### 2.5 Effect of finite open-loop gain and bandwidth on circuit performance

Reading Assignment: 89-93

Bad News! $\rightarrow$ Real Op-Amps are not ideal!

In the "real world", op-amp have a slew (pun intended) of problems that limit their performance and application.

It is vital that we electrical engineers understand these limitations.

## HO: THE GAIN OF REAL OP AMPS

An approximation of can simplify the transfer function.
HO: A USEFUL APPROXIMATION OF THE OP-AMP TRANSFER
FUNCTION

We find the gain-bandwidth product to be a very useful value!

## EXAMPLE: THE GAIN-BANDWIDTH PRODUCT

An amplifier built with an op-amp must have a gain (i.e., the closedloop gain) less than that of the op amp. We find that the resulting amplifier bandwidth is easily determined!

HO: THE CLOSED-LOOP BANDWIDTH

## EXAMPLE: AMPLIFIER BANDWIDTH

## LM741

## Operational Amplifier

## General Description

The LM741 series are general purpose operational amplifiers which feature improved performance over industry standards like the LM709. They are direct, plug-in replacements for the 709C, LM201, MC1439 and 748 in most applications.

The amplifiers offer many features which make their application nearly foolproof: overload protection on the input and
output, no latch-up when the common mode range is exceeded, as well as freedom from oscillations.

The LM741C is identical to the LM741/LM741A except that the LM741C has their performance guaranteed over a $0^{\circ} \mathrm{C}$ to $+70^{\circ} \mathrm{C}$ temperature range, instead of $-55^{\circ} \mathrm{C}$ to $+125^{\circ} \mathrm{C}$.

## Features

## Connection Diagrams



00934102
Note 1: LM741H is available per JM38510/10101
Order Number LM741H, LM741H/883 (Note 1), LM741AH/883 or LM741CH
See NS Package Number H08C
Ceramic Flatpak


Order Number LM741W/883
See NS Package Number W10A

## Typical Application



00934107

## Absolute Maximum Ratings (Note 2)

If Military/Aerospace specified devices are required, please contact the National Semiconductor Sales Office/ Distributors for availability and specifications.
(Note 7)

|  | LM741A | LM741 | LM741C |
| :--- | :---: | :---: | :---: |
| Supply Voltage | $\pm 22 \mathrm{~V}$ | $\pm 22 \mathrm{~V}$ | $\pm 18 \mathrm{~V}$ |
| Power Dissipation (Note 3) | 500 mW | 500 mW | 500 mW |
| Differential Input Voltage | $\pm 30 \mathrm{~V}$ | $\pm 30 \mathrm{~V}$ | $\pm 30 \mathrm{~V}$ |
| Input Voltage (Note 4) | $\pm 15 \mathrm{~V}$ | $\pm 15 \mathrm{~V}$ | $\pm 15 \mathrm{~V}$ |
| Output Short Circuit Duration | Continuous | Continuous | Continuous |
| Operating Temperature Range | $-55^{\circ} \mathrm{C}$ to $+125^{\circ} \mathrm{C}$ | $-55^{\circ} \mathrm{C}$ to $+125^{\circ} \mathrm{C}$ | $0^{\circ} \mathrm{C}$ to $+70^{\circ} \mathrm{C}$ |
| Storage Temperature Range | $-65^{\circ} \mathrm{C}$ to $+150^{\circ} \mathrm{C}$ | $-65^{\circ} \mathrm{C}$ to $+150^{\circ} \mathrm{C}$ | $-65^{\circ} \mathrm{C}$ to $+150^{\circ} \mathrm{C}$ |
| Junction Temperature | $150^{\circ} \mathrm{C}$ | $150^{\circ} \mathrm{C}$ | $100^{\circ} \mathrm{C}$ |
| Soldering Information |  |  |  |
| N-Package (10 seconds) | $260^{\circ} \mathrm{C}$ | $260^{\circ} \mathrm{C}$ | $260^{\circ} \mathrm{C}$ |
| J- or H-Package (10 seconds) | $300^{\circ} \mathrm{C}$ | $300^{\circ} \mathrm{C}$ | $300^{\circ} \mathrm{C}$ |
| M-Package |  |  |  |
| Vapor Phase (60 seconds) | $215^{\circ} \mathrm{C}$ | $215^{\circ} \mathrm{C}$ | $215^{\circ} \mathrm{C}$ |
| Infrared (15 seconds) | $215^{\circ} \mathrm{C}$ | $215^{\circ} \mathrm{C}$ | $215^{\circ} \mathrm{C}$ |

