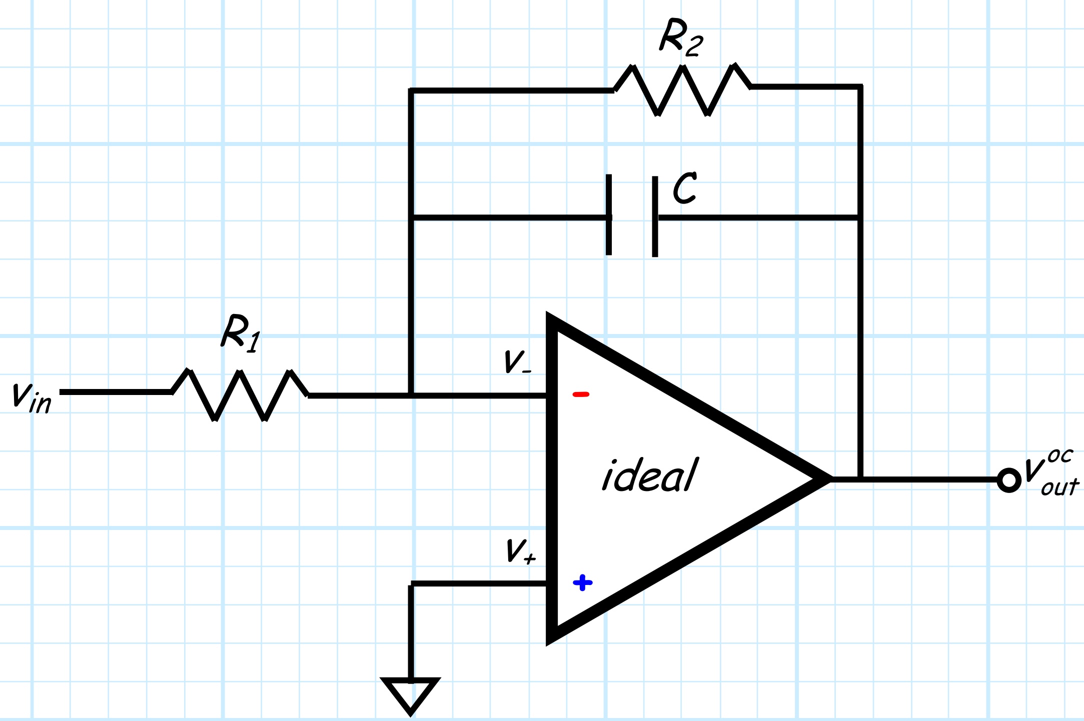
Example: An Inverting Network

Now let's determine the complex **transfer function** of this circuit:



It's the inverting configuration!

Note this circuit uses the **inverting** configuration, so that:

$$G(\omega) = -\frac{Z_2(\omega)}{Z_1(\omega)}$$

where $Z_1 = R_1$, and:

$$Z_2 = R_2 \parallel {}^{1}\!/\!_{j\omega C} = \frac{R_2}{1 + j\omega R_2 C}$$

Therefore, the **transfer function** of this circuit is:

$$G(\omega) = \frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = -\frac{R_2}{R_1}\,\frac{1}{1 + j\omega R_2 C}$$

Another low-pass filter

Thus, the transfer function **magnitude** is:

$$|G(\omega)|^2 = \left(-\frac{R_2}{R_1}\right)^2 \frac{1}{1+\left(\omega/\omega_0\right)^2}$$

