2.8 Integrators and Differentiators

Reading **Assignment**: 105-113

Op-amp circuits can also (and often do) implement reactive elements such as inductors and capacitors.

HO: OP-AMP CIRCUITS WITH REACTIVE ELEMENTS

One important op-amp circuit is the inverting differentiator.

HO: THE INVERTING DIFFERENTIATOR

Likewise the inverting integrator.

HO: THE INVERTING INTEGRATOR

HO: AN APPLICATION OF THE INVERTING INTEGRATOR

Let's do some examples of op-amp circuit analysis with reactive elements.

EXAMPLE: A NON-INVERTING NETWORK

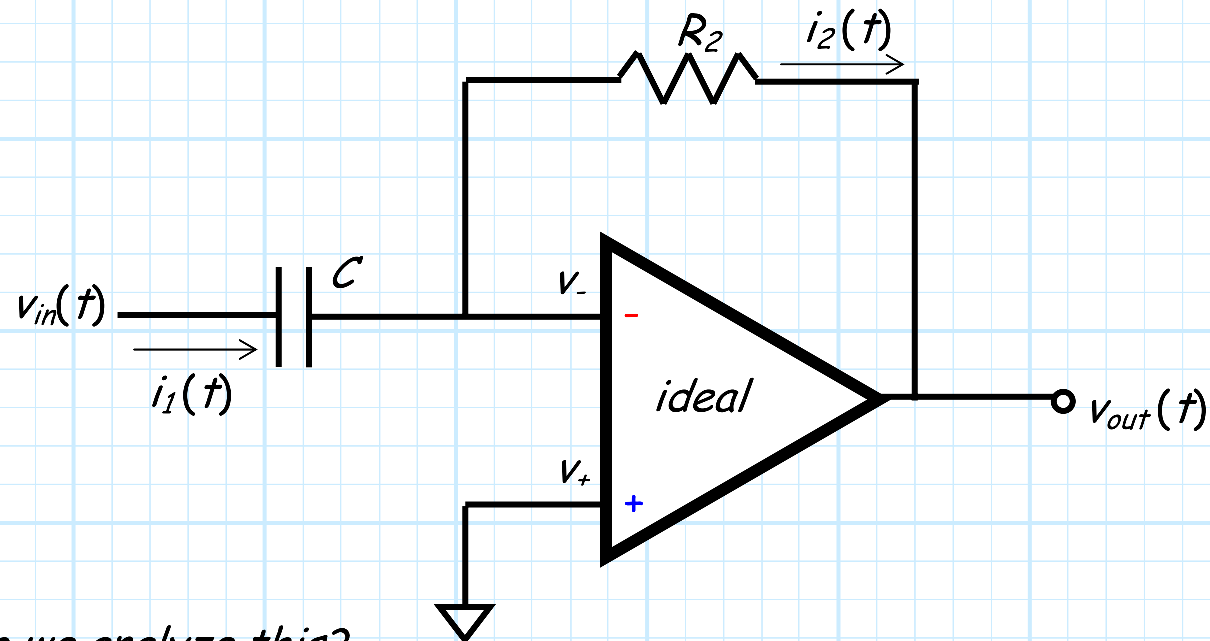
EXAMPLE: AN INVERTING NETWORK

Example: Another Inverting Network

Example: A Complex Processing Circuit

Op-Amp circuits with reactive elements

Now let's consider the case where the op-amp circuit includes **reactive elements**:



Q: *Yikes! **How** do we analyze this?*

A: Don't panic! Remember, the relationship between v_{out} and v_{in} is **linear**, so we can express the output as a **convolution**:

$$v_{out}(t) = \mathcal{L}\left[v_{in}(t)\right] = \int_{-\infty}^{t} g(t-t')\, v_{in}(t')\, dt'$$

Just find the Eigen value

Q: *I'm **still** panicking—**how** do we determine the impulse response $g(t)$ of this circuit?*

A: Say the input voltage $v_{in}(t)$ is an **Eigen function** of linear, time-invariant systems:

$$v_{in}(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t}$$

Then, the **output** voltage is just a **scaled** version of this **input**:

$$v_{out}(t) = \mathcal{L}\left[e^{-st}\right] = \int_{-\infty}^{t} g(t-t')\, e^{-st} dt' = G(s)\, e^{-st}$$

where the "scaling factor" $G(s)$ is the complex **Eigen value** of the linear operator $\mathcal{L}$.

Express the input as a superposition of eigen values (i.e., the Laplace transform)

Q: *First of all, **how** could the input (and output) be this **complex** function e^{st}? Voltages are **real-valued**!*

A: True, but the **real-valued** input and output functions can be expressed as a weighted superposition of these **complex** Eigen functions!

$$v_{in}(s) = \int_0^{+\infty} v_{in}(t)\, e^{-st}\, dt$$

The **Laplace transform**→

$$v_{out}(s) = \int_0^{+\infty} v_{out}(t)\, e^{-st}\, dt$$

Such that:

