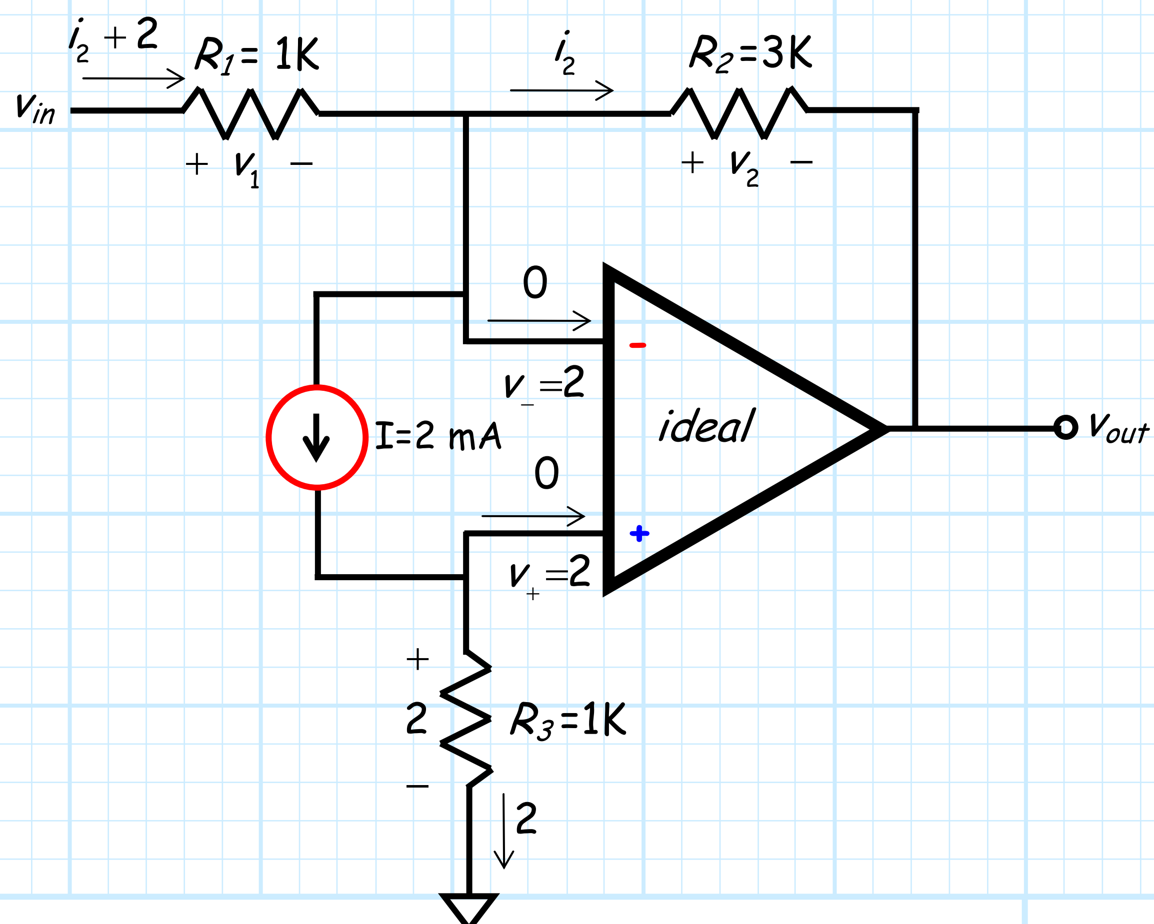
And now $i_1 = i_2 + 2$

Now, inserting another **device equation**:

$$i_- = 0$$

into the **first KCL** equation:

$$i_1 = i_2 + i_- + I \quad \Rightarrow \quad i_1 = i_2 + 0 + 2 \quad \Rightarrow \quad i_1 = i_2 + 2$$