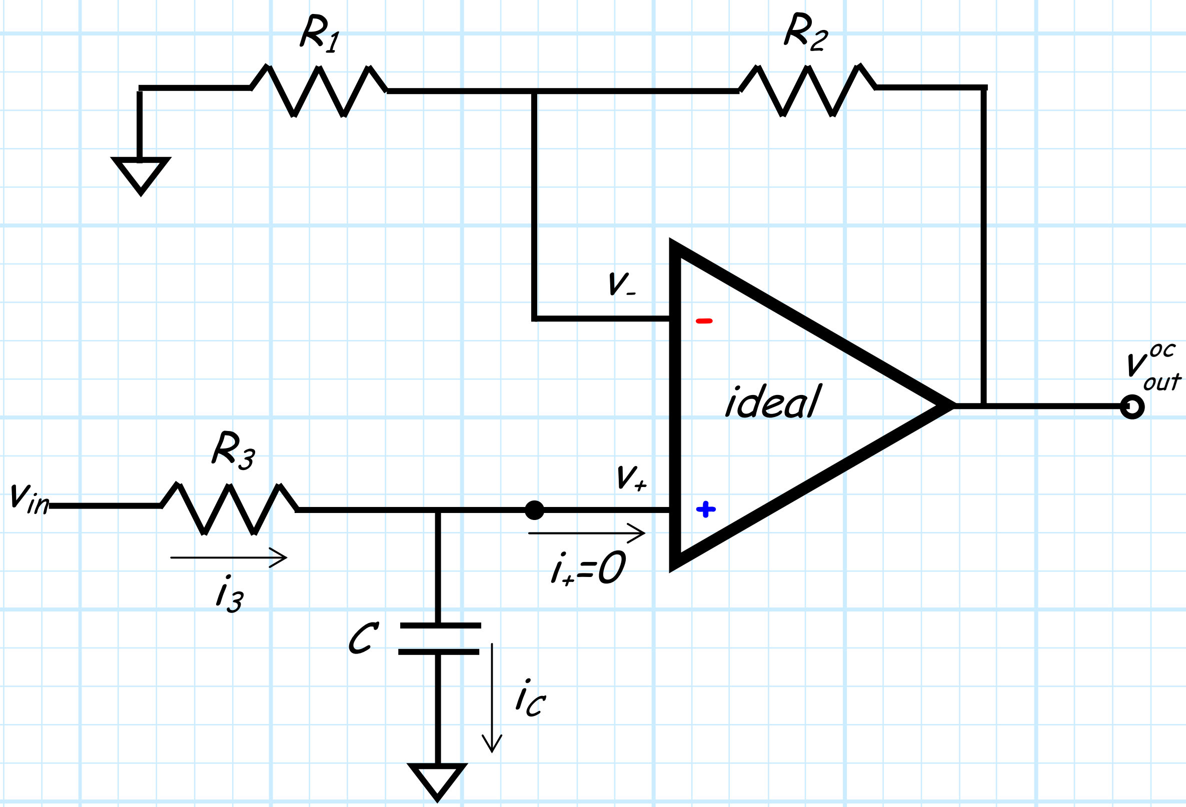
where:

$$\omega_0 = \frac{1}{R_2 C}$$

Thus, just as with the previous example, this circuit is a **low-pass filter**, with **cutoff** frequency ω_0 and pass-band **gain** $\left(R_2/R_1\right)^2$.

Example: A Non-Inverting Network

Let's determine the **transfer function** $G(\omega) = v_{out}^{oc}(\omega) / v_{in}(\omega)$ for the following circuit:



Some enjoyable circuit analysis

From **KCL**, we know:

$$i_3(\omega) = i_C(\omega) + i_+(\omega) = i_C(\omega) + 0 = i_C(\omega)$$

where:

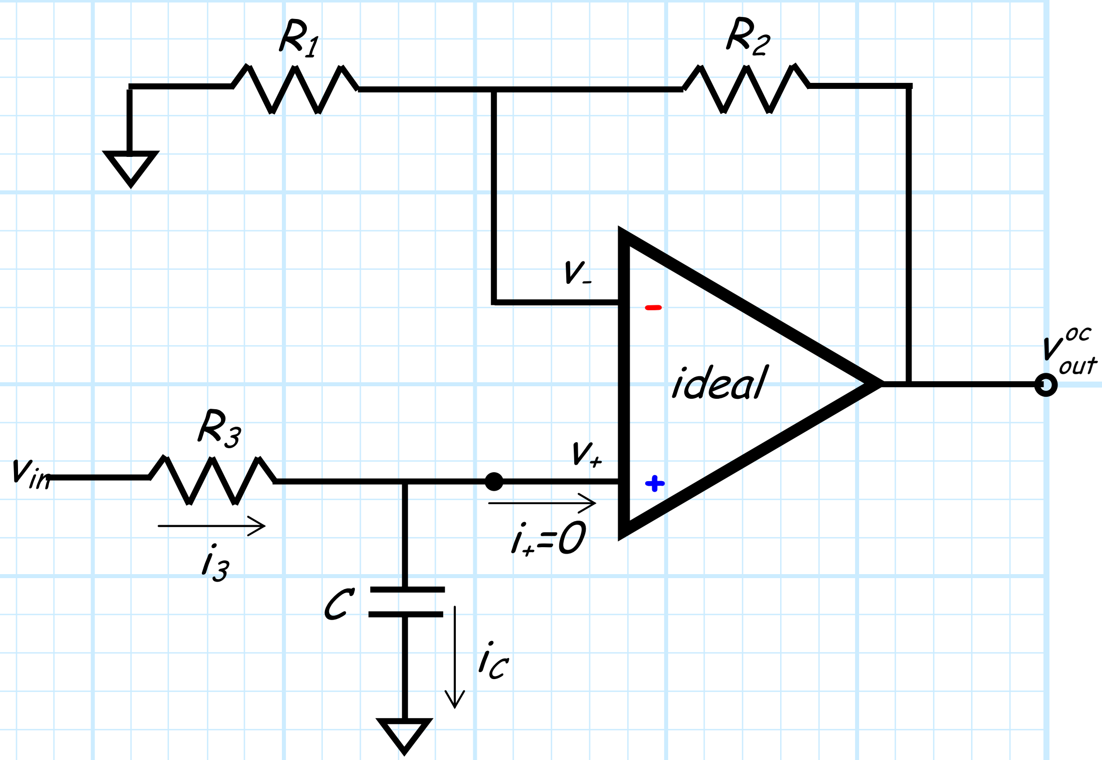
$$i_3(\omega) = \frac{v_{in}(\omega) - v_+(\omega)}{R_3} \quad \text{and} \quad i_C(\omega) = \frac{v_+(\omega) - 0}{\left(1/j\omega C\right)} = j\omega C \, v_+(\omega)$$

