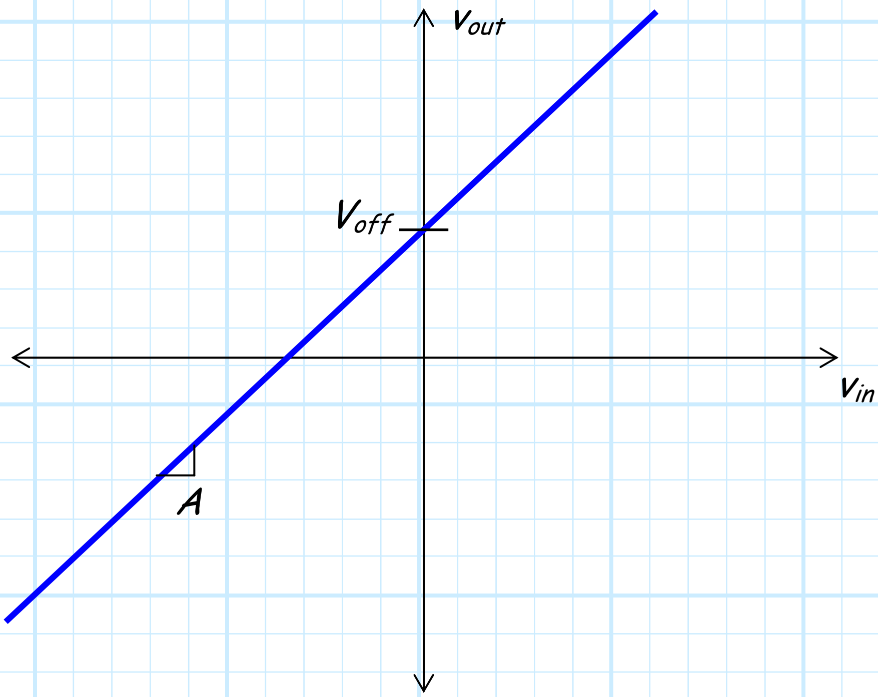
2.7 DC Imperfections

Reading **Assignment**: 98-104

In **addition** to saturation and slew-rate limiting, **another** non-linear behavior of amplifiers is a **DC output offset**!



Amplifiers built with op-amps can/will exhibit this non-linear behavior, mainly for **two** separate reasons!

HO: THE INPUT OFFSET VOLTAGE

EXAMPLE: THE INPUT OFFSET VOLTAGE

The **second** reason for output offsets is there is a constant **bias current** flowing into the input terminals.

HO: THE INPUT BIAS CURRENT

EXAMPLE: THE INPUT BIAS CURRENT

Fortunately, we can **minimize** the DC output offset due the input bias current with **one simple design rule**.

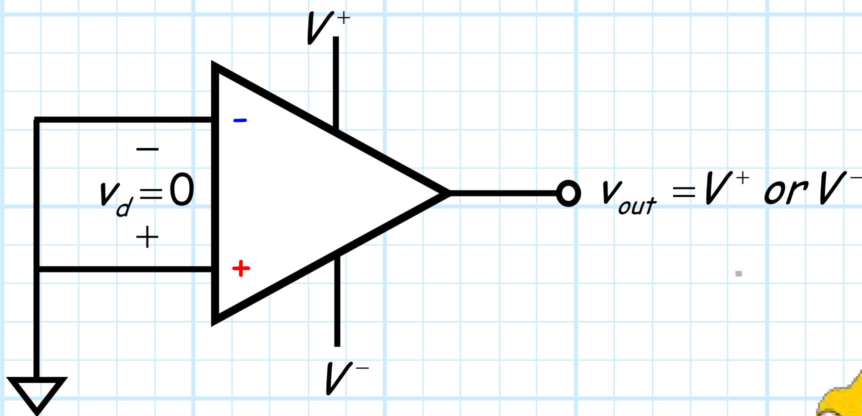
HO: REDUCING THE EFFECT OF THE INPUT BIAS CURRENT

Finally, we find that real op-amps have **input** and **output** resistances that are not **quite** ideal!

HO: REAL OP-AMP INPUT AND OUTPUT RESISTANCES

The Input Offset Voltage

For real op-amps, we typically find that if both inputs are grounded, the output will be—**saturated**!



Q: *What!? Why isn't*

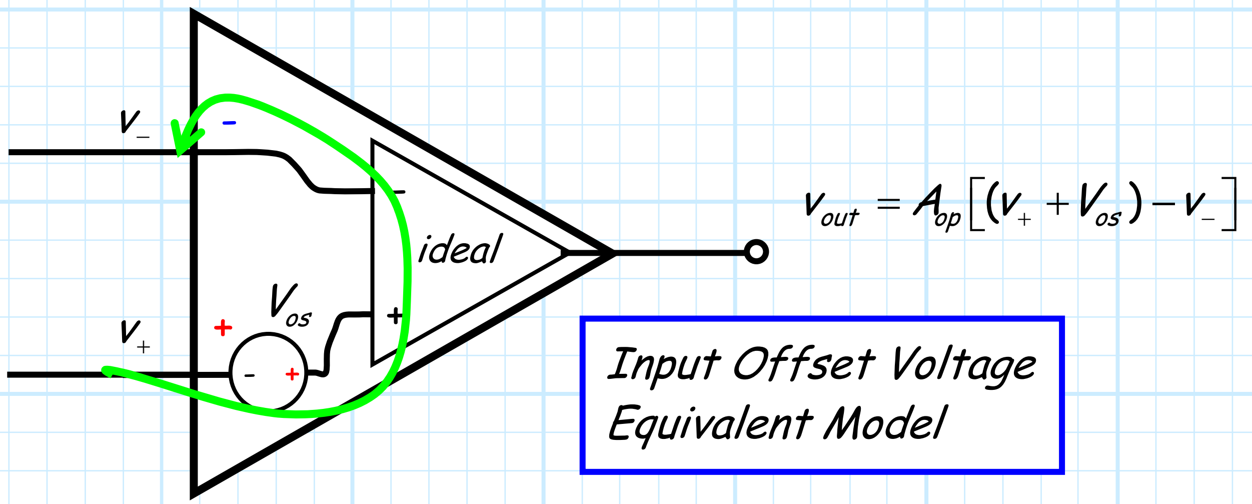
$$\begin{aligned} v_{out}(t) &= A_{op} v_d \\ &= A_{op}(0) \text{ ??} \\ &= 0 \end{aligned}$$

A: The reason the output is saturated is that real op-amps exhibit a phenomenon called the **input offset voltage** V_{os}.

The input offset voltage model

This value can be either positive **or** negative, **typically** with a magnitude of 5 mV or less ($|V_{os}| < 5 \text{ mV}$).

A real op-amp therefore behaves as if it has a small, **internal voltage source** at the non-inverting input:



Applying the concept of a virtual short to the **ideal** op-amp, we find from **KVL**

$$v_- = v_+ + V_{os}$$

Thus, $v_- \neq v_+$!

The new virtual "short"

Recall, however, that the input offset voltage is typically **very small** (i.e., $|V_{os}| < 5\,mV$), so that $v_{-} \approx v_{+}$.

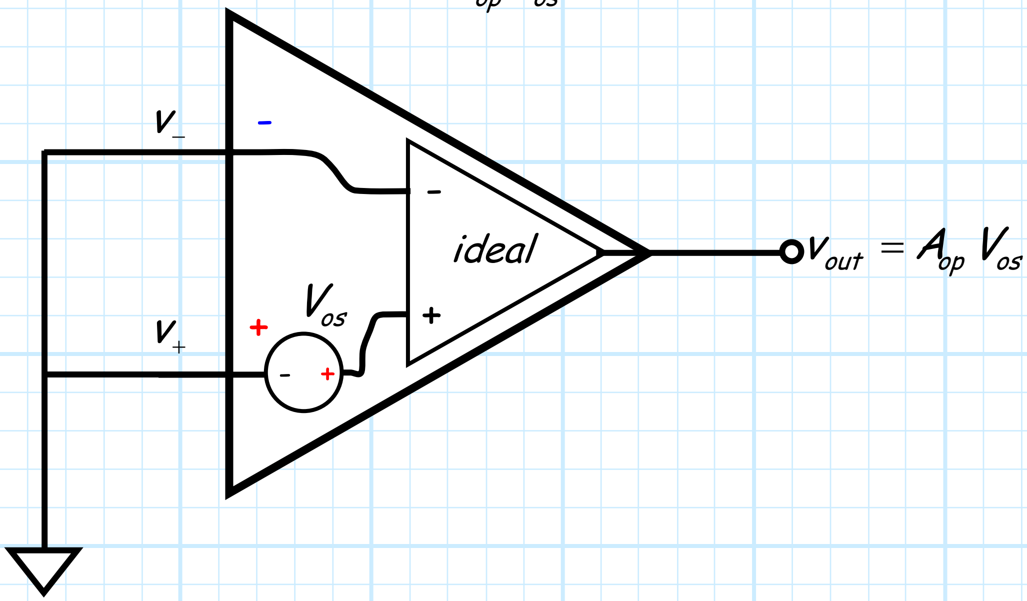
So, for an op-amp with an **input offset voltage**, the virtual "short" equation turns out to be:

$$v_{-} = V_{os} + v_{+}$$

Small, but large enough to saturate!