See AN-450 "Surface Mounting Methods and Their Effect on Product Reliability" for other methods of soldering
surface mount devices.
ESD Tolerance (Note 8)
400V
400V
400V

## Electrical Characteristics (Note 5)

| Parameter | Conditions | LM741A |  |  | LM741 |  |  | LM741C |  |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min | Typ | Max | Min | Typ | Max | Min | Typ | Max |  |
| Input Offset Voltage | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C} \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega \\ & \hline \end{aligned}$ |  | 0.8 | 3.0 |  | 1.0 | 5.0 |  | 2.0 | 6.0 | $\begin{aligned} & \mathrm{mV} \\ & \mathrm{mV} \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }} \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega \end{aligned}$ |  |  | 4.0 |  |  | 6.0 |  |  | 7.5 | $\begin{aligned} & \mathrm{mV} \\ & \mathrm{mV} \end{aligned}$ |
| Average Input Offset Voltage Drift |  |  |  | 15 |  |  |  |  |  |  | $\mu \mathrm{V} /{ }^{\circ} \mathrm{C}$ |
| Input Offset Voltage <br> Adjustment Range | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}, \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V}$ | $\pm 10$ |  |  |  | $\pm 15$ |  |  | $\pm 15$ |  | mV |
| Input Offset Current | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  | 3.0 | 30 |  | 20 | 200 |  | 20 | 200 | nA |
|  | $\mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}$ |  |  | 70 |  | 85 | 500 |  |  | 300 | nA |
| Average Input Offset Current Drift |  |  |  | 0.5 |  |  |  |  |  |  | $n A /{ }^{\circ} \mathrm{C}$ |
| Input Bias Current | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  | 30 | 80 |  | 80 | 500 |  | 80 | 500 | nA |
|  | $\mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}$ |  |  | 0.210 |  |  | 1.5 |  |  | 0.8 | $\mu \mathrm{A}$ |
| Input Resistance | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}, \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V}$ | 1.0 | 6.0 |  | 0.3 | 2.0 |  | 0.3 | 2.0 |  | $\mathrm{M} \Omega$ |
|  | $\begin{aligned} & \mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }} \\ & \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \end{aligned}$ | 0.5 |  |  |  |  |  |  |  |  | $\mathrm{M} \Omega$ |
| Input Voltage Range | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  |  |  |  |  |  | $\pm 12$ | $\pm 13$ |  | V |
|  | $\mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}$ |  |  |  | $\pm 12$ | $\pm 13$ |  |  |  |  | V |

Electrical Characteristics (Note 5) (Continued)