Equating, we find an expression involving $v_{in}(\omega)$ and $v_2(\omega)$ only:

$$\frac{v_{in}(\omega) - v_+(\omega)}{R_3} = j\omega C \, v_+(\omega)$$

and performing a little **algebra**, we find:

$$v_2(\omega) = \frac{v_{in}(\omega)}{1 + j\omega R_3 C}$$

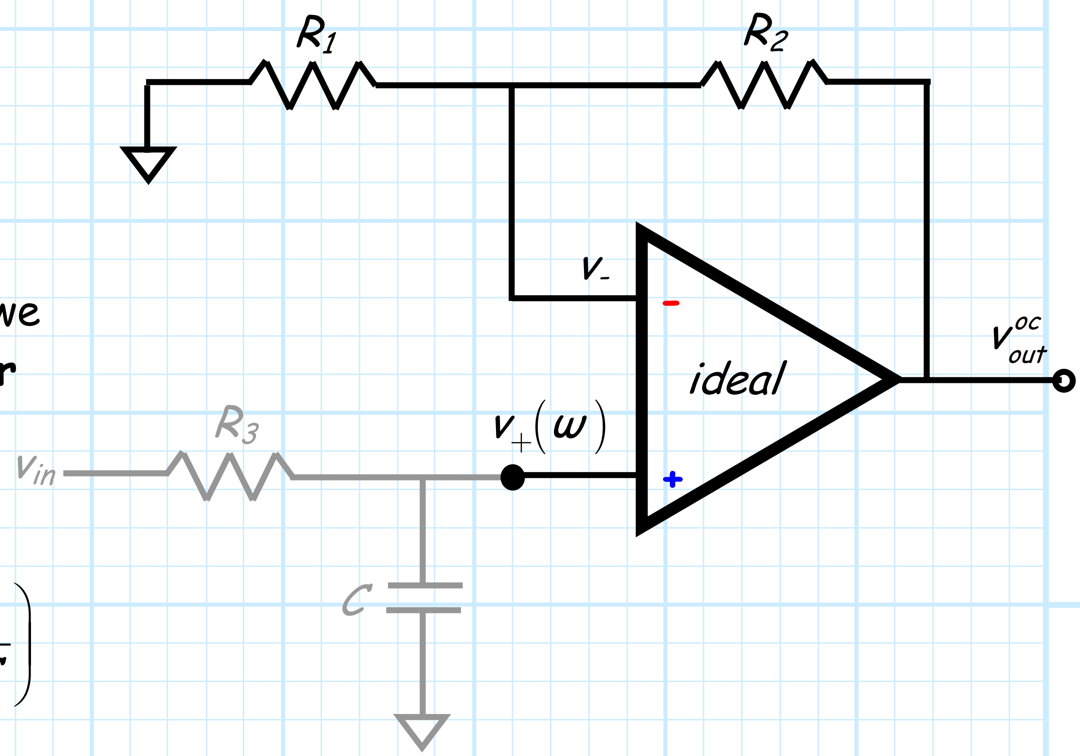


No need to go further: we have a template!

The remainder of the circuit is simply the **non-inverting amplifier** that we studied earlier.

We know that:

$$v_{out}^{oc}(\omega) = \left(1 + \frac{R_2}{R_1}\right) v_+(\omega)$$



Combining these two relationships, we can determine the **complex transfer function** for this circuit:

$$G(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + j\omega R_3 C}\right)$$

It's a low-pass filter!!!