Therefore, **if** $v_- = v_+ = 0$, we find that the output voltage of this op-amp is **ideally** equal to:

$$\begin{aligned} v_{out} &= A_{op}\left(v_+ - v_- + V_{os}\right) \\ &= A_{op}\left(0 - 0 + V_{os}\right) \\ &= A_{op}\, V_{os} \end{aligned}$$



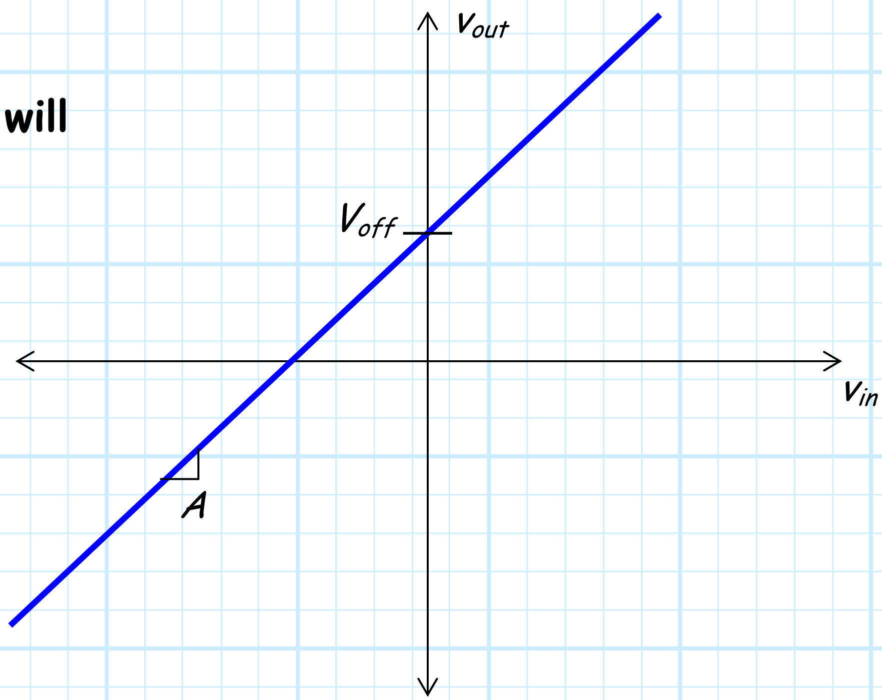
Of course, since the differential voltage A_{op} is **very** large, the product $A_{op}\, V_{os}$ is likewise large, such that the output of **real** op-amps will **saturate**.

This changes our previous results

Q: *Does this mean that V_{os} will cause the output of op-amp **circuits** and **amplifiers** to saturate?*

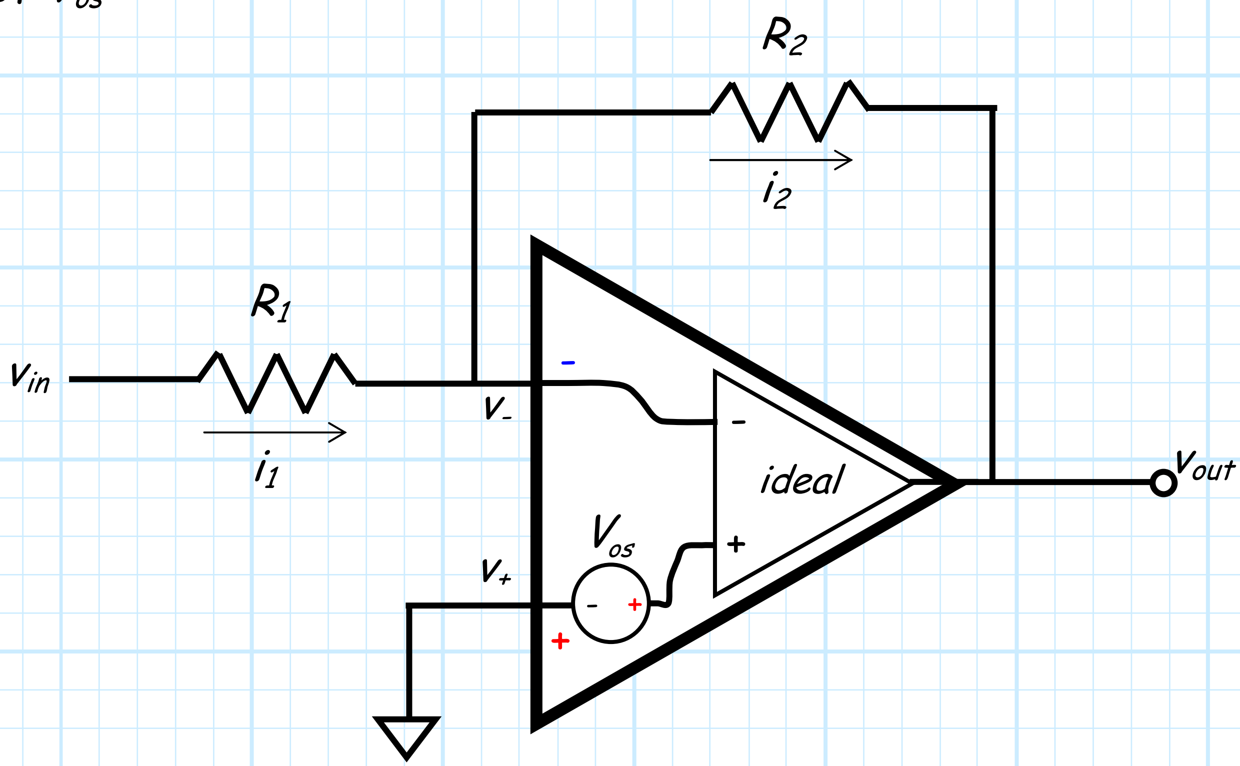
A: Fortunately **no**!

However, the input offset voltage **will** affect the **output** of circuits and amplifiers made with op-amps.



Example: The Input Offset Voltage

Consider an **inverting** amp constructed with an op-amp exhibiting an **input offset voltage** of V_{os}:



v_- not equal to v_+

We know that because of the input offset voltage:

$$v_- = v_+ + V_{os}$$

For the circuit above, the non-inverting terminal of the op-amp is connected to ground (i.e., $v_+ = 0$), and so the virtual "ground" is now described by:

$$v_- = V_{os}$$

The **current** into each terminal of the op-amp is still **zero**, so that from KCL:

$$i_1 = i_2$$

where form KCL and Ohm's Law:

$$i_1 = \frac{v_{in} - v_-}{R_1} = \frac{v_{in} - V_{os}}{R_1}$$

and:

$$i_2 = \frac{v_- - v_{out}}{R_2} = \frac{V_{os} - v_{out}}{R_2}$$

The output has a DC offset!

Combining, we find:

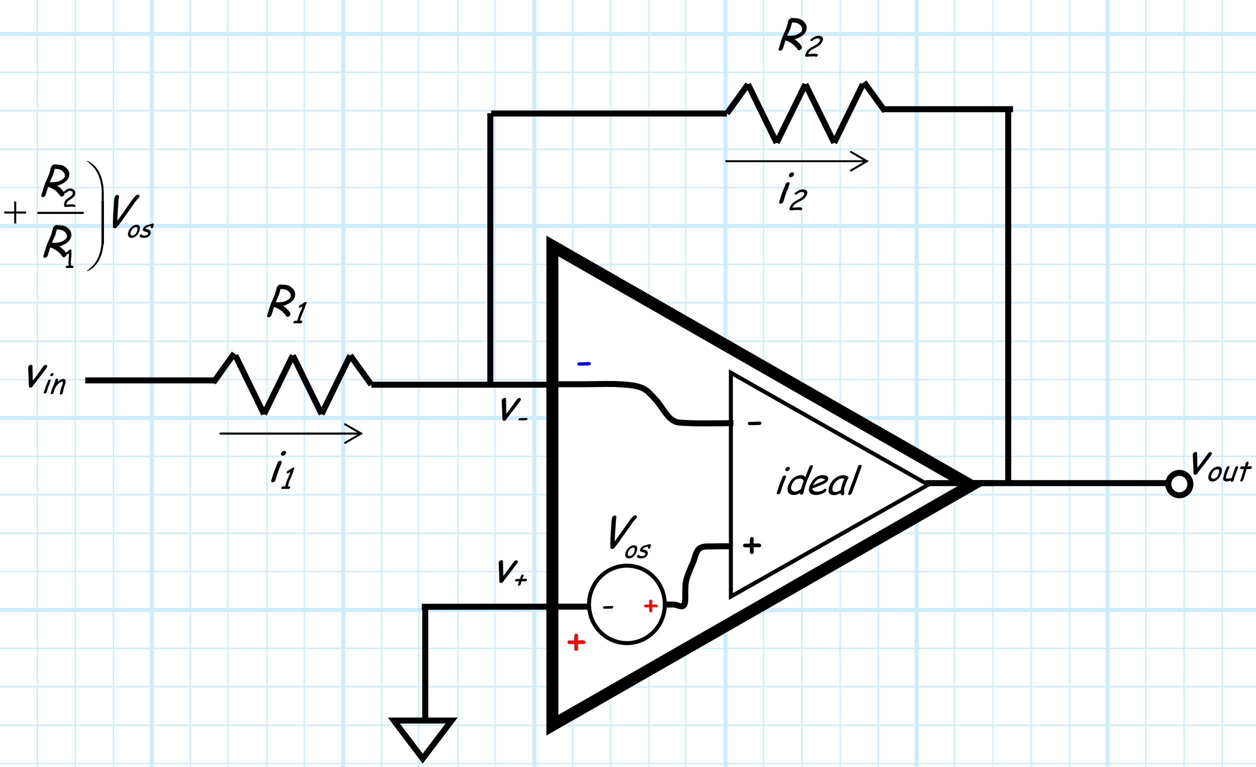
$$\frac{v_{in} - V_{os}}{R_1} = \frac{V_{os} - v_{out}}{R_2}$$