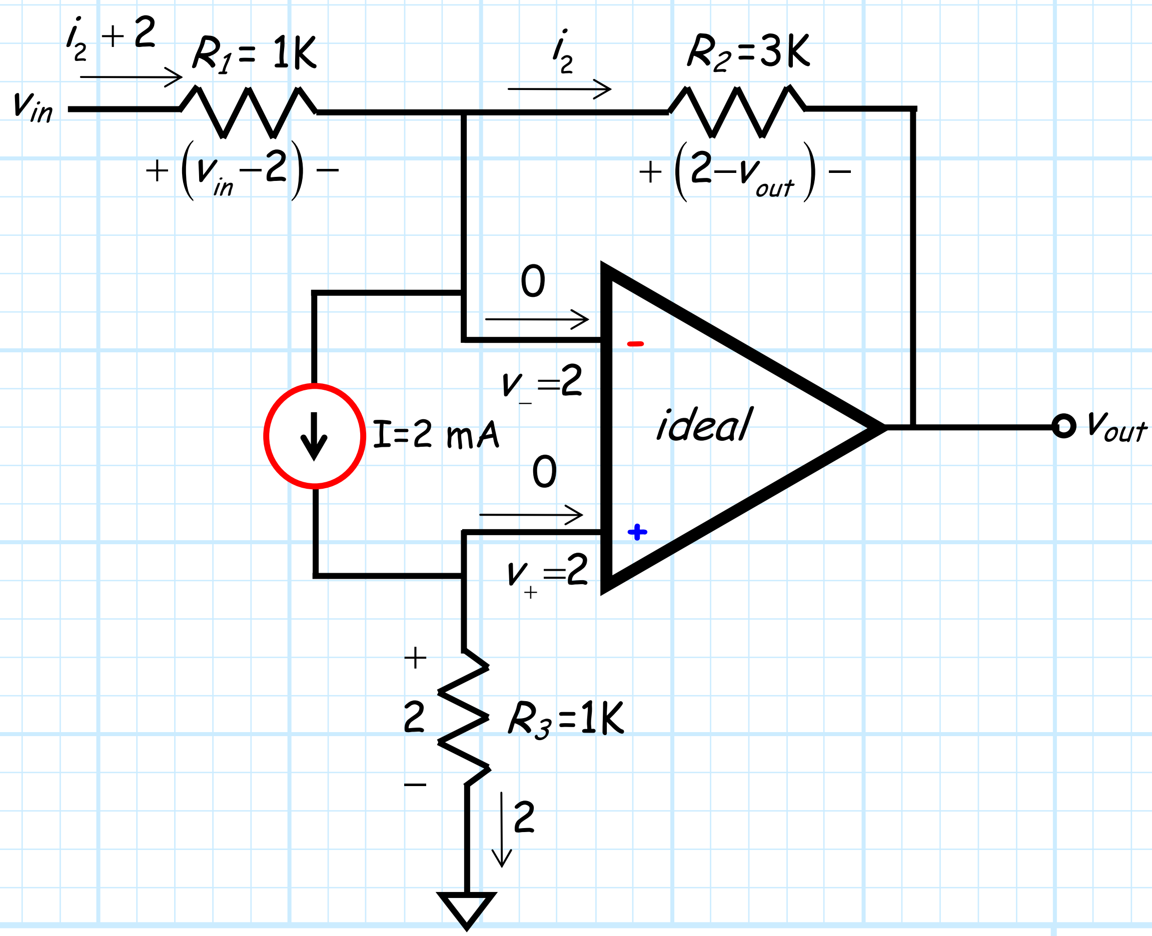
So that $v_2 = 2 - v_{out}$

From **KVL** we find:

$$v_1 = v_{in} - v_- \quad \Rightarrow \quad v_1 = v_{in} - 0 \quad \Rightarrow \quad v_1 = v_{in}$$

and:

$$v_2 = v_- - v_{out} \quad \Rightarrow \quad v_2 = 2 - v_{out}$$



Now we can find v_{out}

So, from **Ohm's law** (one of those device equations!), we find:

$$i_1 = \frac{v_1}{R_1} \quad \Rightarrow \quad i_2 + 2 = \frac{v_{in} - 2}{1} \quad \Rightarrow \quad i_2 = v_{in} - 4$$