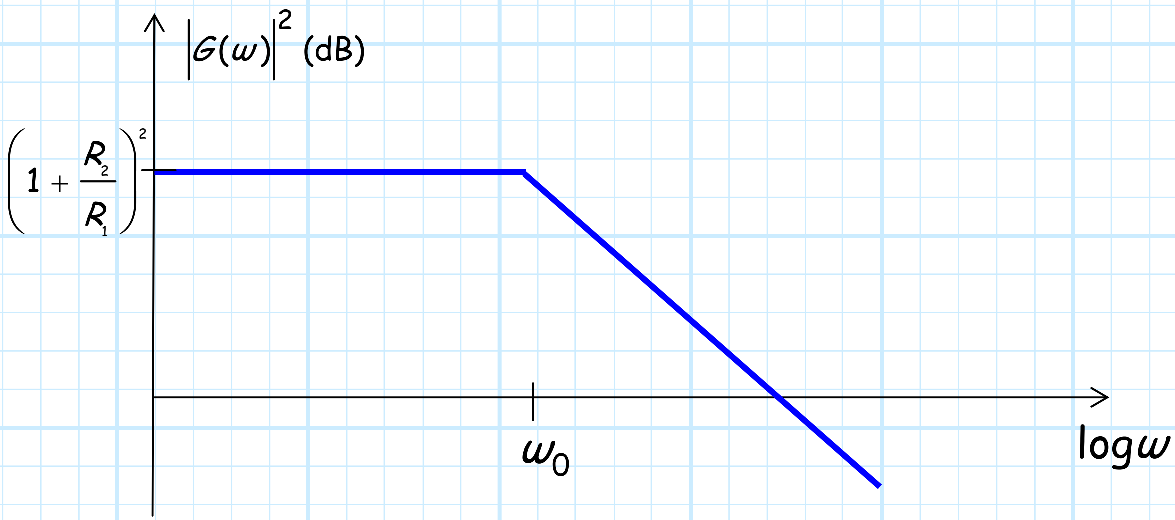
The **magnitude** of this transfer function is therefore:

$$|G(\omega)|^2 = \left(1+\frac{R_2}{R_1}\right)^2 \frac{1}{1+\left(\omega/\omega_0\right)^2}$$

where:

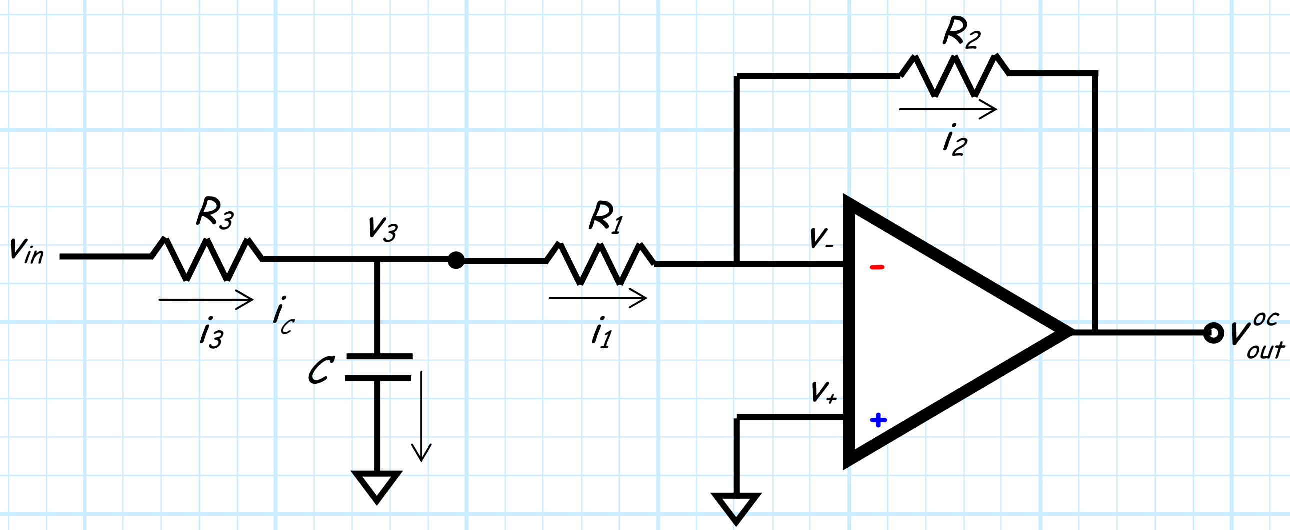
$$\omega_0 = \frac{1}{R_3 C}$$

This is a **low-pass filter**—one with **pass-band gain**!