| Parameter | Conditions | LM741A |  |  | LM741 |  |  | LM741C |  |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min | Typ | Max | Min | Typ | Max | Min | Typ | Max |  |
| Large Signal Voltage Gain | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}, \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 15 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 10 \mathrm{~V} \\ & \hline \end{aligned}$ | 50 |  |  | 50 | 200 |  | 20 | 200 |  | $\begin{aligned} & \mathrm{V} / \mathrm{mV} \\ & \mathrm{~V} / \mathrm{mV} \end{aligned}$ |
|  | $\begin{array}{\|l} \hline \mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}, \\ \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega, \\ \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 15 \mathrm{~V} \\ \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 10 \mathrm{~V} \\ \mathrm{~V}_{\mathrm{S}}= \pm 5 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 2 \mathrm{~V} \\ \hline \end{array}$ | $\begin{aligned} & 32 \\ & 10 \end{aligned}$ |  |  | 25 |  |  | 15 |  |  | $\mathrm{V} / \mathrm{mV}$ <br> $\mathrm{V} / \mathrm{mV}$ <br> $\mathrm{V} / \mathrm{mV}$ |
| Output Voltage Swing | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{L}} \geq 10 \mathrm{k} \Omega \\ & \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \\ & \hline \end{aligned}$ | $\begin{aligned} & \pm 16 \\ & \pm 15 \end{aligned}$ |  |  |  |  |  |  |  |  | $\begin{aligned} & \text { V } \\ & \text { V } \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 15 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{L}} \geq 10 \mathrm{k} \Omega \\ & \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \end{aligned}$ |  |  |  | $\begin{aligned} & \pm 12 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 14 \\ & \pm 13 \end{aligned}$ |  | $\begin{aligned} & \pm 12 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 14 \\ & \pm 13 \end{aligned}$ |  | $\begin{aligned} & \text { V } \\ & \text { V } \end{aligned}$ |
| Output Short Circuit Current | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C} \\ & \mathrm{~T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }} \end{aligned}$ | $\begin{aligned} & 10 \\ & 10 \end{aligned}$ | 25 | $\begin{aligned} & 35 \\ & 40 \end{aligned}$ |  | 25 |  |  | 25 |  | $\begin{aligned} & \mathrm{mA} \\ & \mathrm{~mA} \end{aligned}$ |
| Common-Mode Rejection Ratio | $\begin{aligned} & \mathrm{T}_{\mathrm{AMIN}} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\mathrm{AMAX}} \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega, \mathrm{~V}_{\mathrm{CM}}= \pm 12 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega, \mathrm{~V}_{\mathrm{CM}}= \pm 12 \mathrm{~V} \end{aligned}$ | 80 | 95 |  | 70 | 90 |  | 70 | 90 |  | $\begin{aligned} & \mathrm{dB} \\ & \mathrm{~dB} \end{aligned}$ |
| Supply Voltage Rejection Ratio | $\begin{aligned} & \mathrm{T}_{\mathrm{AMIN}} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\mathrm{AMAX}}, \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \text { to } \mathrm{V}_{\mathrm{S}}= \pm 5 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega \\ & \hline \end{aligned}$ | 86 | 96 |  | 77 | 96 |  | 77 | 96 |  | $\begin{aligned} & \mathrm{dB} \\ & \mathrm{~dB} \end{aligned}$ |
| Transient Response <br> Rise Time <br> Overshoot | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$, Unity Gain |  | $\begin{gathered} 0.25 \\ 6.0 \end{gathered}$ | $\begin{aligned} & 0.8 \\ & 20 \end{aligned}$ |  | $\begin{gathered} 0.3 \\ 5 \end{gathered}$ |  |  | $\begin{gathered} 0.3 \\ 5 \end{gathered}$ |  | $\begin{aligned} & \mu \mathrm{s} \\ & \% \end{aligned}$ |
| Bandwidth (Note 6) | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ | 0.437 | 1.5 |  |  |  |  |  |  |  | MHz |
| Slew Rate | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$, Unity Gain | 0.3 | 0.7 |  |  | 0.5 |  |  | 0.5 |  | V/ $\mathrm{\mu s}$ |
| Supply Current | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  |  |  |  | 1.7 | 2.8 |  | 1.7 | 2.8 | mA |
| Power Consumption | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V} \end{aligned}$ |  | 80 | 150 |  | 50 | 85 |  | 50 | 85 | $\begin{aligned} & \mathrm{mW} \\ & \mathrm{~mW} \end{aligned}$ |
| LM741A | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \\ & \mathrm{~T}_{\mathrm{A}}=\mathrm{T}_{\text {AMIN }} \\ & \mathrm{T}_{\mathrm{A}}=\mathrm{T}_{\text {AMAX }} \\ & \hline \end{aligned}$ |  |  | $\begin{aligned} & 165 \\ & 135 \\ & \hline \end{aligned}$ |  |  |  |  |  |  | $\begin{aligned} & \mathrm{mW} \\ & \mathrm{~mW} \\ & \hline \end{aligned}$ |
| LM741 | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 15 \mathrm{~V} \\ & \mathrm{~T}_{\mathrm{A}}=\mathrm{T}_{\text {AMIN }} \\ & \mathrm{T}_{\mathrm{A}}=\mathrm{T}_{\text {AMAX }} \\ & \hline \end{aligned}$ |  |  |  |  | $\begin{aligned} & 60 \\ & 45 \end{aligned}$ | $\begin{aligned} & 100 \\ & 75 \end{aligned}$ |  |  |  | $\begin{aligned} & \mathrm{mW} \\ & \mathrm{~mW} \\ & \hline \end{aligned}$ |

Note 2: "Absolute Maximum Ratings" indicate limits beyond which damage to the device may occur. Operating Ratings indicate conditions for which the device is functional, but do not guarantee specific performance limits.