and:

$$i_2 = \frac{v_2}{R_2} \quad \Rightarrow \quad i_2 = \frac{2 - v_{out}}{3}$$

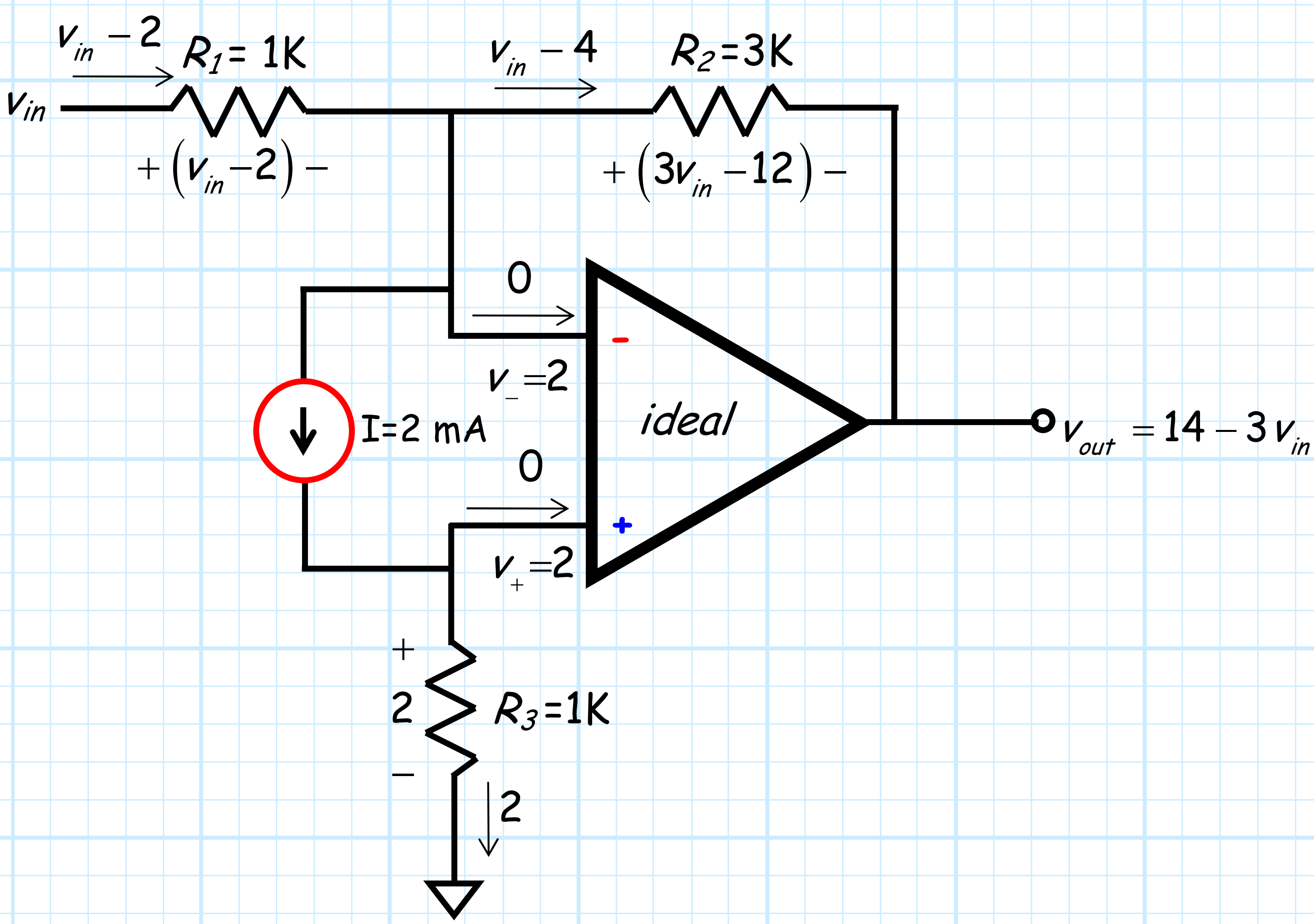
Equating these last two results:

$$v_{in} - 4 = \frac{2 - v_{out}}{3} \quad \Rightarrow \quad v_{out} = 14 - 3\,v_{in}$$

The "right equation" !