Performing a little algebra, we can solve this equation for **output voltage** v_{out} :

$$v_{out} = \frac{V_{os}R_1 + V_{os}R_2 - v_{in}R_2}{R_1}$$

and rearranging:

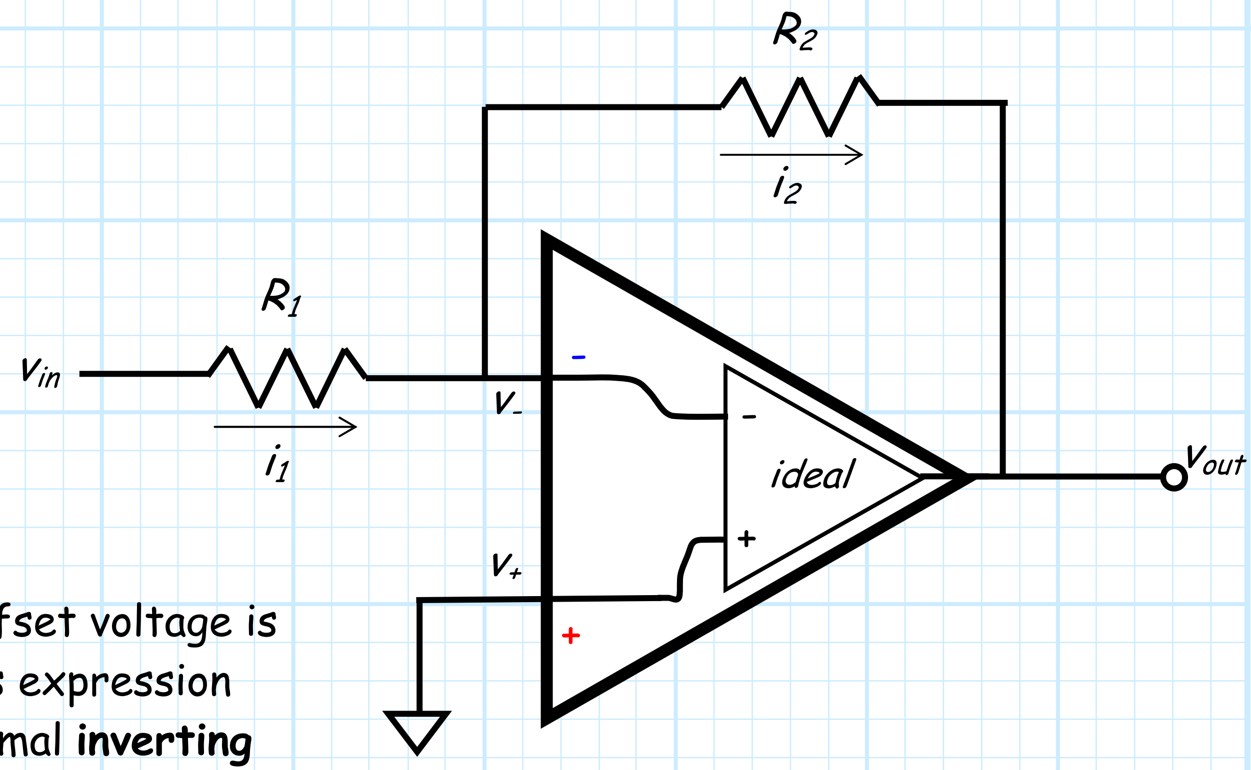
$$v_{out} = -\left(\frac{R_2}{R_1}\right)v_{in} + \left(1 + \frac{R_2}{R_1}\right)V_{os}$$



Superposition is your friend

Q: Hey! Couldn't we have easily found this result by applying **superposition**?

A: Absolutely!

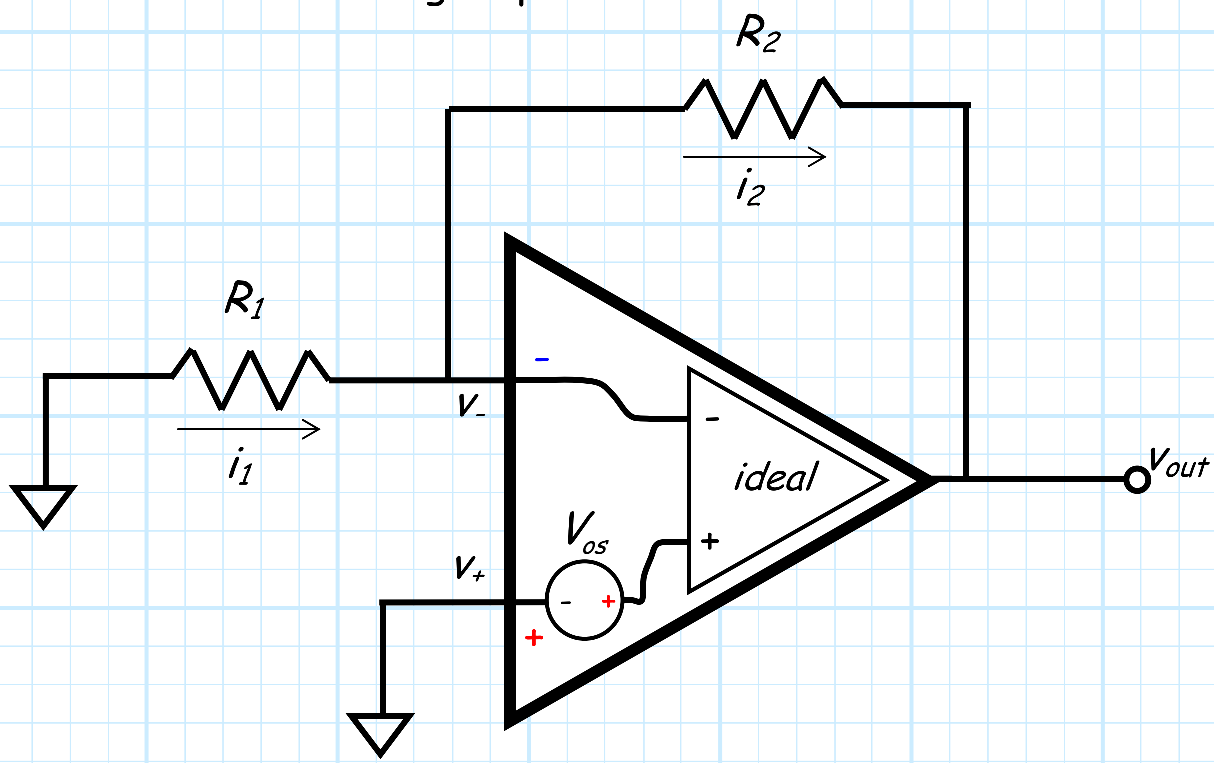


Note that **if** the input offset voltage is zero (its **ideal** value), this expression simply reduces to the normal **inverting amplifier** expression:

$$v_{out} = -\left(\frac{R_2}{R_1}\right)v_{in}$$

It's the non-inverting amplifier!

Likewise, if we set the input voltage source to ground potential (i.e., $v_{in} = 0$), it is evident that we have a non-inverting amplifier:



And so the output voltage is:

$$v_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{os}$$

Look at the DC offset!

