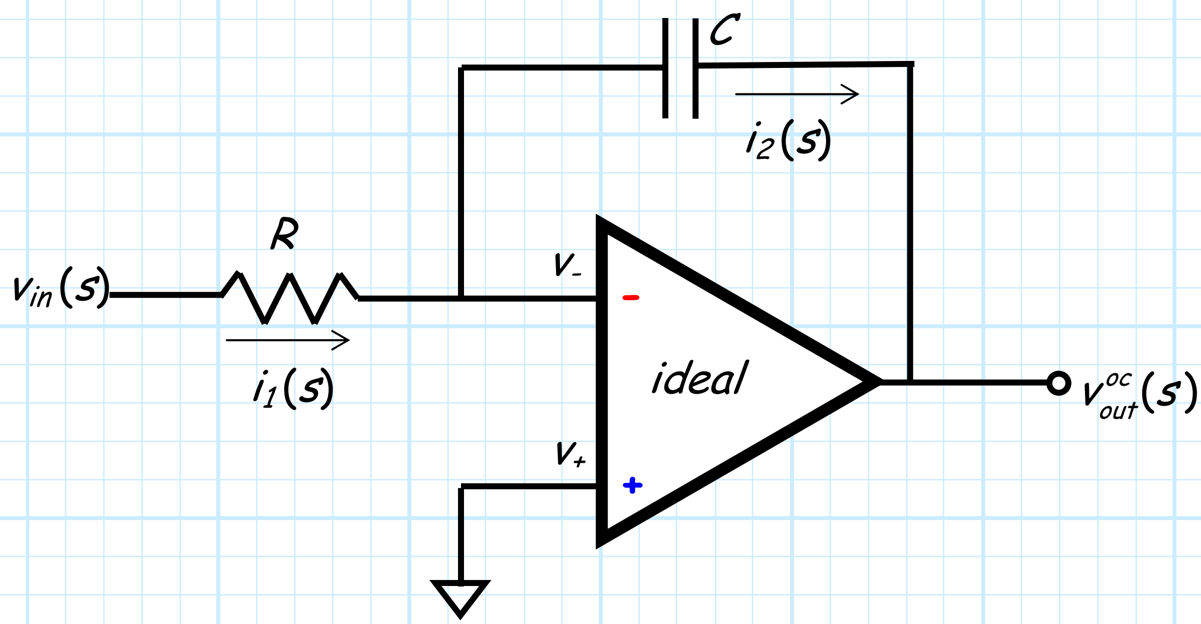
The Inverting Integrator

The circuit shown below is the inverting **integrator**.



It's the inverting configuration!

Since the circuit uses the **inverting** configuration, we can conclude that the circuit transfer function is:

$$G(s) = \frac{v_{out}^{oc}(s)}{v_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{(1/sC)}{R} = \frac{-1}{sRC}$$

In other words, the output signal is related to the input as:

$$v_{out}^{oc}(s) = \frac{-1}{RC}\,\frac{v_{in}(s)}{s}$$

From our knowledge of **Laplace Transforms**, we know this means that the output signal is proportional to the **integral** of the input signal!

The circuit integrates the input

Taking the **inverse** Laplace Transform, we find:

$$v_{out}^{oc}(t) = \frac{-1}{RC} \int_0^t v_{in}(t')\, dt'$$

For example, if the **input** is:

$$v_{in}(t) = \sin \omega t$$

then the **output** is:

$$v_{out}^{oc}(t) = \frac{-1}{RC} \int_0^t \sin \omega t \, dt' = \frac{-1}{RC} \frac{-1}{\omega} \cos \omega t = \frac{1}{\omega RC} \cos \omega t$$

Or, in the Fourier domain

We likewise could have determined this result using **Fourier Analysis** (i.e., frequency domain):

$$G(\omega) = \frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = -\frac{Z_2(\omega)}{Z_1(\omega)} = -\frac{\left(1/j\omega C\right)}{R} = \frac{j}{\omega RC}$$

Thus, the **magnitude** of the transfer function is:

$$|G(\omega)| = \left|\frac{j}{\omega RC}\right| = \frac{1}{\omega RC}$$

And since:

$$j = e^{j(\pi/2)} = \cos\left(\pi/2\right) + j\sin\left(\pi/2\right)$$

the **phase** of the transfer function is:

$$\angle G(\omega) = \pi/2 \ \text{ radians} = 90^\circ$$

Magnitude and phase

Given that:

$$\left|v_{out}^{oc}(\omega)\right| = \left|G(\omega)\right| \left|v_{in}(\omega)\right|$$

and:

$$\angle v_{out}^{oc}(\omega) = \angle G(\omega) + \angle v_{in}(\omega)$$

we find for the **input**:

$$v_{in}(t) = \sin\omega t$$

where:

$$\left|v_{in}(\omega)\right| = 1 \qquad \text{and} \qquad \angle v_{in}(\omega) = 0$$

that the **output** of the inverting integrator is:

$$\left|v_{out}^{oc}(\omega)\right| = \left|G(\omega)\right| \left|v_{in}(\omega)\right| = \frac{1}{\omega RC}$$

and:

$$\angle v_{out}^{oc}(\omega) = \angle G(\omega) + \angle v_{in}(\omega) = 90^{\circ} + 0 = 90^{\circ}$$