Example: Another Inverting Network

Consider now the transfer function of **this** circuit:

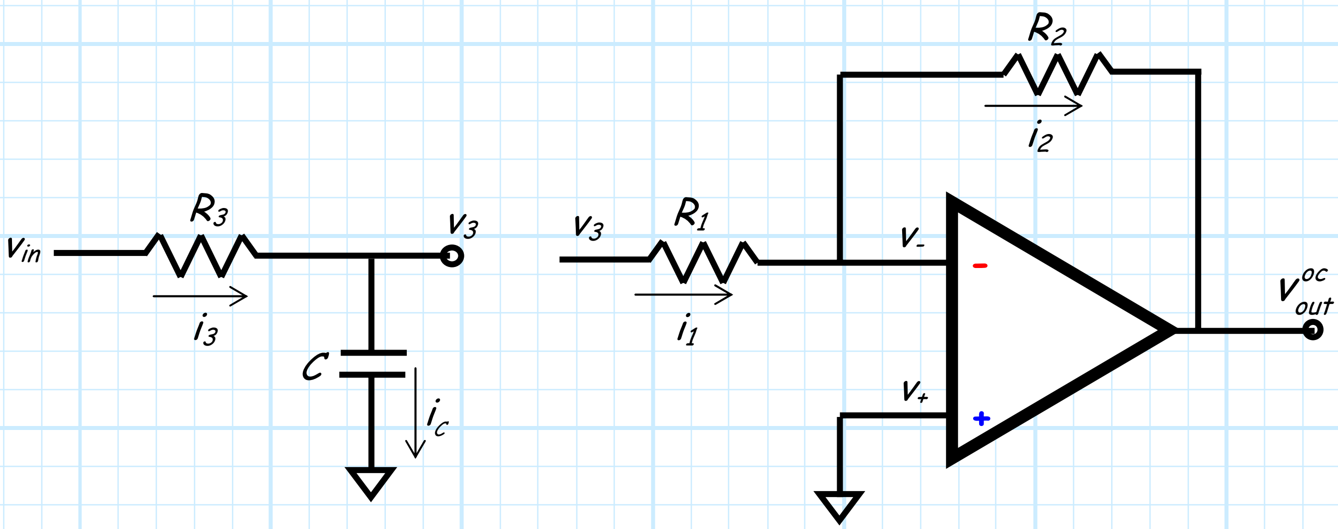


Some more enjoyable circuit analysis

To accomplish this analysis, we must first…

__Wait!__ You don't need to explain this to __me__.

It is obvious that we can divide this is circuit into __two__ pieces—the first being a complex __voltage divider__ and the second a __non-inverting__ amplifier.



Can we analyze the circuit this way?

The transfer function of the complex voltage divider is:

$$\frac{v_3(\omega)}{v_{in}(\omega)} = \frac{1/j\omega C}{R_3 + 1/j\omega C} = \frac{1}{1 + j\omega R_3 C}$$

and that of the inverting amplifier:

$$\frac{v_{out}^{oc}(\omega)}{v_3(\omega)} = -\frac{R_2}{R_1}$$

And so of course ***I*** *have correctly determined that the transfer function of this circuit is:*

Joe Marquette / AP file

$$\frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = \frac{v_{out}^{oc}(\omega)}{v_3(\omega)} \frac{v_3(\omega)}{v_{in}(\omega)} = -\frac{R_2}{R_1} \frac{1}{1 + j\omega R_3 C}$$

No, we cannot

NO! This is **not** correct:

$$\frac{v_o(\omega)}{v_i(\omega)} \neq -\frac{R_2}{R_1}\frac{1}{1+j\omega R_3 C}$$

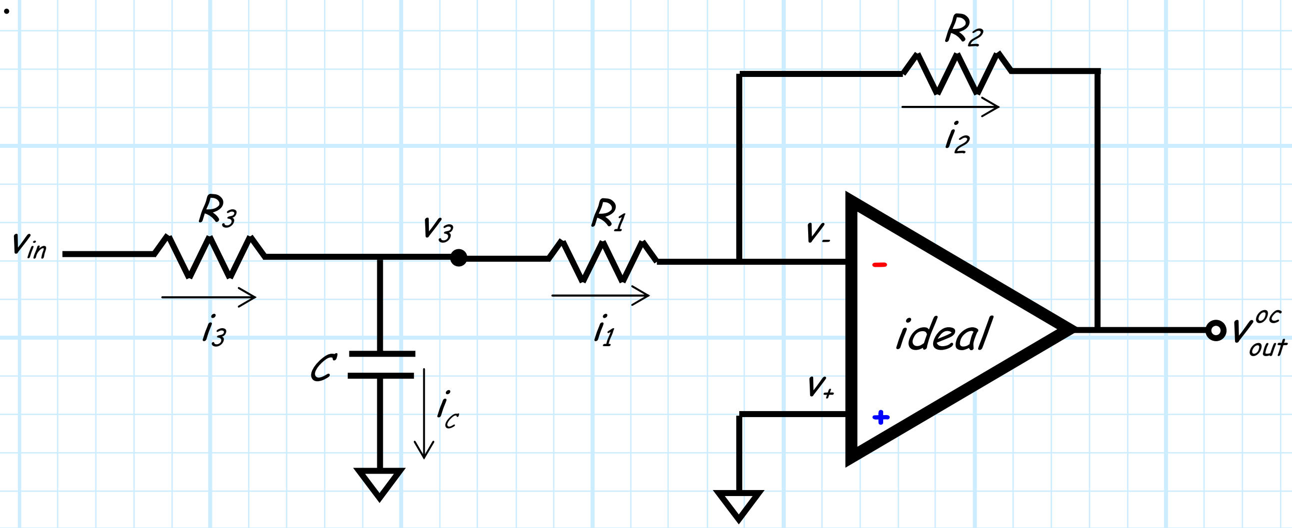
The problem with the above "analysis" is that we **cannot** apply **this** complex voltage divider equation to determine $v_3(\omega)$:

