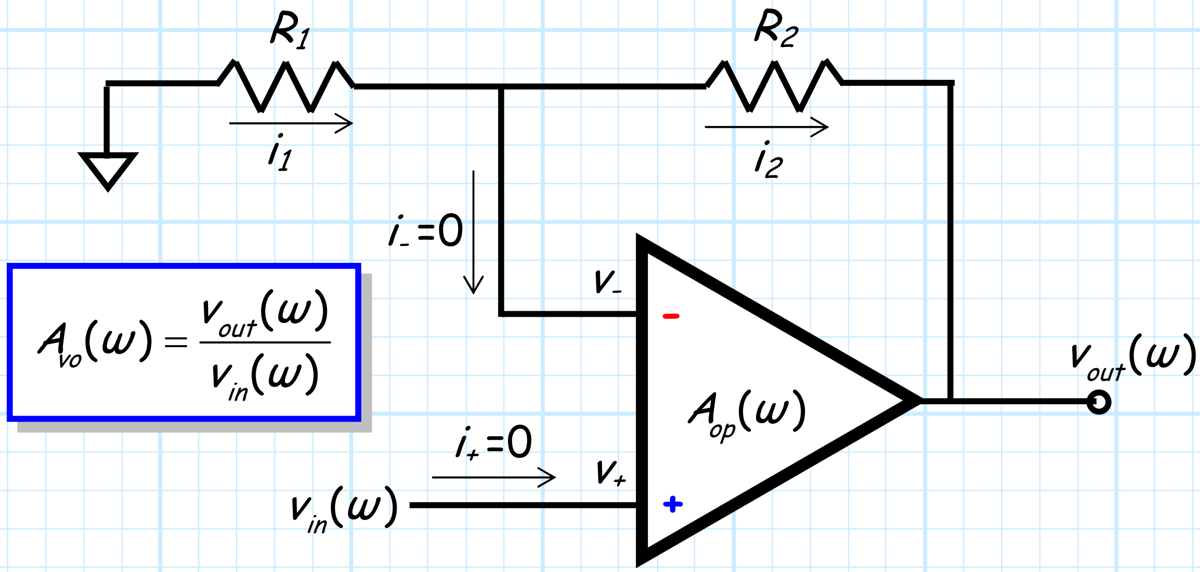
Closed-Loop Bandwidth

Say we build in the **lab** (i.e., the op-amp is not ideal) **this** amplifier:



We know that the open-circuit voltage gain (i.e., the closed-loop gain) of this amplifier **should** be:

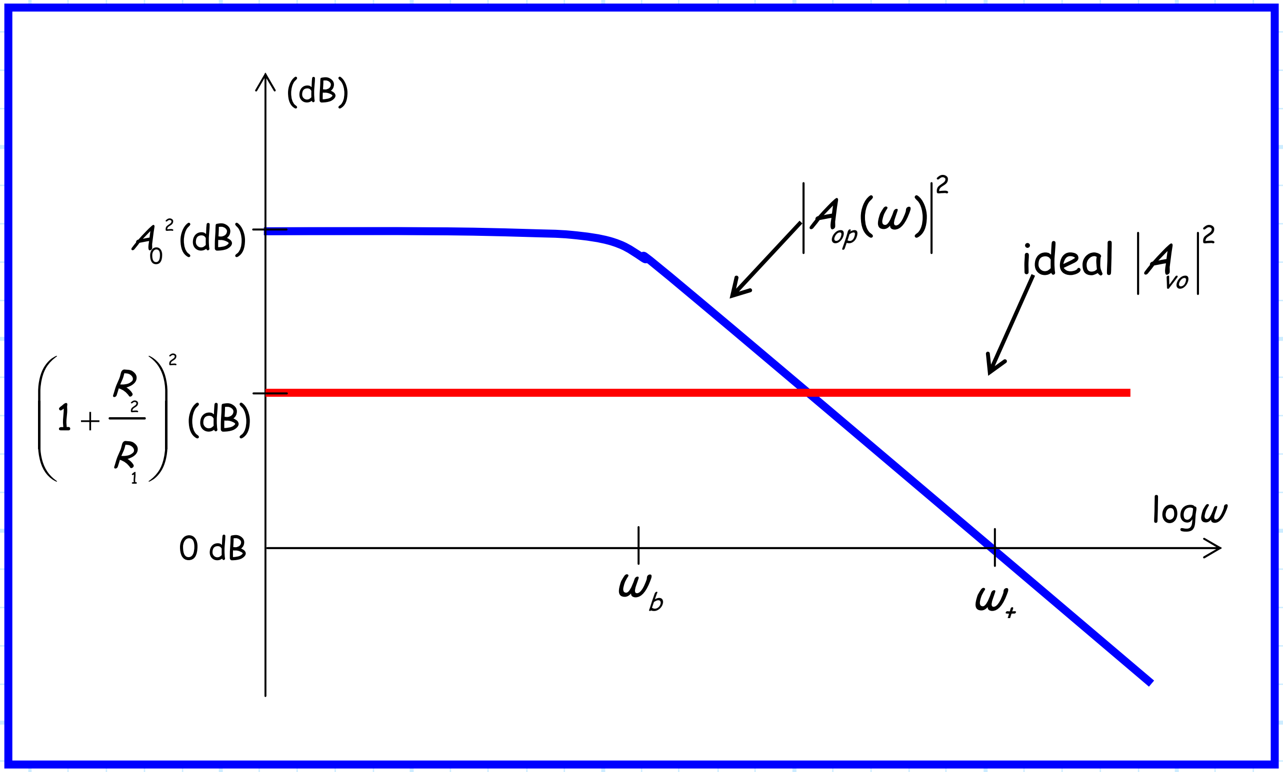
$$A_{vo}(\omega)=\frac{v_{out}(\omega)}{v_{in}(\omega)}=1+\frac{R_2}{R_1} \quad ???$$