The sum of these two results provides our previous answer:

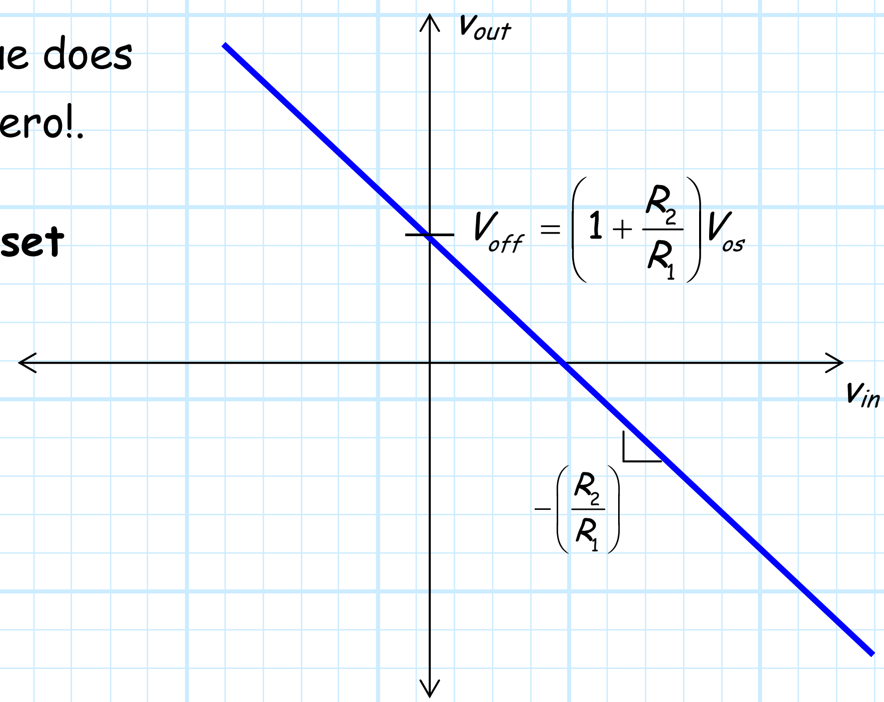
$$v_{out} = -\left(\frac{R_2}{R_1}\right)v_{in} + \left(1+\frac{R_2}{R_1}\right)V_{os}$$

Note the term:

$$\left(1+\frac{R_2}{R_1}\right)V_{os}$$

is a **constant** with respect to v_{in}—its value does not change, **even** if the input voltage is zero!.

Thus, the term represents an **output offset** voltage.



How do we define gain?

Q: *But what is the* ***gain*** *of this amplifier? The ratio* v_{out}/v_{in} *is not a constant!*

$$\frac{v_{out}}{v_{in}} = -\left(\frac{R_2}{R_1}\right) + \left(1 + \frac{R_2}{R_1}\right)\frac{V_{os}}{v_{in}} \quad ????$$

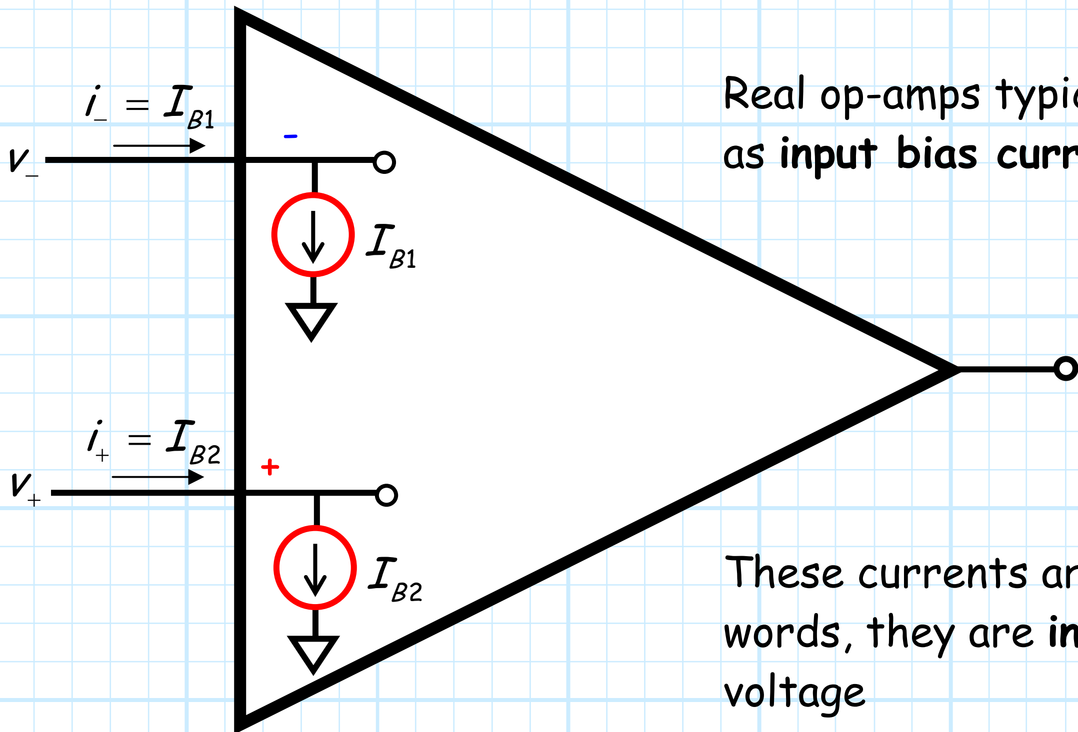
A: Remember, it is more accurate and more general to define gain in terms of the **derivative**:

$$A_{vo} \doteq \frac{d\,v_{out}}{d\,v_{in}}$$

Which for this case provides the **same result** for the inverting amplifier:

$$A_{vo} = -\left(\frac{R_2}{R_1}\right)$$

The Input Bias Current



Input Bias Current Op-Amp Model

Real op-amps typically exhibit a phenomenon known as **input bias current**.

We find that there is a **small** amount of current flowing into each of the op-amp inputs (i.e., $i_+ \neq 0$, and $i_+ \neq 0$)!

These currents are **constant** currents—in other words, they are **independent** of the input terminal voltage

Real op-amps act like there are small **current sources** at the inputs!!!!!!!

The input offset current

The values of bias currents I_{B1} and I_{B2} are **approximately**—but **not exactly**—equal.

As a result, we typically express these currents in terms of their common-mode (i.e., average) and differential modes.

The common mode is called the Input **Bias** Current:

$$I_B = \frac{I_{B1} + I_{B2}}{2} \doteq \text{Input } \textbf{Bias} \text{ Current}$$

The differential mode is called the Input **Offset** Current:

$$I_{os} = \left| I_{B1} - I_{B2} \right| \doteq \text{Input } \textbf{Offset} \text{ Current}$$

They seem so small, yet…

Thus, the two bias currents can be expressed as:

$$I_{B1} = I_B \pm \frac{I_{os}}{2} \qquad\qquad I_{B2} = I_B \mp \frac{I_{os}}{2}$$

Typical values of these parameters are, for example, I_B = 100nA and I_{os} =10nA.

Q: *These bias current values are so **tiny**, we do we even care about them????*

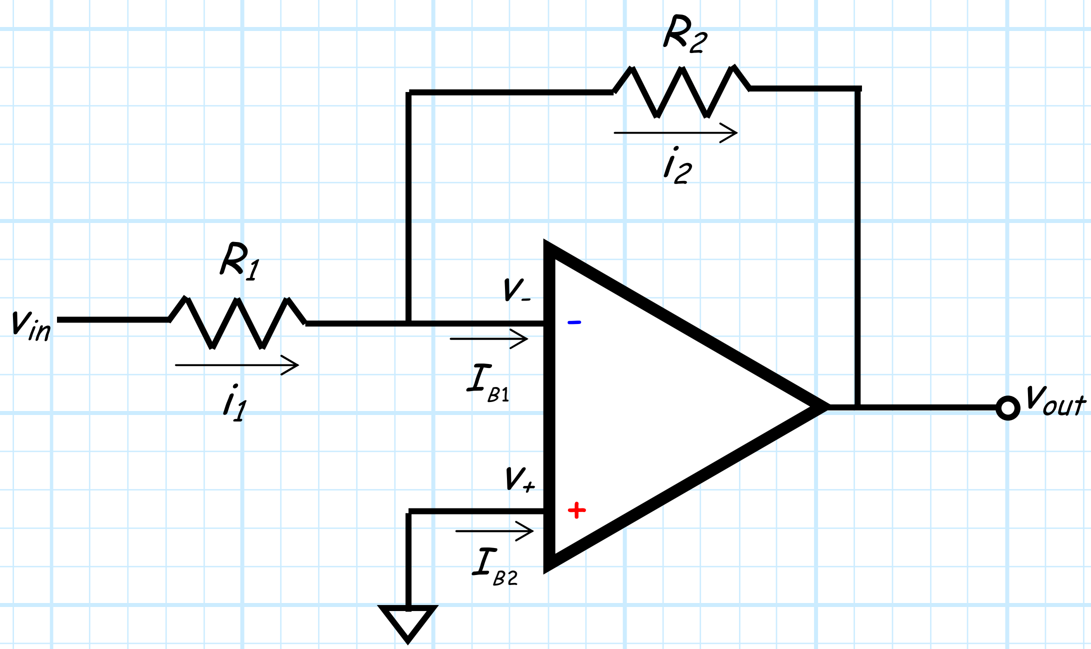
A: Because they can cause **offset voltages** in op-amp circuits!