$$v_3(\omega) \neq \frac{1/j\omega C}{R_3 + 1/j\omega C} v_{in}(\omega)$$

The reason of course is that the output of this voltage divider is **not** open-circuited, and thus current $i_3(\omega) \neq i_c(\omega)$.

My computer suspiciously crashed while writing this (really, it did!)

We **cannot** divide this circuit into two independent pieces, we must analyze it as **one** circuit.

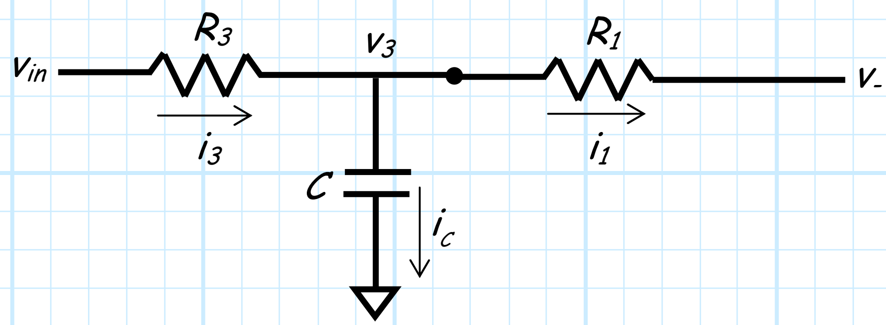


*Of course what I **meant** to say was that we should determine the **impedance** Z_1 of input network, and **then** use the inverting configuration equation* $T(\omega) = -Z_2/Z_1$.

An even worse idea than Vista

NO! This idea is as bad as the last one!

We **cannot** specify an impedance for the input network:

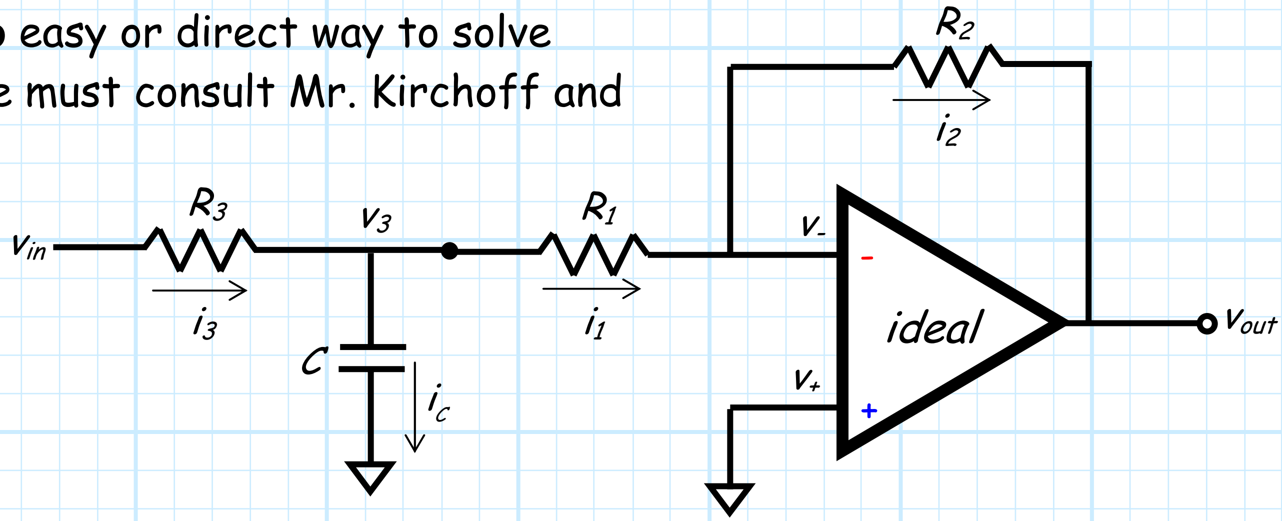


After all, would we define this impedance as:

$$Z_1 = \frac{v_{in} - v_-}{i_3} \quad \text{or} \quad Z_1 = \frac{v_{in} - v_-}{i_1} \quad ???$$

Don't look for templates: trust what you know

So, there is **no** easy or direct way to solve this circuit, we must consult Mr. Kirchoff and his laws!