Electrical Characteristics (Note 5) (Continued)
Note 3: For operation at elevated temperatures, these devices must be derated based on thermal resistance, and $T_{j}$ max. (listed under "Absolute Maximum Ratings"). $T_{j}=T_{A}+\left(\theta_{j A} P_{D}\right)$.

| Thermal Resistance | Cerdip (J) | DIP (N) | HO8 (H) | SO-8 (M) |
| :--- | :---: | :---: | :---: | :---: |
| $\theta_{\mathrm{jA}}$ (Junction to Ambient) | $100^{\circ} \mathrm{C} / \mathrm{W}$ | $100^{\circ} \mathrm{C} / \mathrm{W}$ | $170^{\circ} \mathrm{C} / \mathrm{W}$ | $195^{\circ} \mathrm{C} / \mathrm{W}$ |
| $\theta_{\mathrm{jC}}$ (Junction to Case) | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $25^{\circ} \mathrm{C} / \mathrm{W}$ | $\mathrm{N} / \mathrm{A}$ |

Note 4: For supply voltages less than $\pm 15 \mathrm{~V}$, the absolute maximum input voltage is equal to the supply voltage.
Note 5: Unless otherwise specified, these specifications apply for $V_{S}= \pm 15 \mathrm{~V},-55^{\circ} \mathrm{C} \leq \mathrm{T}_{\mathrm{A}} \leq+125^{\circ} \mathrm{C}$ (LM741/LM741A). For the LM741C/LM741E, these specifications are limited to $0^{\circ} \mathrm{C} \leq \mathrm{T}_{\mathrm{A}} \leq+70^{\circ} \mathrm{C}$.
Note 6: Calculated value from: BW (MHz) $=0.35 /$ Rise Time $(\mu \mathrm{s})$.
Note 7: For military specifications see RETS741X for LM741 and RETS741AX for LM741A.
Note 8: Human body model, $1.5 \mathrm{k} \Omega$ in series with 100 pF .

## Schematic Diagram



## The Gain of Real Op-Amps

The open-circuit voltage gain $A_{o p}$ (a differential gain!) of a real (i.e., nonideal) operational amplifier is very large at D.C. (i.e., $w=0$ ), but gets smaller as the signal frequency $\omega$ increases!

In other words, the differential gain of an op-amp (i.e., the open-loop gain of a feedback amplifier) is a function of frequency $\omega$.

We will thus express this gain as a complex function in the frequency domain (i.e., $A_{\text {op }}(\omega)$ ).

## Gain is a complex function frequency

Typically, this op-amp behavior can be described mathematically with the complex function:

$$
A_{o p}(\omega)=\frac{A_{0}}{1+j\left(\omega / \omega_{b}\right)}
$$

or, using the frequency definition $\omega=2 \pi f$, we can write:

$$
A_{o p}(f)=\frac{A_{0}}{1+j\left(f / f_{b}\right)}
$$

where $\omega$ is frequency expressed in units of radians/sec, and $f$ is signal frequency expressed in units of cycles/sec.

## $D C$ is when the signal frequency is zero

Note the squared magnitude of the op-amp gain is therefore the real function:

$$
\begin{aligned}
\left|A_{o p}(\omega)\right|^{2} & =\frac{A_{0}}{1+j\left(\omega / w_{b}\right)} \frac{A_{0}}{1-j\left(\omega / \omega_{b}\right)} \\
& =\frac{A_{0}^{2}}{1+\left(\omega / \omega_{b}\right)^{2}}
\end{aligned}
$$

Therefore at D.C. $(\omega=0)$ the op-amp gain is:

$$
A_{o p}(\omega=0)=\frac{A_{b}}{1+j\left(\% \omega_{b}\right)}=A_{b}
$$

and thus:

$$
\left|A_{o p}(\omega=0)\right|^{2}=A_{o}^{2}
$$

Where:

$$
A_{0}=\text { op-amp D.C. gain }
$$

## The break frequency

Again, note that the D.C. gain $A_{0}$ is:

1) an open-circuit voltage gain
2) a differential gain
3) also referred to as the open-loop D.C. gain

The open-loop gain of real op-amps is very large, but fathomable - typically between $10^{5}$ and $10^{8}$.