Thus, we have **at last** arrived at the result:

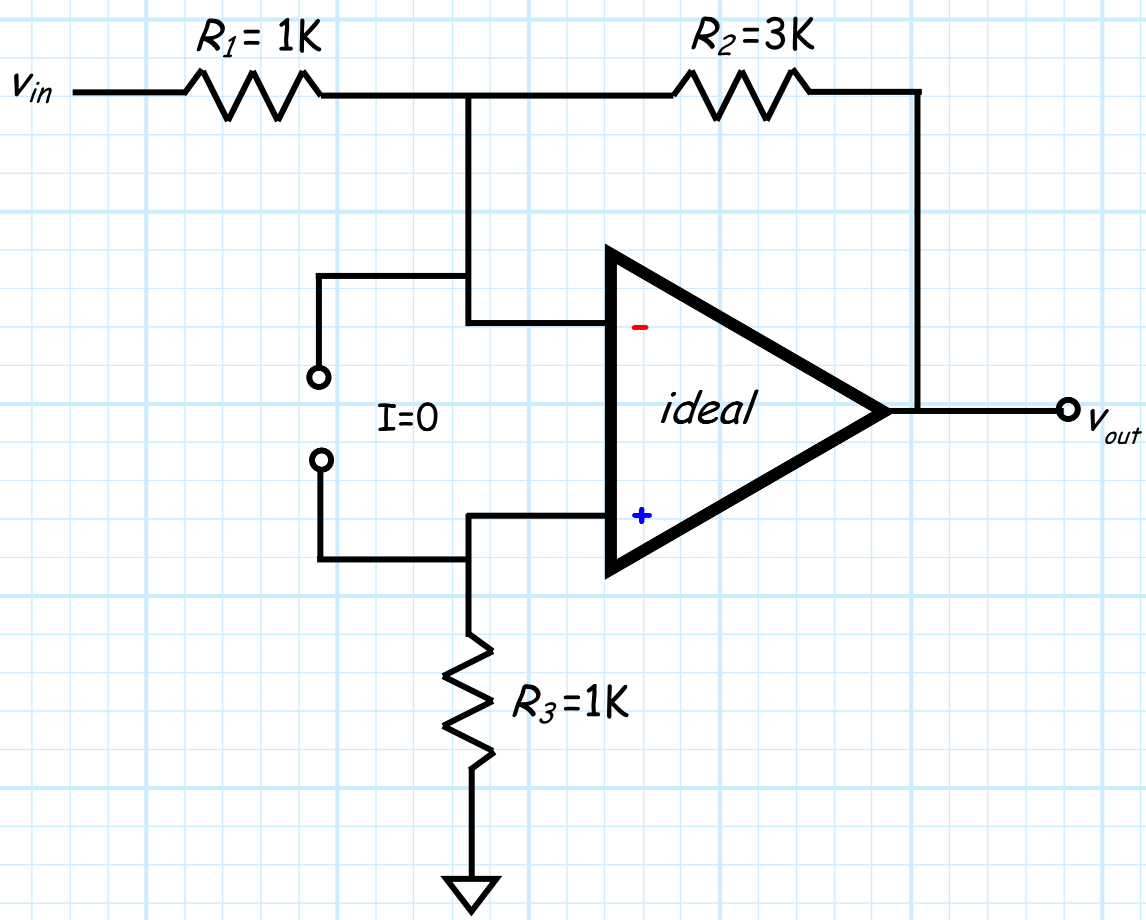
$$v_{out} = 14 - 3\,v_{in}$$



An alternative: superposition

Note an **alternative** method for determining this result is the application of **superposition**.

First we turn off the current source (e.g., $I = 0$)—note that this is an **open circuit**!!!!!!!!