We know that $i_1 = i_2$, where:

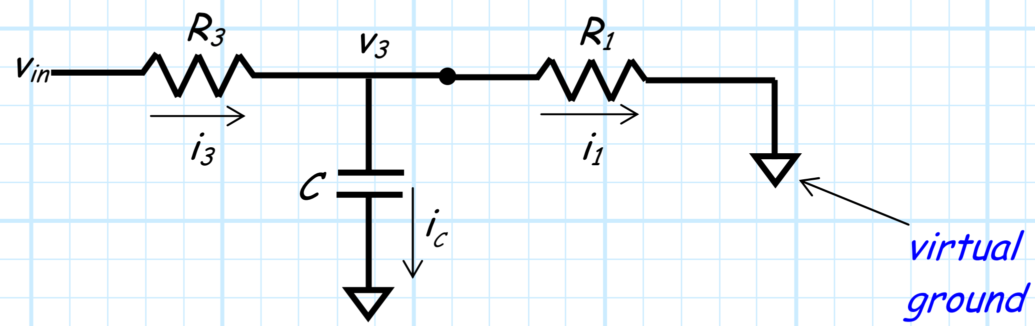
$$i_1 = \frac{v_3 - v_-}{R_1} = \frac{v_3}{R_1} \qquad \text{and} \qquad i_2 = \frac{v_+ - v_{out}}{R_2} = \frac{-v_{out}}{R_2}$$

Combining these equations, we get the **expected** result:

$$v_{out} = -\frac{R_2}{R_1} v_3$$

Don't forget virtual ground!

We must therefore determine v_3 in terms of v_i:



Note R_1 and C are connected in **parallel**!

Thus, from **voltage division**, we find:

$$v_3 = \frac{R_1 \parallel \frac{1}{j\omega C}}{R_3 + \left(R_1 \parallel \frac{1}{j\omega C}\right)} v_{in}$$

where:

$$R_1 \parallel \frac{1}{j\omega C} = \frac{R_1\left(\frac{1}{j\omega C}\right)}{R_1 + \frac{1}{j\omega C}} = \frac{R_1}{1 + j\omega R_1 C}$$

The Eigen value at last!

Performing some **algebra**, we find:

$$v_3 = \left(\frac{R_1}{(R_1 + R_3) + j\omega R_1 R_3 C} \right) v_{in}$$

and since:

$$v_{out} = \frac{-R_2}{R_1} v_3$$

we **finally** discover that:

$$G(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \left(\frac{-R_2}{(R_1 + R_3) + j\,\omega R_1 R_3 C} \right)$$

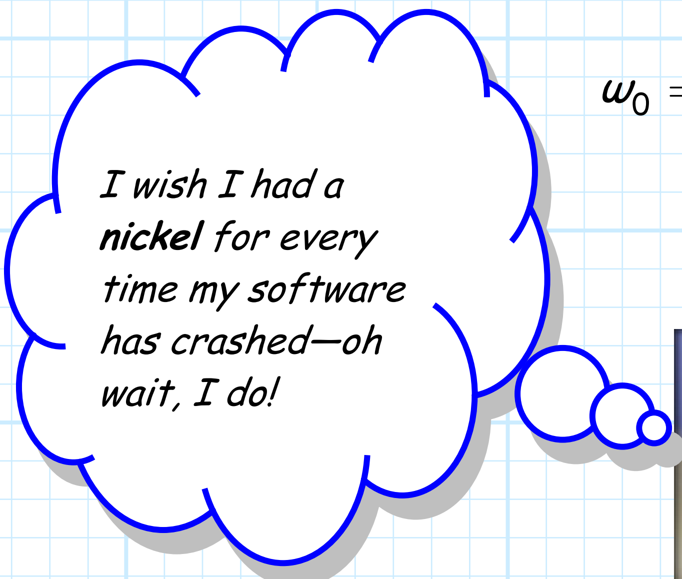
This again is a low-pass filter

We can rearrange this transfer function to find that this circuit is likewise a **low-pass filter** with **pass-band gain**:

$$G(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \frac{-R_2}{R_1 + R_3}\left(\frac{1}{1 + j\left(\omega / \omega_o\right)}\right)$$

where the **cutoff frequency** ω_0 is:

$$\omega_0 = \frac{1}{\left(\frac{R_1 R_3}{R_1 + R_3}\right) C} = \frac{1}{\left(R_1 \| R_3\right) C}$$