Q: So just what does the value $w_{b}$ indicate?

A: The value $w_{b}$ is the op-amp's break frequency.

Typically, this value is very small (e.g. $f_{b}=10 \mathrm{~Hz}$ ).

## The 3dB bandwidth

To see why this value is important, consider the op-amp gain at $\omega=\omega_{b}$ :

$$
A_{o p}\left(\omega=\omega_{b}\right)=\frac{A_{0}}{1+j\left(\omega / \omega_{b}\right)}=\frac{A_{0}}{1+j}=\frac{A_{b}}{2}-j \frac{A_{0}}{2}=\frac{\left|A_{b}\right|}{\sqrt{2}} e^{-j \pi / 4}
$$

The squared magnitude of this gain is therefore:

$$
\left|A_{o p}\left(\omega=\omega_{b}\right)\right|^{2}=\frac{A_{b}}{1+j} \frac{A_{b}}{1-j}=\frac{A_{b}^{2}}{1-j^{2}}=\frac{A_{b}^{2}}{2}
$$

As a result, the break frequency $\omega_{b}$ is also referred to as the "half-power" frequency, or the " 3 dB " frequency.

## This value is very important!

If we plot $\left|A_{o p}(\omega)\right|^{2}$ on a "log-log" scale, we get something like this:

Q: Hey! You have defined a new frequency - $\omega_{+}$. What is this frequency and why is it important?

## The unity gain frequency

A: Note that $\omega_{t}$ is the frequency where the magnitude of the gain is "unity" (i.e., where the gain is 1 ). I.E.,

$$
\left|A_{o p}\left(\omega=\omega_{+}\right)\right|^{2}=1
$$

Note that when expressed in dB , unity gain is:

$$
10 \log _{10}\left|A_{o p}\left(\omega=\omega_{+}\right)\right|^{2}=10 \log _{10}(1)=0 \mathrm{~dB}
$$

Therefore, on a "log-log" plot, the gain curve crosses the horizontal axis at frequency $\omega_{t}$.

We thus refer to the frequency $\omega_{+}$as the "unity-gain frequency" of the operational amplifier.

## It's the product of the gain and the bandwidth!

Note that we can solve for this frequency in terms of break frequency $\omega_{b}$ and D.C. gain $A_{0}$ :

$$
1=\left|A_{o p}\left(\omega=\omega_{f}\right)\right|^{2}=\frac{A_{b}^{2}}{1+\left(\omega_{f} / \omega_{b}\right)^{2}}
$$

meaning that:

$$
\omega_{t}^{2}=\omega_{b}^{2}\left(A_{b}^{2}-1\right)
$$

But recall that $A_{b} \gg 1$, therefore $A_{b}^{2}-1 \approx A_{b}^{2}$ and:

$$
\omega_{t}=\omega_{b}\left|A_{b}\right|
$$

Note since the frequency $\omega_{b}$ defines the 3 dB bandwidth of the op-amp, the unity gain frequency $\omega_{t}$ is simply the product of the op-amp's D.C. gain $\left|A_{0}\right|$ and its bandwidth $\omega_{b}$.

## It's not rocket science!

As a result, $\omega_{+}$is alternatively referred to as the gain-bandwidth product!
and

$$
w_{t} \doteq \text { Unity Gain Frequency }
$$

$$
w_{t} \doteq \text { Gain - Bandwidth Product }
$$

This is so simple perhaps even I can remember it:

The gain-bandwidth-product is the product of the gain and the bandwidth!