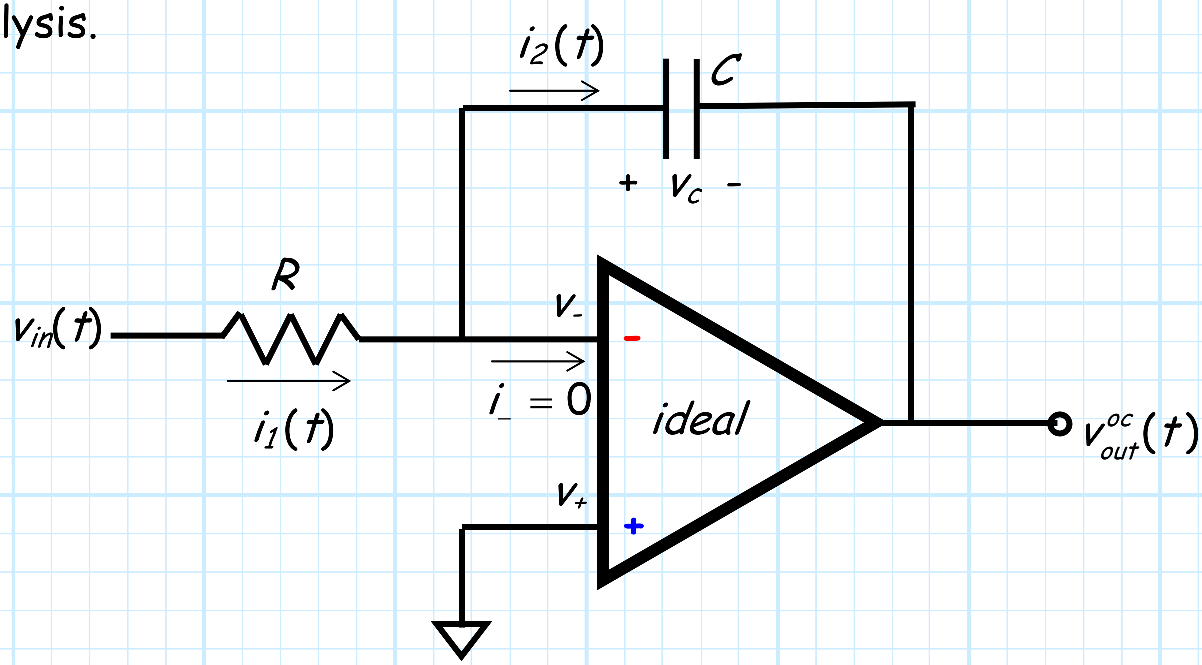
See, it's an integrator

Therefore:

$$v_{out}^{oc}(t) = \frac{1}{\omega RC} \sin\left(\omega t + 90^\circ\right)$$
$$= \frac{1}{\omega RC} \cos\omega t$$

Exactly the **same result** as before!

If you are still **unconvinced** that this circuit is an integrator, consider this **time-domain** analysis.



The time-domain solution

From our elementary **circuits knowledge**, we know that the voltage across a capacitor is:

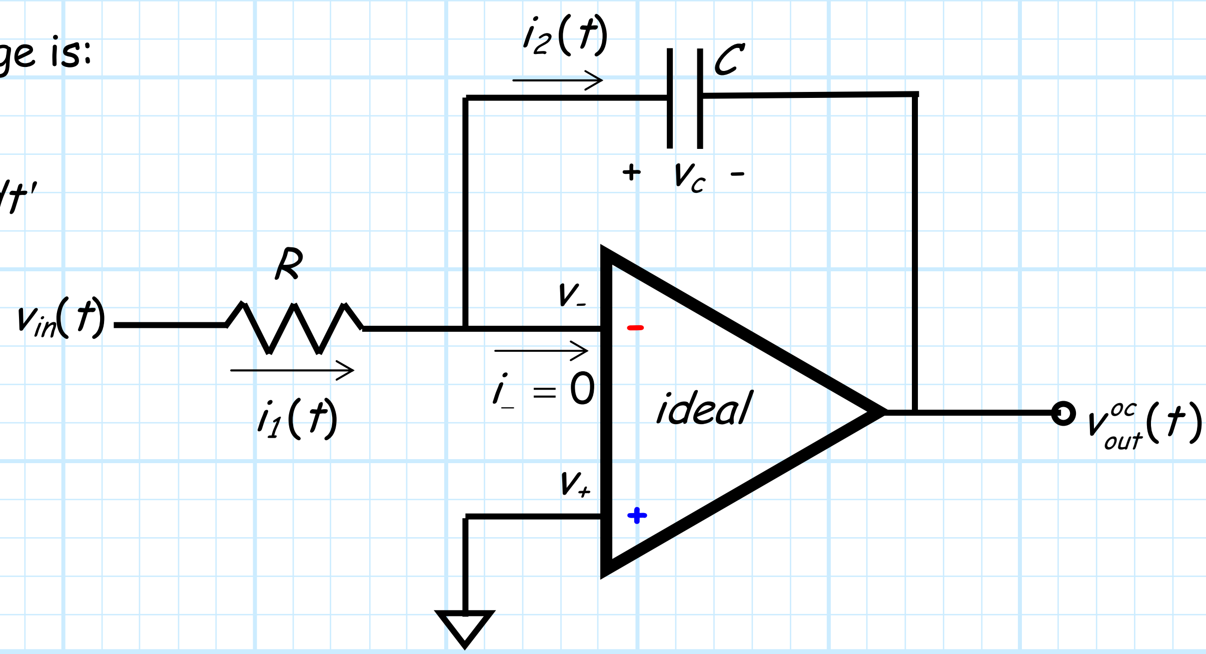
$$v_c(t) = \frac{1}{C}\int_0^t i_2(t')\,dt'$$