Example: A Complex Processing Circuit using the Inverting Configuration

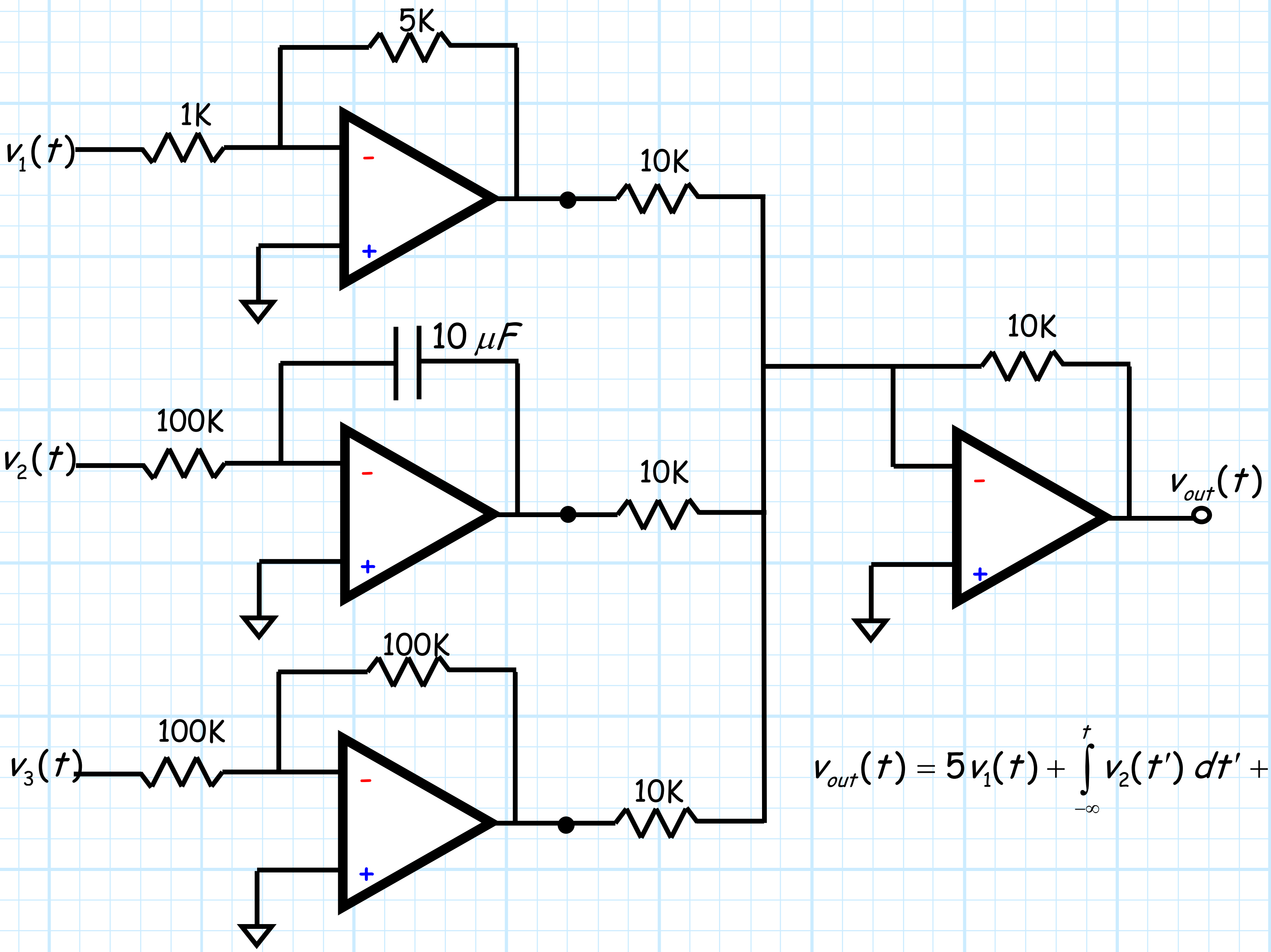
Note that we can combine inverting amplifiers to form a more **complex** processing system.

For **example**, say we wish to take **three** input signals $v_1(t)$, $v_2(t)$, and $v_3(t)$, and process them such that the open-circuit output voltage is:

$$v_{out}(t) = 5\, v_1(t) + \int_{-\infty}^{t} v_2(t')\, dt' + \frac{d\, v_3(t)}{dt}$$

Assuming that we use **ideal** (or near ideal) op-amps, with an **output resistance equal to zero** (or at least very small), we can realize the above signal processor with the following circuit:

This circuit performs this operation!



$$v_{out}(t) = 5v_1(t) + \int_{-\infty}^{t} v_2(t')\,dt' + \frac{d\,v_3(t)}{dt}$$