Example: The Input Bias Current

Q: *How do **input bias currents** I_{B1} and I_{B2} affect amplifier operation?*

A: Consider **both** inverting and non-inverting configurations.

Inverting Configuration



KCL is now a bit more tricky!

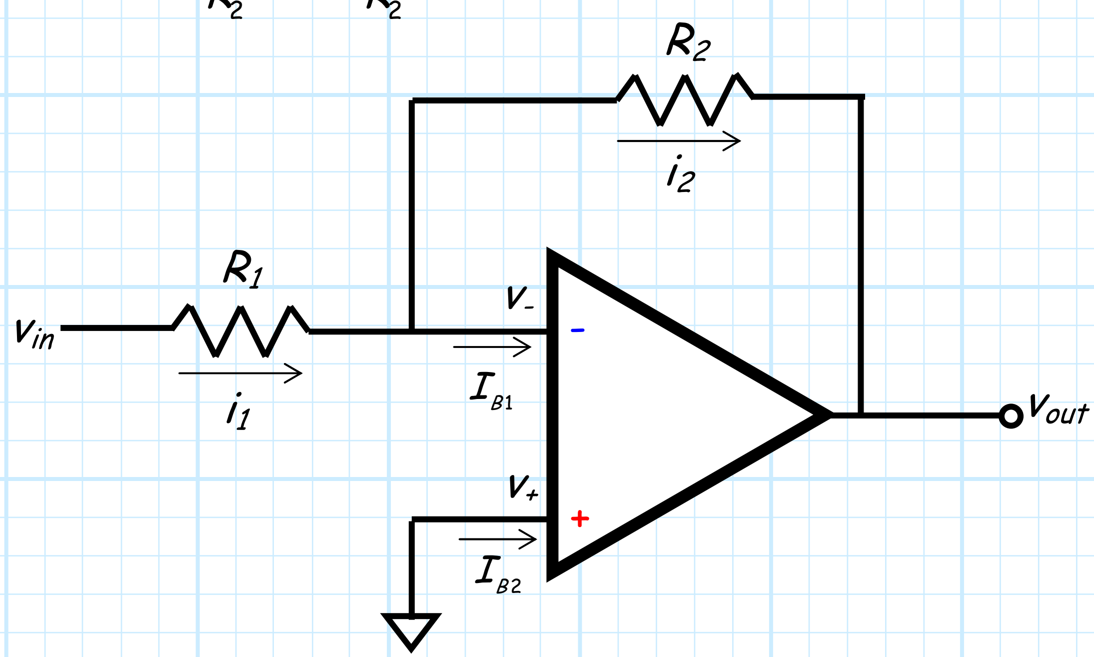
In this case, we apply **KCL** and we find:

$$i_1 = i_2 + I_{B1}$$

However, we still find $v_- \simeq v_+ = 0$ (**neglecting** the input offset voltage) by virtue of the virtual short.

Therefore, from KVL and Ohm's Law:

$$i_1 = \frac{v_{in} - v_-}{R_1} = \frac{v_{in}}{R_1} \qquad \text{and} \qquad i_2 = \frac{v_- - v_{out}}{R_2} = \frac{-v_{out}}{R_2}$$



Combining these results:

$$\frac{v_{in}}{R_1} = \frac{-v_{out}}{R_2} + I_{B1}$$

The output voltage is thus:

$$v_{out} = -\left(\frac{R_2}{R_1}\right)v_{in} + R_2 I_{B1}$$

Should we make R_1 really small?

Note again that if $I_{B1} = 0$, the result reduces to the expected **inverting amplifier** equation:

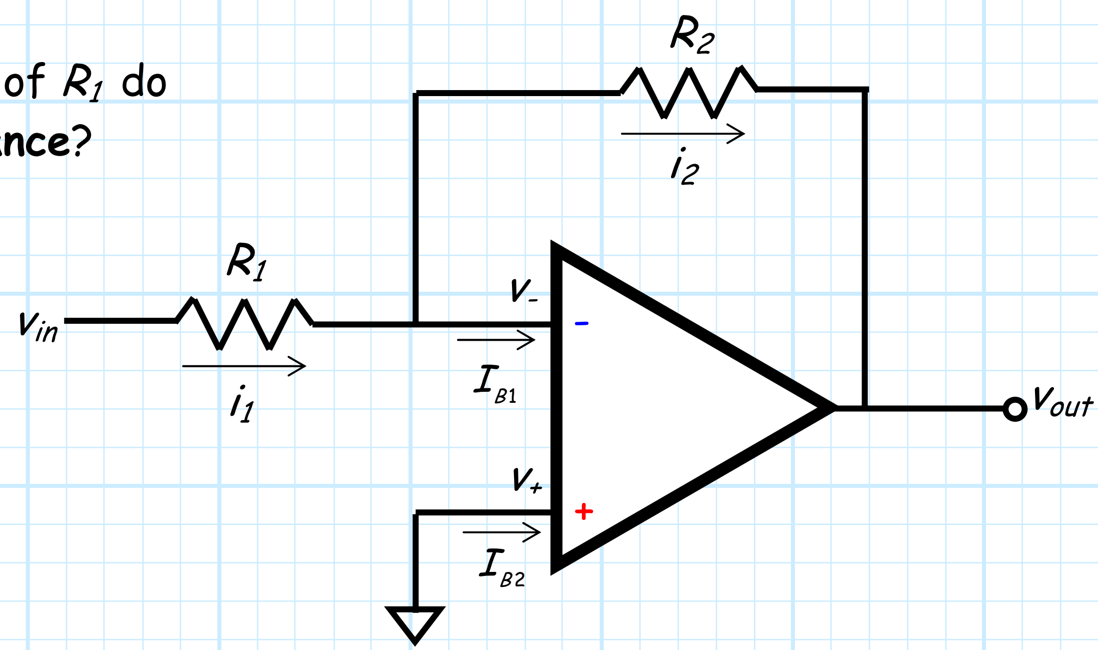
$$v_{out} = -\left(\frac{R_2}{R_1}\right)v_{in}$$

The second term in the above expression ($I_{B1} R_1$) therefore represents another **output offset voltage**!

It appears that we should keep the value of R_1 **small** to minimize the output offset voltage.

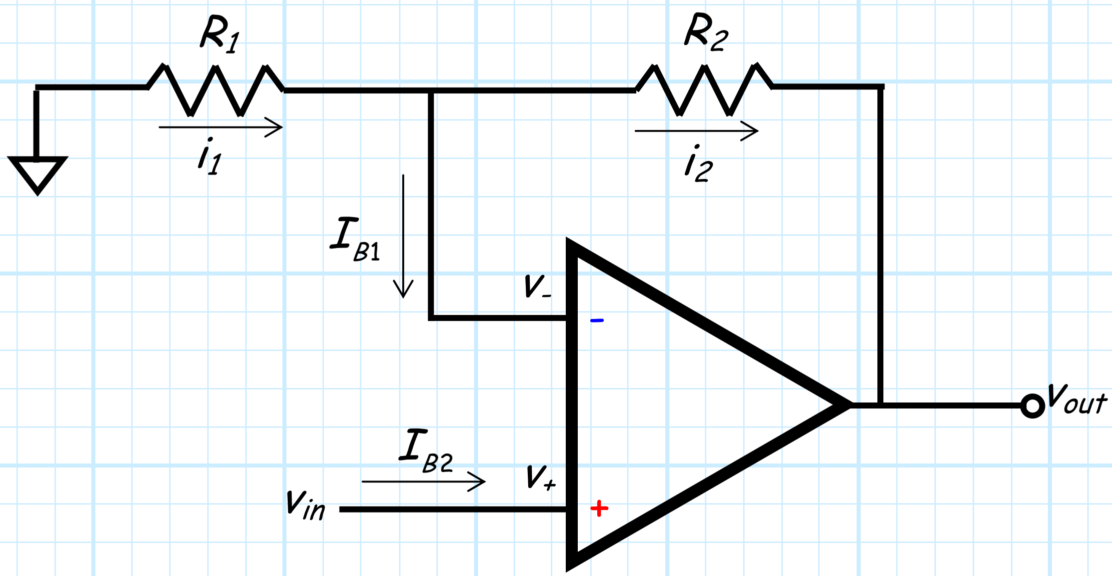
Q: What would a small value of R_1 do to the amplifier **input resistance**?

A:



Please welcome the non-inverting config.