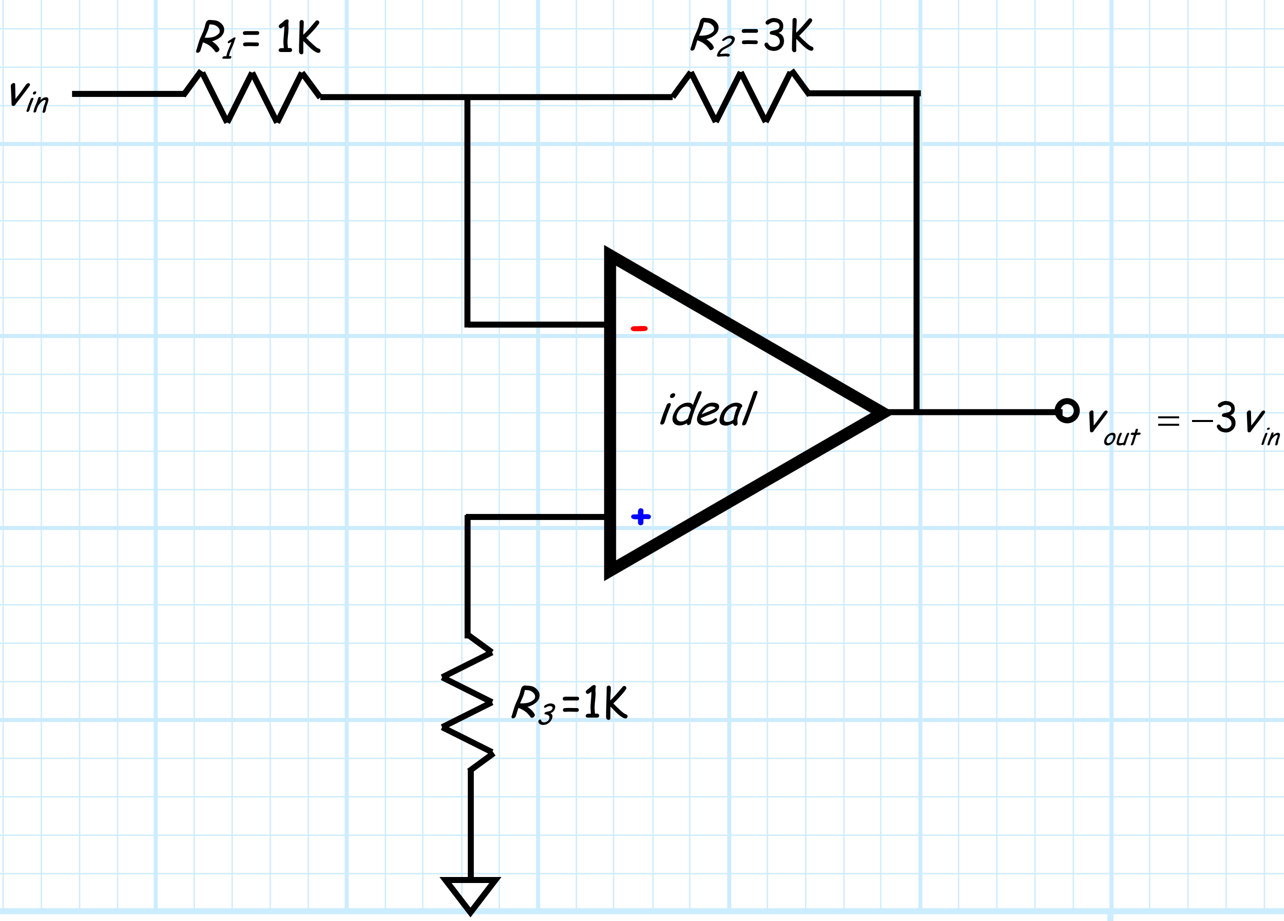


It's just an inverting amp

Note this is the **same** configuration as that of an **inverting amplifier**!