## An Approximation of the Op-Amp

## Transfer Function

Recall the complex transfer function describing the differential gain of an opamp is:

$$
A_{o p}(\omega)=\frac{v_{\text {out }}(\omega)}{v_{d}(\omega)}=\frac{A_{0}}{1+j\left(\omega / \omega_{b}\right)}
$$

For frequencies much less than the break frequency, we find that $\omega / \omega_{b} \ll 1$ and thus this gain is approximately equal to $A_{0}$ :

$$
A_{o p}\left(\omega \ll \omega_{b}\right) \approx A_{0}
$$

## For "large" frequencies, the math gets simple

Likewise, for frequencies much greater than the break frequency, we find that $\omega / \omega_{b} \gg 1$ and thus this gain is approximately equal to:

$$
A_{o p}\left(\omega \gg \omega_{b}\right)=\frac{A_{0}}{1+j\left(\omega / \omega_{b}\right)} \approx \frac{A_{0}}{j\left(\omega / \omega_{b}\right)}=-j \frac{A_{b} \omega_{b}}{\omega}
$$

But, we recall that the product of the op-amp D.C. gain $A_{0}$ and the op-amp bandwidth $\omega_{b}$ is the gain-bandwidth product $\omega_{t}$ (aka the unity gain frequency).

Thus, we can likewise write the previous approximation as:

$$
A_{o p}\left(\omega \gg \omega_{b}\right) \approx-j \frac{A_{b} \omega_{b}}{\omega}=-j \frac{\omega_{t}}{\omega}
$$

## A useful approx. of the transfer function

Recall also that when the signal frequency is equal to the op-amp break frequency (i.e., $\omega=\omega_{b}$ ), the transfer function is:

$$
A_{o p}\left(\omega=\omega_{b}\right)=\frac{A_{0}}{1+j\left(\omega / \omega_{b}\right)}=\frac{A_{b}}{1+j}
$$

such that $\left|A_{o p}\left(\omega=\omega_{b}\right)\right|=A^{6} / \sqrt{2}$.
Expressed in terms of the magnitude of this complex transfer function, we can express these approximations as:

$$
\left|A_{o p}(f)\right| \approx \begin{cases}A_{b} & \text { if } f \ll f_{b} \\ A_{b} / \sqrt{2} & \text { if } f \approx f_{b} \\ \frac{f_{f}}{f} & \text { if } f \gg f_{b}\end{cases}
$$

## Example: The Gain -Bandwidth Product

An op-amp has a D.C. differential gain of $A_{0}=10^{5}$.
At a frequency of $1 \mathrm{MHz}\left(f=10^{6}\right)$, the differential op-amp gain drops to 10 (i.e., $\left|A_{o p}\left(f=10^{6}\right)\right|=10$ ).

Q: What is the break frequency and unity-gain frequency of this op-amp?
A: We know that if $f>f_{b}$ :

$$
\left|A_{o p}(f)\right|=\frac{A_{b} f_{b}}{f}
$$

and thus at a frequency of 1 MHz , we find for the parameters of this problem:

$$
\left|A_{o p}\left(f=10^{6}\right)\right|=10=\frac{10^{5} f_{b}}{10^{6}}
$$

## It's 10 MHz

It is apparent then that the break frequency of this op-amp must be:

$$
f_{b}=\frac{(10)\left(10^{6}\right)}{10^{5}}=100 \mathrm{~Hz}
$$

and since the unity-gain bandwidth $f_{t}$ is related to the break frequency and D.C. gain as:
we find that:

$$
f_{t}=A_{b} f_{b}
$$

$$
\begin{aligned}
f_{t} & =A_{b} f_{b} \\
& =10^{5}(100) \\
& =10^{7}
\end{aligned}
$$

Thus, the unity-gain frequency (i.e., the gain-bandwidth product) for this problem is 10 MHz .

## The gain depends on frequency

Q: What is the differential gain of this op-amp at a frequency of 10 kHz (i.e., $\left|A_{o p}\left(f=10^{4}\right)\right|$ )?

A: We know that:

$$
\left|A_{o p}(f)\right|=\frac{A_{b} f_{b}}{f}=\frac{f_{t}}{f}
$$

therefore, using the values of this example:

$$
\begin{aligned}
\left|A_{o p}\left(f=10^{4}\right)\right| & =\frac{f_{f}}{f} \\
& =\frac{10^{7}}{10^{4}} \\
& =10^{3}
\end{aligned}
$$

Hence, the differential op-amp gain at 10 kHz is 1000.

## Closed-Loop Bandwidth

Say we build in the lab (i.e., the op-amp is not ideal) this amplifier:


We know that the open-circuit voltage gain (i.e., the closed-loop gain) of this amplifier should be:

$$
A_{\text {vo }}(\omega)=\frac{v_{\text {out }}(\omega)}{v_{\text {in }}(\omega)}=1+\frac{R_{2}}{R_{1}} \quad ? ? ?
$$

This gain will certainly be accurate for input signals $v_{\text {in }}(w)$ at low frequencies $\omega$.