Non-Inverting Configuration



Neglecting the input offset voltage, we can use the **virtual short** to determine that:

$$v_- \simeq v_{in}$$

and **KCL** provides the same result as that of the inverting amplifier:

$$i_1 = i_2 + I_{B1}$$

Again, a DC output offset

From **KVL and Ohm's Law**:

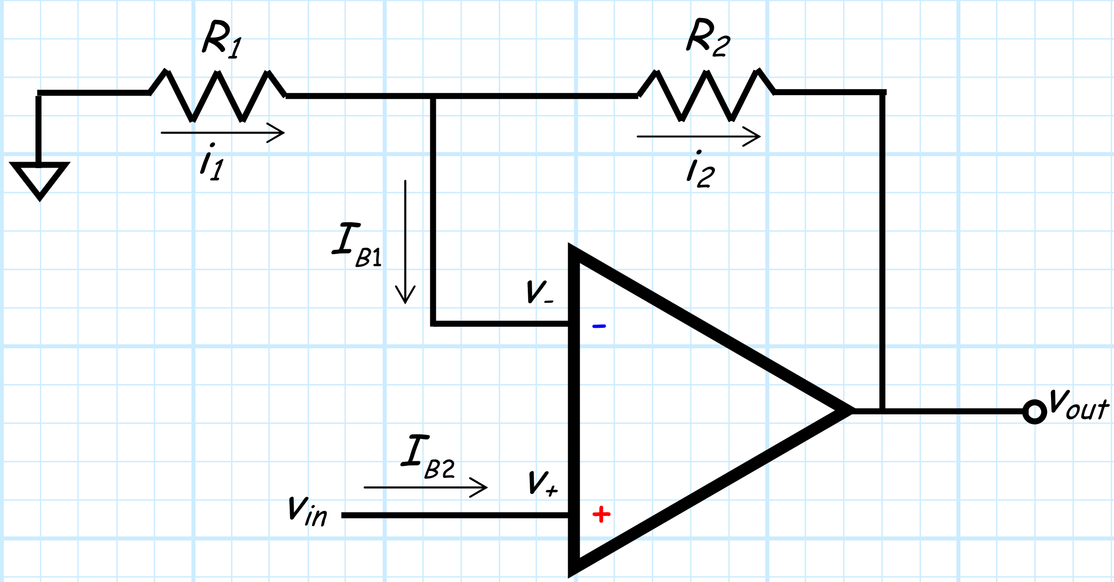
$$i_1 = \frac{0 - v_-}{R_1} = \frac{-v_{in}}{R_1}$$

and likewise:

$$i_2 = \frac{v_- - v_{out}}{R_2} = \frac{v_{in} - v_{out}}{R_2}$$

Combining, we find:

$$\frac{-v_{in}}{R_1} = \frac{v_{in} - v_{out}}{R_2} + I_{B1}$$



or rearranging:

$$v_{out} = \left(1 + \frac{R_2}{R_1}\right) v_{in} + I_{B1} R_2$$

We have another trick or two up our sleeve

Again, we find that this result is simply the ideal **non-inverting** expression:

$$v_{out} = \left(1 + \frac{R_2}{R_1}\right) v_{in}$$

with an added **output offset voltage** term:

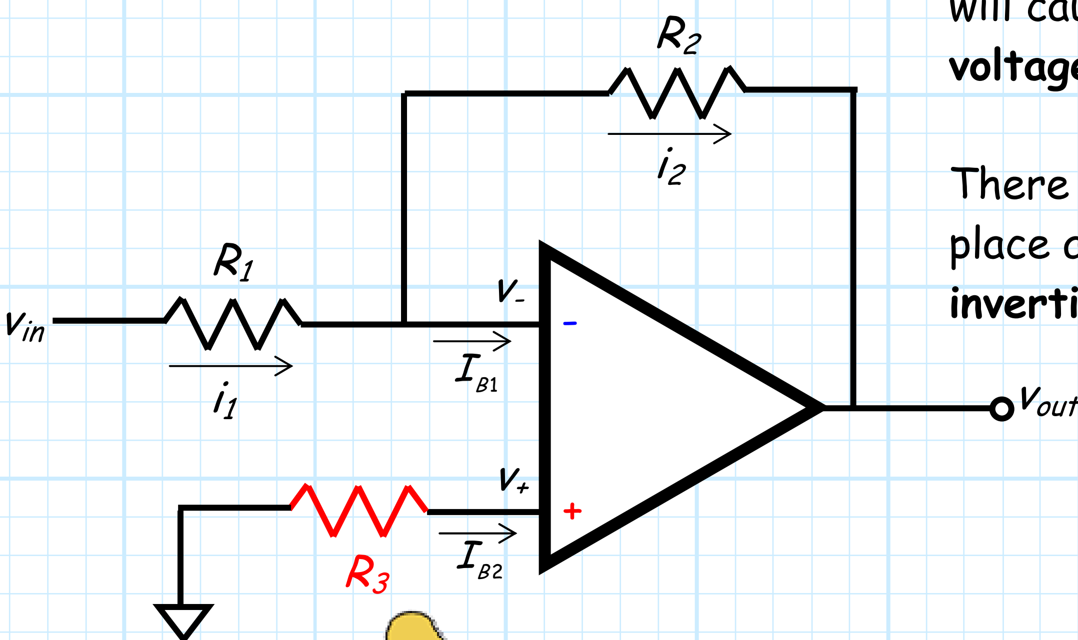
$$I_{B1} R_2$$

In this case, we find that this offset voltage is minimized by making feedback resistor R_2 **small**.

In general, we find that the effects of the input bias currents can be minimized by using **small** resistor values.

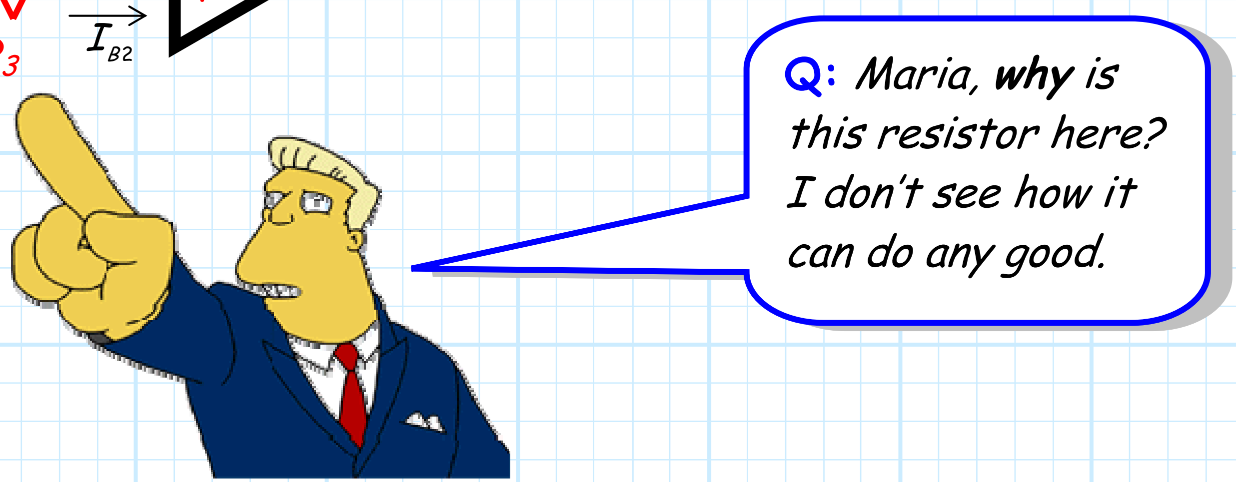
However, we will find that there is an **additional strategy** for minimizing the effects of input bias currents!

Reducing the Effect of Input Bias Current



We found that the input bias current will cause an **offset** in the **output voltage**.

There is a **solution** to this problem—place a **resistor** (R_3) on the **non-inverting input**!



The voltage v_+ is non-zero!

A: Let's **analyze** this circuit to determine how this new resistor helps.

First, notice that the voltage at the **non**-inverting terminal is now **non-zero**!