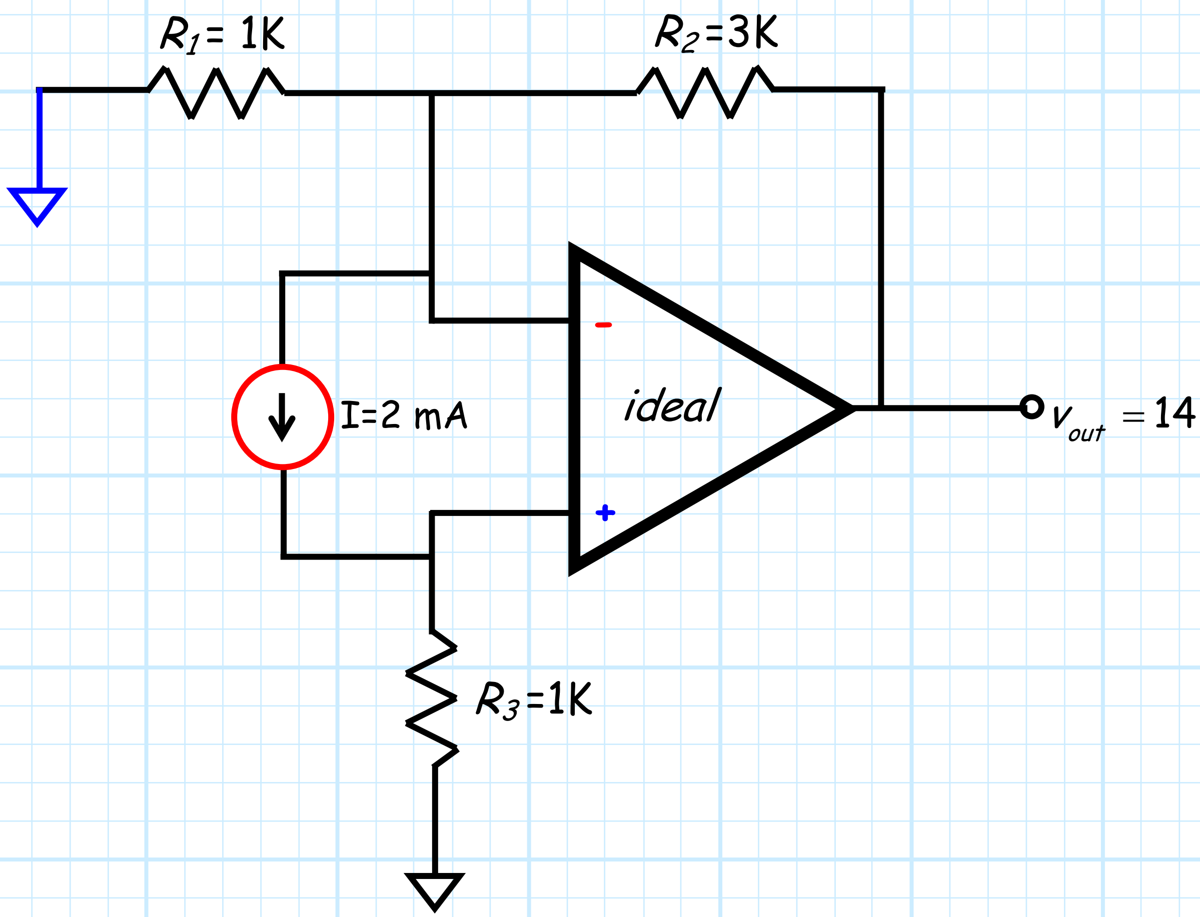
Thus, we can quickly determine (since we already know!) that:

$$v_{out} = -\frac{R_2}{R_1} v_{in} = -\frac{3}{1} v_{in} = -3 v_{in}$$



See if you can prove this result!

Likewise, if we **instead** set the **input** source to zero ($v_{in} = 0$—ground potential!), we will find that the output voltage is **14 volts** (with respect to ground):



Look; the answer is the same!

From **superposition**, we conclude that the output voltage is the **sum** of these two results:

$$v_{out} = 14 - 3v_{in}$$

The **same** result as before!