## As the signal frequency increases

But remember, the Op-amp (i.e., open-loop gain) gain $A_{o p}(\omega)$ decreases with frequency.

If the signal frequency $\omega$ becomes too large, the open-loop gain $A_{\text {op }}(\omega)$ will become less than the ideal closed-loop gain!


## The amp gain cannot exceed the op-amp gain

Note as some sufficiently high frequency ( $\omega^{\prime}$ say), the open-loop (op-amp) gain will become equal to the ideal closed-loop (non-inverting amplifier) gain:

$$
\left|A_{o p}\left(\omega=\omega^{\prime}\right)\right|=1+\frac{R_{2}}{R_{1}}
$$

Moreover, if the input signal frequency is greater than frequency $\omega^{\prime}$, the opamp (open-loop) gain will in fact be smaller that the ideal non-inverting (closedloop) amplifier gain:

$$
\left|A_{o p}\left(\omega>\omega^{\prime}\right)\right|<1+\frac{R_{2}}{R_{1}}
$$

Q: If the signal frequency is greater than $\omega^{\prime}$, will the non-inverting amplifier still exhibit an open-circuit voltage (closed-loop) gain of $A_{v o}(\omega)=1+R_{2} / R_{1}$ ?

A: Allow my response to be both direct and succinct-NEVER!

## Closed-loop gain < or = open-loop gain

The gain $A_{\text {so }}(\omega)$ of any amplifier constructed with an op-amp can never exceed the gain $A_{o p}(\omega)$ of the op-amp itself.

In other words, the closed-loop gain of any amplifier can never exceed its openloop gain.

* We find that if the input signal frequency exceeds $\omega^{\prime}$, then the amplifier (closed-loop) gain $A_{10}(\omega)$ will equal the op-amp (open-loop) gain $A_{\text {op }}(\omega)$.
* Of course, if the signal frequency is less than $\omega^{\prime}$, the closed-loop gain will be equal to its ideal value $A_{10}(\omega)=1+R_{2} / R_{1}$, since the op-amp (openloop) gain is much larger than this ideal value $\left(\left|A_{o p}\left(\omega<\omega^{\prime}\right)\right| \gg 1+R_{2} / R_{1}\right)$.
* We now refer to the value $1+R_{2} / R_{1}$ as the mid-band gain of the amplifier.


## $1+R_{2} / R_{1}$ is the midband gain

Therefore, we find for this non-inverting amplifier that:

$$
\left|A_{v o}(\omega)\right| \approx\left\{\begin{array}{cc}
1+\frac{R_{2}}{R_{1}} & \omega<\omega^{\prime} \\
\left|A_{o p}(\omega)\right| & \omega>\omega^{\prime}
\end{array}\right.
$$



## Can we determine this bandwidth?

Now for one very important fact: the transition frequency $\omega^{\prime}$ is the break frequency of the amplifier closed-loop gain $\left|A_{v_{0}}(w)\right|$.

Thus, we come to conclusion that $\omega^{\prime}$ is the 3dB bandwidth of this non-inverting amplifier (i.e., $\omega^{\prime}=\omega_{3 a B}$ )!

Q: Is there some way to numerically determine this value?
A: Of course!
Recall we defined frequency $\omega^{\prime}$ as the value where the open-loop (op-amp) gain and the ideal closed-loop (non-inverting amplifier) gains were equal:

$$
\left|A_{o p}\left(\omega=\omega^{\prime}\right)\right|=1+\frac{R_{2}}{R_{1}}
$$

Recall also that for $\omega>\omega_{b}$, we can approximate the op-amp (open-loop) gain as:

$$
\left|A_{o p}(\omega)\right| \approx \frac{A_{0} \omega_{b}}{\omega}
$$

## Divide the gain-bandwidth product by gain, and you have determined the bandwidth!

Combining these results, we find:

$$
\left|A_{o p}\left(\omega=\omega^{\prime}\right)\right|=1+\frac{R_{2}}{R_{1}} \simeq \frac{A_{0} \omega_{b}}{\omega^{\prime}}
$$

and thus:

$$
\omega^{\prime}=\left(1+\frac{R_{2}}{R_{1}}\right)^{-1}\left(A_{b} \omega_{b}\right)
$$

But remember, we found that this frequency is equal to the breakpoint of the non-inverting amplifier (closed-loop) gain $A_{10}(\omega)$.