The bias current I_{B2} means that, by virtue of KVL:

$$v_+ = 0 - R_3 I_{B2} = -R_3 I_{B2}$$

Now, because of the **virtual short**:

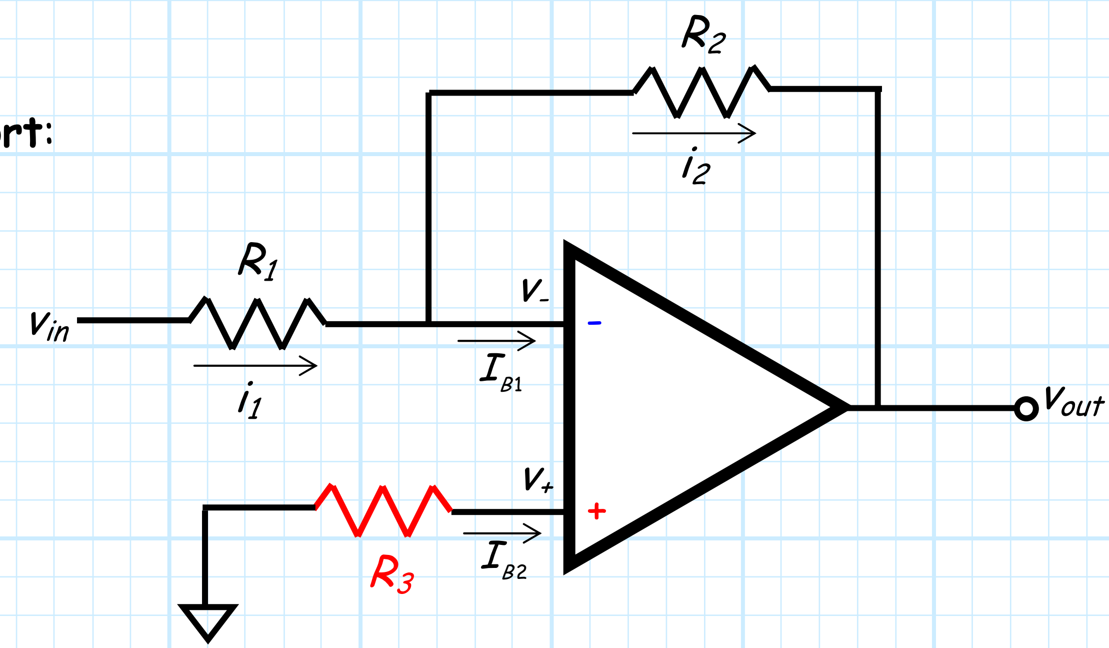
$$v_- = v_+ = -R_3 I_{B2}$$

And from KCL:

$$i_1 = i_2 + I_{B1}$$

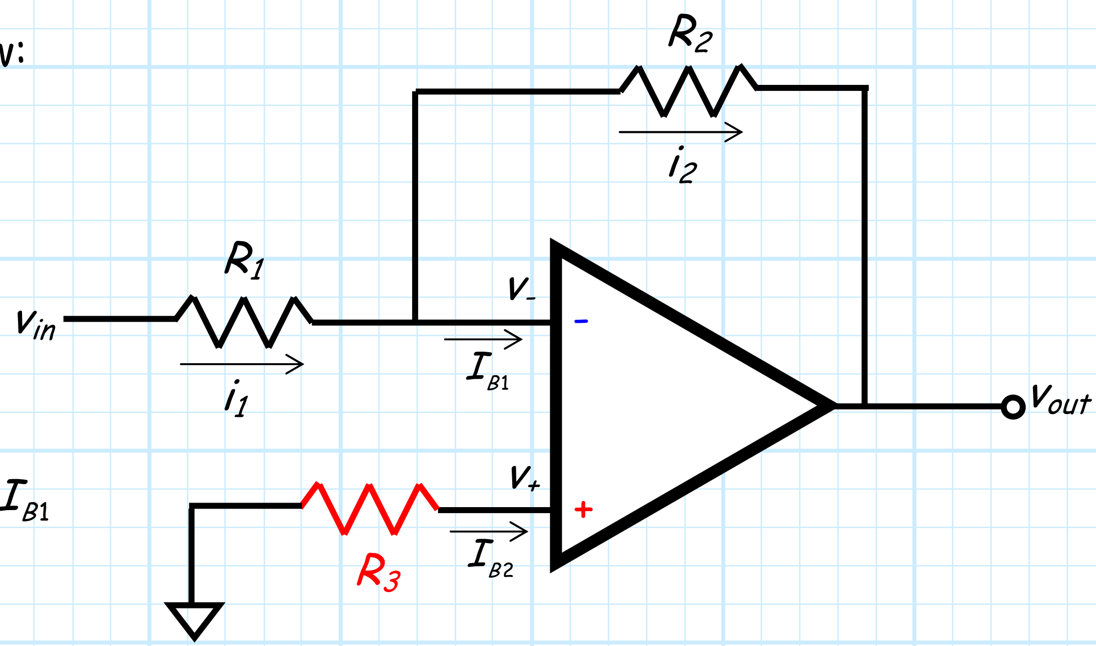
where from KCL and Ohm's Law:

$$i_1 = \frac{v_{in} - v_-}{R_1} = \frac{v_{in} + R_3 I_{B2}}{R_1}$$



It seems like this just made the offset even larger

And also from KCL and Ohm's Law:



$$i_2 = \frac{v_{in} - v_{out}}{R_2} = \frac{-(R_3 I_{B2} + v_{out})}{R_2}$$

Combining these results:

$$\frac{v_{in} + R_3 I_{B2}}{R_1} = \frac{-(R_3 I_{B2} + v_{out})}{R_2} + I_{B1}$$

Performing the usual algebraic gymnastics, we **rearrange** this result and find that the **output voltage** is:

$$v_{out} = \left(-\frac{R_2}{R_1}\right) v_{in} - \left(R_3 I_{B2} + \frac{R_2 R_3}{R_1} I_{B2} - R_2 I_{B1}\right)$$

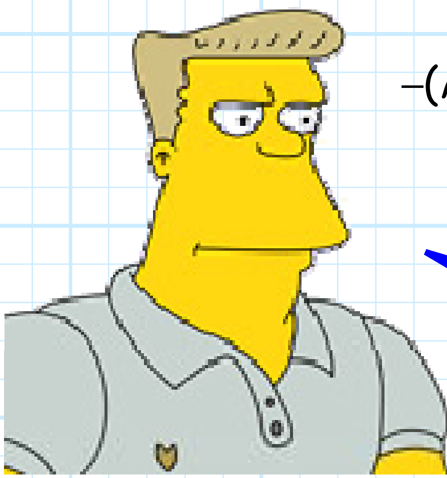
Awl be baak

Again we find the output consists of **two** terms. The first term is the **ideal** inverting amplifier result:

$$-\frac{R_2}{R_1} v_{in}$$

and the second is an output **D.C. offset**:

$$-\left(R_3 I_{B2} + \frac{R_2 R_3}{R_1} I_{B2} - R_2 I_{B1}\right)$$



Q: *Resistor R_3 was supposed to **reduce** the D.C. offset, but it seems to have made things even **worse**. Fix this or I shall be forced to pummel you.*

We must choose the proper value of R_3...

A: Say we **set** the value of resistor R_3 to equal $R_3 = R_1 \| R_2$, i.e.:

$$R_3 = \frac{R_1 R_2}{R_1 + R_2}$$

In this case, the **D.C. offset** becomes:

$$\begin{aligned} & -\left(\frac{R_1 R_2}{R_1 + R_2} I_{B2} + \frac{R_2^{\,2}}{R_1 + R_2} I_{B2} - R_2 I_{B1} \right) \\ &= -\left(\frac{(R_1 + R_2) R_2}{R_1 + R_2} I_{B2} - R_2 I_{B1} \right) \\ &= R_2 (I_{B1} - I_{B2}) \\ &= R_2 I_{os} \end{aligned}$$

Typically, the bias currents I_{B1} and I_{B2} are approximately equal, so that **offset current** $I_{B1} - I_{B2} = I_{os}$ is very **tiny**.

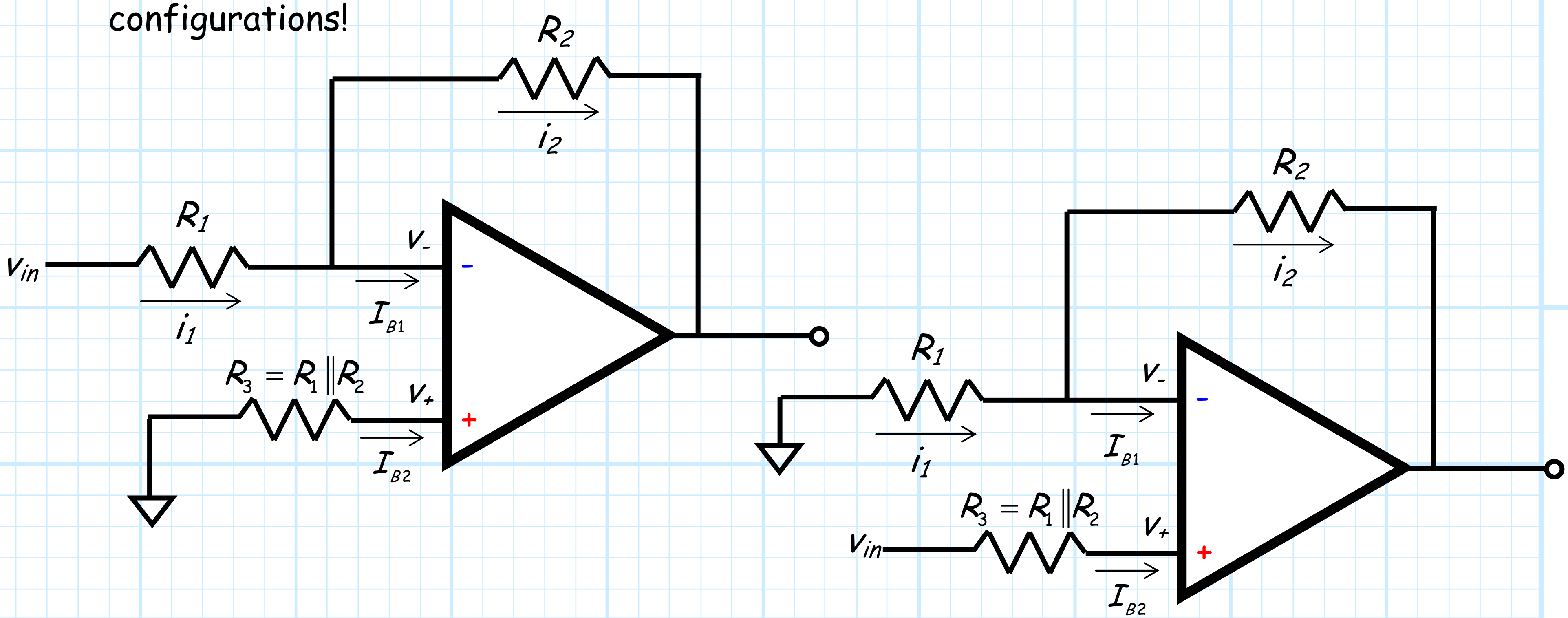
Therefore, the resulting output **offset voltage** is likewise very **tiny**!