This gain **will** certainly be accurate for input signals $v_{in}(\omega)$ **at low frequencies** ω.

As the signal frequency increases

But remember, the Op-amp (i.e., open-loop gain) gain $A_{op}(\omega)$ **decreases with frequency**.

If the signal frequency ω becomes **too large**, the **open-loop** gain $A_{op}(\omega)$ will become **less** than the **ideal closed-loop gain**!



The amp gain cannot exceed the op-amp gain

Note as some sufficiently high frequency (ω' say), the open-loop (op-amp) gain will become **equal** to the ideal closed-loop (non-inverting amplifier) gain:

$$\left|A_{op}(\omega = \omega')\right| = 1 + \frac{R_2}{R_1}$$

Moreover, if the input signal frequency is greater than frequency ω', the op-amp (**open-loop**) gain will in fact be smaller that the **ideal** non-inverting (**closed-loop**) amplifier gain:

$$\left|A_{op}(\omega > \omega')\right| \quad < \quad 1 + \frac{R_2}{R_1}$$

Q: *If the signal frequency is greater than ω', will the non-inverting amplifier still exhibit an open-circuit voltage (closed-loop) gain of $A_{vo}(\omega) = 1 + R_2/R_1$?*

A: Allow my response to be both direct and succinct—**NEVER!**

Closed-loop gain < or = open-loop gain

The gain $A_{vo}(\omega)$ of **any** amplifier constructed with an op-amp can **never** exceed the gain $A_{op}(\omega)$ of the op-amp itself.

In other words, the closed-loop gain of any amplifier can **never** exceed its open-loop gain.

* We find that if the input signal frequency **exceeds** ω', then the amplifier (closed-loop) gain $A_{vo}(\omega)$ will **equal** the **op-amp** (open-loop) gain $A_{op}(\omega)$.

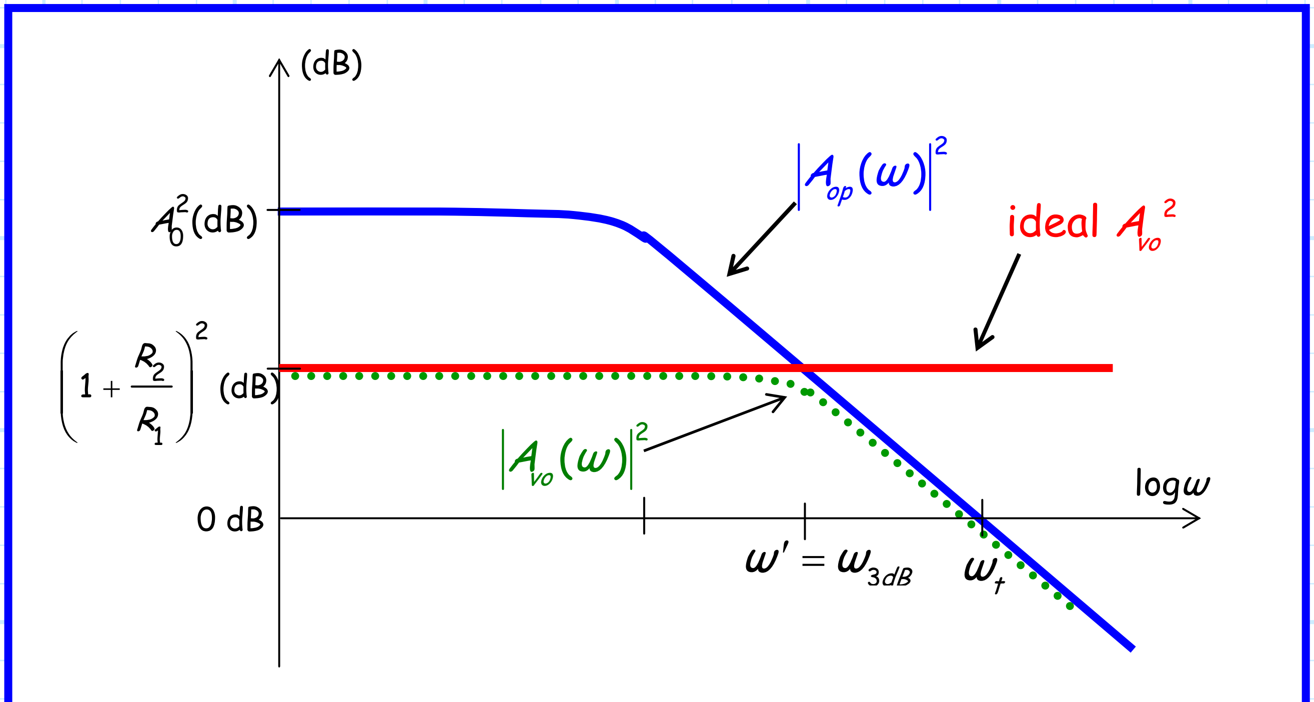
* Of course, if the signal frequency is **less** than ω', the closed-loop gain will be **equal** to its **ideal** value $A_{vo}(\omega) = 1 + R_2/R_1$, since the op-amp (open-loop) gain is much **larger** than this **ideal** value ($\left|A_{op}(\omega < \omega')\right| \gg 1 + R_2/R_1$).