Therefore, the 3dB, closed-loop bandwidth of this amplifier is:

$$
\omega_{3 d B} \simeq\left(1+\frac{R_{2}}{R_{1}}\right)^{-1}\left(A_{b} \omega_{b}\right)
$$

## This is not rocket science

Recall also that $A_{b} \omega_{b}=\omega_{+}$, so that:

$$
w_{3 A B} \simeq\left(1+\frac{R_{2}}{R_{1}}\right)^{-1} w_{t}
$$



If we rewrite this equation, we find something interesting:

$$
w_{3 d B}\left(1+\frac{R_{2}}{R_{1}}\right) \simeq w_{+}
$$

Look what this says: the PRODUCT of the amplifier (mid-band) GAIN and the amplifier BANDWIDTH is equal to the GAIN-BANDWIDTH PRODUCT.

This result should not be difficult to remember!

## The gain-bandwidth product

## is an op-amp parameter

The above approximation is valid for virtually all amplifiers built using operational amplifiers, i.e.:
where:

$$
\left|A_{v o}\left(\omega_{m}\right)\right| \omega_{3 d B}=\omega_{t}
$$

$$
\left|A_{v o}\left(\omega_{m}\right)\right| \doteq \text { mid-band gain }
$$

In other words, $w_{m}$ is some frequency within the bandwidth of the amplifier (e.g., $0<\omega_{m}<\omega_{3 d B}$ ). We of course can equivalently say:

$$
\left|A_{v o}\left(f_{m}\right)\right| f_{3 d B}=f_{t}
$$

The product of the amplifier gain and the amplifier bandwidth is equal to the op-amp gain-bandwidth product!

## Example: Amplifier Bandwidth

Say we build the following amplifier in the lab:


The op-amp in this circuit happens to have a unity-gain bandwidth of 1 MHz .

Q: What is the $3 d B$ bandwidth of this amplifier?

## $x y=10^{6}$ and $x=20$; you figure it out

A: We know that the mid-band gain of this amplifier is:

$$
\left|A_{0}\left(\omega_{m}\right)\right|=\left|\frac{-R_{2}}{R_{1}}\right|=\frac{R_{2}}{R_{1}}=\frac{10}{0.5}=20 \quad(26 \mathrm{~dB})
$$

Since we know that $f_{t}=10^{6}$, we can directly determine the amplifier bandwidth:

$$
f_{3 \Delta B}=\frac{f_{t}}{\left|A_{o}\left(f_{m}\right)\right|}=\frac{10^{6}}{20}=5 \times 10^{4}
$$

Since the product of the amplifier gain and bandwidth is equal to the gainbandwidth product, we find that the gain-bandwidth product $f_{t}$ divided by the mid-band gain equals the amplifier bandwidth $f_{3 a B}$ !

## If I want more bandwidth...

In this case, the amplifier bandwidth is $f_{3 d B}=50 \mathrm{kHz}$.


Q: Is there any way to increase the bandwidth of this amplifier to 500 kHz ?
A: Sure! But we must decrease its mid-band gain.

## ... I must accept less gain

The gain-bandwidth product $f_{t}=10^{6}$ is a constant-if we increase the bandwidth, we must decrease the gain.

Therefore, if we want the amplifier bandwidth to equal 500 kHz , we must decrease the mid-band gain to:

$$
\begin{equation*}
\left|A_{o}\left(f_{m}\right)\right|=\frac{f_{f}}{f_{3 a B}}=\frac{10^{6}}{5 \times 10^{5}}=2 \tag{6dB}
\end{equation*}
$$

A gain of 2-quite a decrease!
But this of course makes sense.

To increase the bandwidth 10 times, we must decrease the gain by a factor of 10.

## There's no free lunch

Note we could accomplish this by simply changing the feedback resistor from $R_{2}=10 \mathrm{~K}$ to $R_{2}=1 \mathrm{~K}$.