…and this is that proper value

Therefore, when designing an amplifier with **real** op-amps, **always** include a resistor R_3 equal to the value:

$$R_3 = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

This is true **regardless** of whether we use the **inverting** or **non**-inverting configurations!



This is just the type of subtle point that shows up on an exam

If the impedances are **complex** (i.e., $Z_1(\omega)$ and $Z_2(\omega)$), then set the resistor R_3 based on the D.C. values of the impedances:

$$R_3 = Z_1(\omega = 0) \| Z_2(\omega = 0)$$

In other words, set the **capacitors** to **open** circuits and **inductors** to **short** circuits.

Real Op-Amp Input and Output Resistances

The **input** resistances of real op-amps are **very large**, but of course **not** infinite!

Typical values of input resistances range from several hundred **K** Ohms to tens of **Mega** Ohms.

As a result, there is a **small** amount of current flowing into **input** terminals of a real op-amp.

Q: *Well of course! We **just studied** this topic.*

*We already know that there is a **bias current** I_B flowing into (or out of) real op-amp terminals!*

A: This is true! However, there is an **additional** amount of current flowing into the input terminals. This current is **not** a constant bias current, but instead is directly **proportional** to the input terminal voltage.

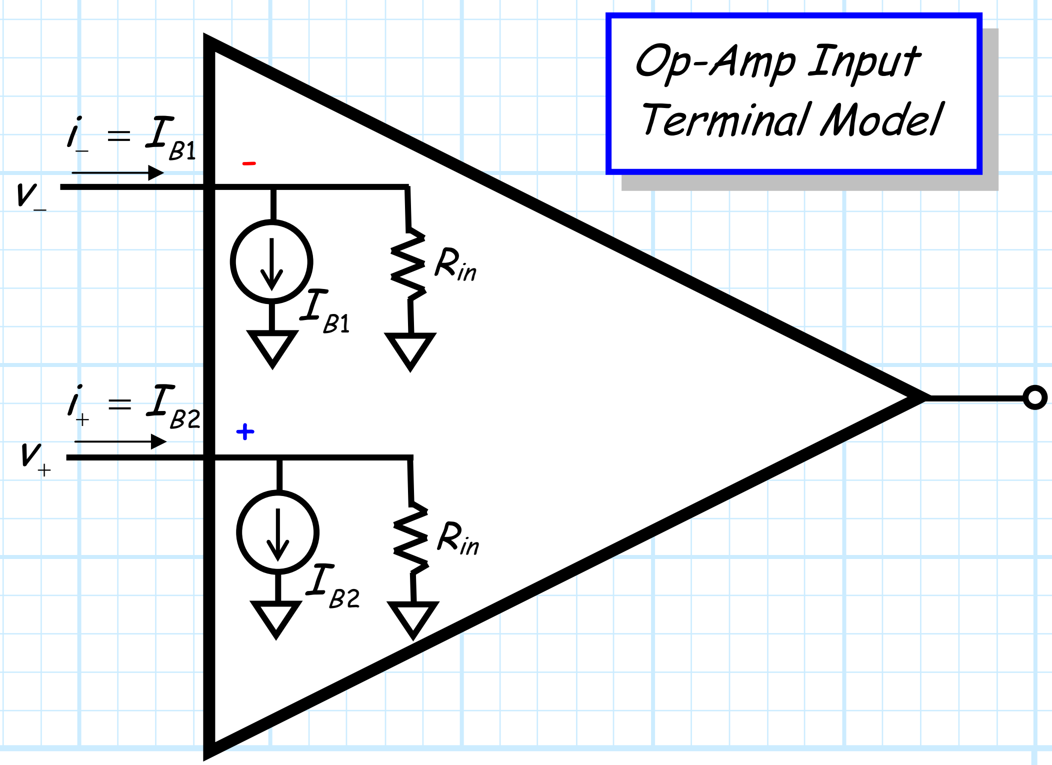
The input resistance is large, but finite

Because the **input resistance** is finite, the **total** current into real op-amp terminals are:

$$i_+ = I_{B2} + v_+ / R_{in}$$

$$i_- = I_{B1} + v_- / R_{in}$$

As such, our input terminal **circuit model** is:



Don't use resistors that are too large!

We find that the input current v_-/R_{in} or v_+/R_{in} will be **insignificant** (i.e., we can ignore its effect), provided that **all** other resistors used in an op-amp circuit are significantly **less** than the op-amp input resistance R_{in}.

Q: *But this would imply that we should* ***never*** *use resistor values greater than* ***100K*** *in our op-amp circuits!*

A: That's **exactly** right!

If the resistor values that **you** use in your op-circuit design are of the order of R_{in}, you may find that **your** circuit behaves quite **differently** from what you expected!

Worse even than finding haggis on the menu

Now let's examine the real values of op-amp **output** resistance.

Instead of the ideal value of zero, we find that the output resistances of real op-amps are **non-zero** (i.e., $R_{out}^{op} > 0$)!

Recall that the output resistance of **both** the inverting and non-inverting configurations is approximately equal to the op-amp output resistance (i.e., $R_{out} = R_{out}^{op}$).

Thus, we find that the **output resistance** of real inverting and non-inverting amplifiers are likewise **non-zero**!

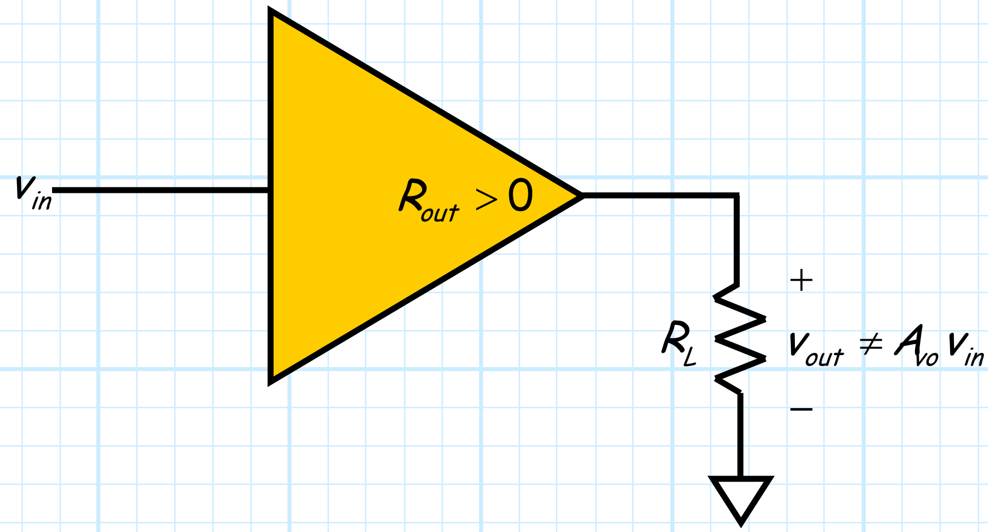
Q: **NO!** *The amplifier output resistance is **not** zero?!?*

*This means that the amplifier output will **not** be equal to the **open-circuit** voltage if a finite **load** is attached!*

A: This is absolutely correct!

Still, R_{out} is usually pretty darn small

Remember, the output voltage of an amplifier is equal to the input voltage times the **open-circuit** voltage gain **only** when the amplifier output is connected to an **open circuit**.



But, recall that the output voltage will be **approximately equal** to the open-circuit voltage **if** the output resistance is much **smaller** than the load resistance. I.E.,:

$$v_{out} \simeq A_{vo} v_{in} \quad \text{if} \quad R_{out} \ll R_L$$

Typical values of real op-amp output resistances are **less than 5 Ohms**!