* We now refer to the value $1 + R_2/R_1$ as the **mid-band gain** of the amplifier.

$1+R_2/R_1$ is the midband gain

Therefore, we find for **this** non-inverting amplifier that:

$$\left|A_{vo}(\omega)\right| \approx \begin{cases} 1+\dfrac{R_2}{R_1} & \omega < \omega' \\ \left|A_{op}(\omega)\right| & \omega > \omega' \end{cases}$$



Can we determine this bandwidth?

Now for **one** very **important** fact: the transition frequency ω' is the **break** frequency of the amplifier **closed-loop** gain $|A_{vo}(\omega)|$.

Thus, we come to conclusion that ω' is the **3dB bandwidth** of this non-inverting amplifier (i.e., $\omega' = \omega_{3dB}$)!

Q: *Is there some way to numerically determine this value ?*

A: Of course!

Recall we defined frequency ω' as the value where the open-loop (op-amp) gain and the **ideal** closed-loop (non-inverting amplifier) gains were equal:

$$|A_{op}(\omega = \omega')| = 1 + \frac{R_2}{R_1}$$

Recall also that for $\omega > \omega_b$, we can approximate the op-amp (open-loop) gain as:

$$|A_{op}(\omega)| \approx \frac{A_0 \omega_b}{\omega}$$

Divide the gain-bandwidth product by gain, and you have determined the bandwidth!

Combining these results, we find:

$$\left|A_{op}(\omega = \omega')\right| = 1 + \frac{R_2}{R_1} \simeq \frac{A_0 \omega_b}{\omega'}$$

and thus:

$$\omega' = \left(1 + \frac{R_2}{R_1}\right)^{-1} \left(A_0 \omega_b\right)$$

But remember, we found that this frequency is equal to the **breakpoint** of the non-inverting amplifier (closed-loop) gain $A_{vo}(\omega)$.

Therefore, the 3dB, closed-loop **bandwidth** of this amplifier is:

$$\omega_{3dB} \simeq \left(1 + \frac{R_2}{R_1}\right)^{-1} \left(A_0 \omega_b\right)$$

This is not rocket science

Recall also that $A_0 \omega_b = \omega_t$, so that:

$$\omega_{3dB} \simeq \left(1 + \frac{R_2}{R_1}\right)^{-1} \omega_t$$

If we rewrite this equation, we find something interesting:

$$\omega_{3dB} \left(1 + \frac{R_2}{R_1}\right) \simeq \omega_t$$

Look what this says: the **PRODUCT** of the amplifier (mid-band) ***GAIN*** and the amplifier **BANDWIDTH** is equal to the **GAIN-BANDWIDTH PRODUCT**.

This result should **not** be difficult to remember !

The gain-bandwidth product is an op-amp parameter

The above approximation is valid for virtually **all** amplifiers built using operational amplifiers, i.e.:

$$\left|A_{vo}(\omega_m)\right| \omega_{3dB} = \omega_t$$

where:

$$\left|A_{vo}(\omega_m)\right| \doteq \text{mid-band gain}$$

In other words, ω_m is some frequency **within the bandwidth** of the amplifier (e.g., $0 < \omega_m < \omega_{3dB}$). We of course can equivalently say:

$$\left|A_{vo}(f_m)\right| f_{3dB} = f_t$$

The product of the **amplifier** gain and the amplifier **bandwidth** is equal to the **op-amp** gain-bandwidth product!