and from the circuit we see that:

$$v_c(t) = v_-(t) - v_{out}^{oc}(t) = -v_{out}^{oc}(t)$$

therefore the **output** voltage is:

$$v_{out}^{oc}(t) = -\frac{1}{C}\int_0^t i_2(t')\,dt'$$



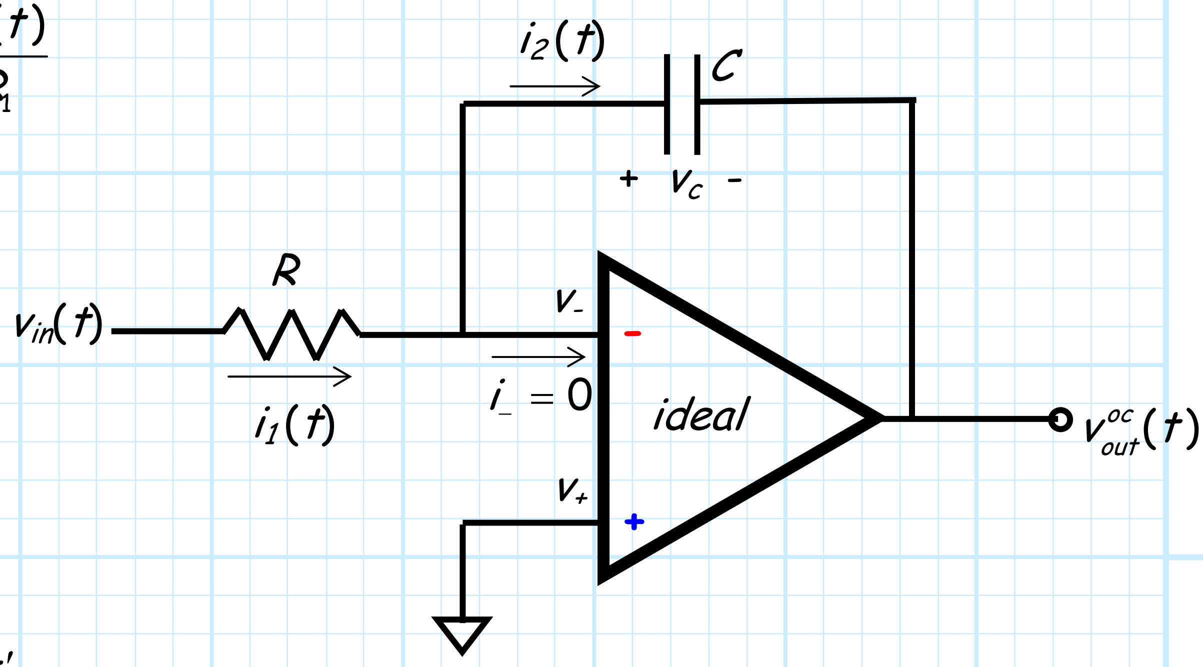
The same result no matter how we do it!

From KCL, we likewise know that:

$$i_1(t) = i_2(t)$$

and from Ohm's Law:

$$i_1(t) = \frac{v_{in}(t) - v_-(t)}{R_1} = \frac{v_{in}(t)}{R_1}$$



Therefore:

$$i_2(t) = \frac{v_{in}(t)}{R_1}$$

and thus:

$$v_{out}^{oc}(t) = \frac{-1}{C}\int_0^t i_2(t')\,dt'$$

$$= \frac{-1}{RC}\int_0^t v_{in}(t')\,dt'$$

The **same** result as before!