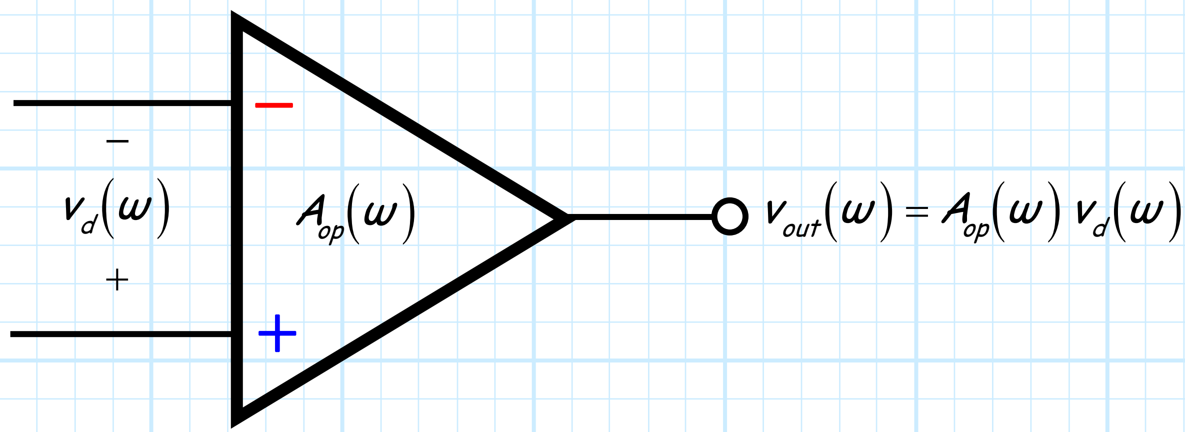
The Gain of Real Op-Amps

The **open-circuit** voltage gain A_{op} (a **differential** gain!) of a **real (i.e., non-ideal)** operational amplifier is **very** large at D.C. (i.e., $\omega = 0$), but gets **smaller** as the signal frequency ω **increases**!

In other words, the **differential** gain of an op-amp (i.e., the **open-loop** gain of a feedback amplifier) is a function of frequency ω.

We will thus express this gain as a **complex** function in the **frequency domain** (i.e., $A_{op}(\omega)$).



Gain is a complex function frequency

Typically, this op-amp behavior can be described mathematically with the **complex** function:

$$A_{op}(\omega) = \frac{A_0}{1 + j\left(\omega / \omega_b\right)}$$

or, using the frequency definition $\omega = 2\pi f$, we can write:

$$A_{op}(f) = \frac{A_0}{1 + j\left(f / f_b\right)}$$

where ω is frequency expressed in units of **radians/sec**, and f is signal frequency expressed in units of **cycles/sec**.

DC is when the signal frequency is zero

Note the squared **magnitude** of the op-amp gain is therefore the **real** function:

$$\begin{aligned}\left|A_{op}(\omega)\right|^2 &= \frac{A_0}{1+j\left(\omega/\omega_b\right)}\,\frac{A_0}{1-j\left(\omega/\omega_b\right)} \\ &= \frac{A_0^2}{1+\left(\omega/\omega_b\right)^2}\end{aligned}$$

Therefore at **D.C.** $(\omega = 0)$ the op-amp gain is:

$$A_{op}(\omega=0) = \frac{A_0}{1+j\left(0/\omega_b\right)} = A_0$$

and thus:

$$\left|A_{op}(\omega=0)\right|^2 = A_0^2$$

Where:

$A_0 =$ op-amp D.C. gain

The break frequency

Again, note that the D.C. gain A_0 is:

1) an **open-circuit** voltage gain
2) a **differential** gain
3) also referred to as the **open-loop** D.C. gain

Q: *So just what **does** the value ω_b indicate ?*

A: The value ω_b is the op-amp's **break frequency**.

Typically, this value is very **small** (e.g. $f_b = 10Hz$).

The 3dB bandwidth

To see **why** this value is **important**, consider the op-amp gain at $\omega = \omega_b$:

$$A_{op}\left(\omega = \omega_b\right) = \frac{A_0}{1 + j\left(\omega/\omega_b\right)} = \frac{A_0}{1+j} = \frac{A_0}{2} - j\,\frac{A_0}{2} = \frac{\left|A_0\right|}{\sqrt{2}}\, e^{-j\pi/4}$$

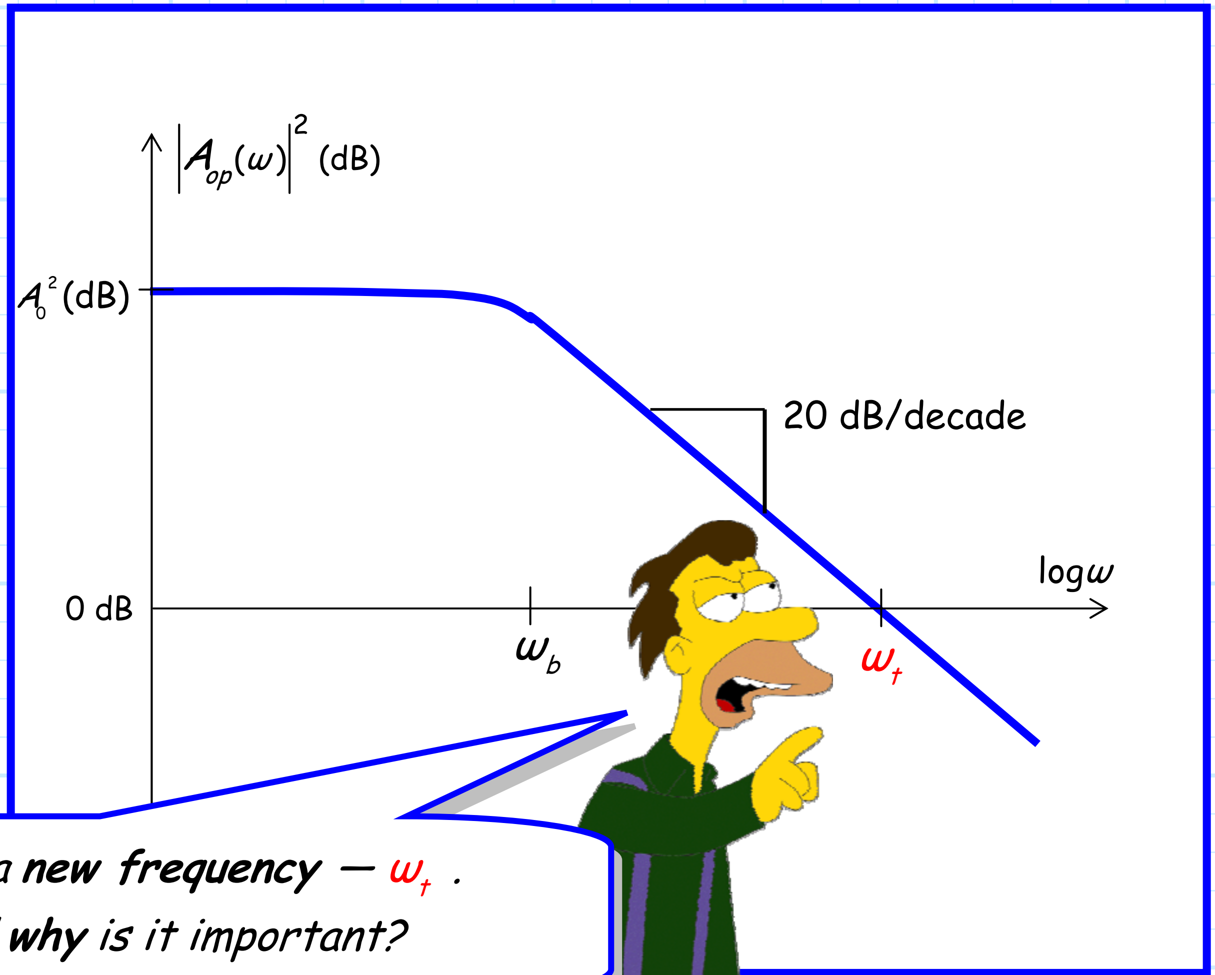
The **squared magnitude** of this gain is therefore:

$$\left|A_{op}(\omega = \omega_b)\right|^2 = \frac{A_0}{1+j}\,\frac{A_0}{1-j} = \frac{A_0^{\,2}}{1-j^2} = \frac{A_0^{\,2}}{2}$$

As a result, the **break** frequency ω_b is also referred to as the "**half-power**" frequency, or the "**3 dB**" frequency.

This value is very important!

If we **plot** $\left|A_{op}(\omega)\right|^2$ on a "**log-log**" scale, we get something like this:



Q: *Hey! You have defined a **new frequency** — ω_t .*
***What** is this frequency and **why** is it important?*

The unity gain frequency

A: Note that ω_t is the frequency where the magnitude of the gain is "**unity**" (i.e., where the gain is **1**). I.E.,

$$\left|A_{op}(\omega = \omega_t)\right|^2 = 1$$

Note that when expressed in dB, **unity** gain is:

$$10 \log_{10} \left|A_{op}(\omega = \omega_t)\right|^2 = 10 \log_{10} (1) = 0 \text{ dB}$$

Therefore, on a "log-log" plot, the gain curve crosses the **horizontal axis** at frequency ω_t.

We thus refer to the frequency ω_t as the "**unity-gain frequency**" of the operational amplifier.

It's the product of the gain and the bandwidth!

Note that we can **solve** for this frequency in terms of **break frequency** ω_b and **D.C. gain** A_o:

$$1 = \left| A_{op}(\omega = \omega_t) \right|^2 = \frac{A_0^2}{1 + \left(\omega_t / \omega_b \right)^2}$$

meaning that:

$$\omega_t^2 = \omega_b^2 \left(A_0^2 - 1 \right)$$

But recall that $A_0 \gg 1$, therefore $A_0^2 - 1 \approx A_0^2$ and:

$$\omega_t = \omega_b \left| A_0 \right|$$

Note since the frequency ω_b defines the 3 dB **bandwidth** of the op-amp, the unity gain frequency ω_t is simply the **product** of the op-amp's D.C. **gain** $\left| A_0 \right|$ and its **bandwidth** ω_b.

It's not rocket science!

As a result, ω_t is **alternatively** referred to as the **gain-bandwidth product**!

$$\omega_t \doteq \textbf{Unity Gain Frequency}$$

and

$$\omega_t \doteq \textbf{Gain - Bandwidth Product}$$

*This is so **simple** perhaps even I can remember it:*

*The **gain-bandwidth-product** is the **product** of the **gain** and the **bandwidth**!*