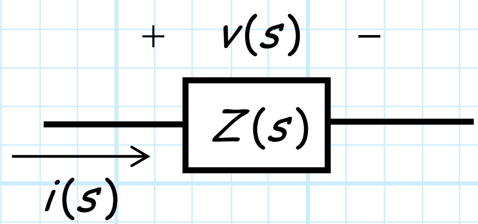
$$v_{out}(s) = G(s)\, v_{in}(s)$$

Find the eigen value from circuit theory and impedance

Q: *Still, I don't know **how** to find the eigen value $G(s)$!*

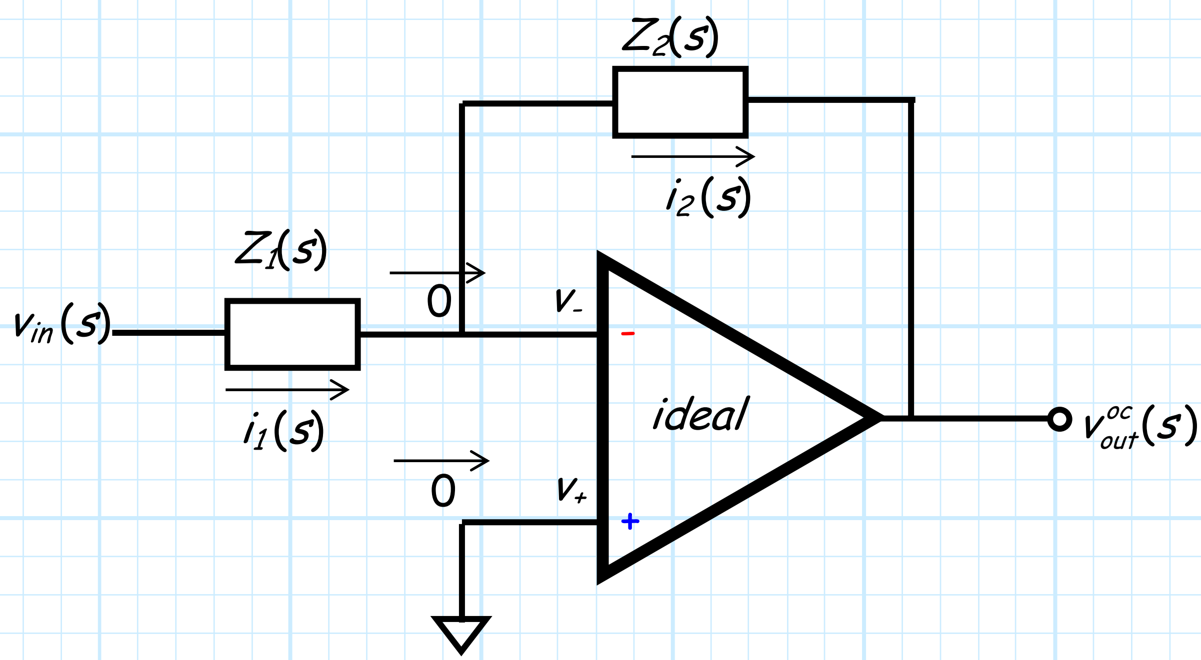
A: Remember, we can find $G(s)$ by analyzing the circuit using the Eigen value of **each linear circuit element**—a value we know as complex **impedance**!

$$\frac{v(s)}{i(s)} = Z(s)$$



For example

For **example**, consider this amplifier in with the **inverting configuration**, where the resistors have been **replaced** with complex **impedances**:



What is the **open-circuit voltage gain** $A_{vo}(s) = \dfrac{v_{out}^{oc}(s)}{v_{in}(s)}$?

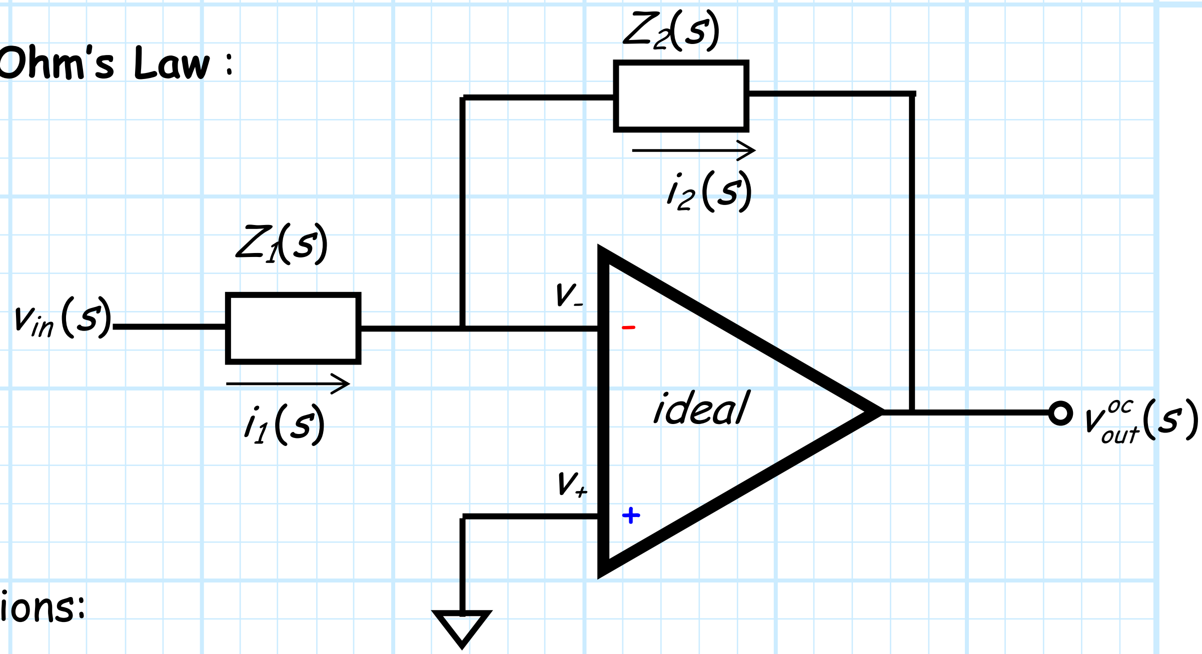
The eigen value of this linear operator

From **KCL**:

$$i_1(s) = i_2(s)$$

Since $v_-(s) = 0$, we find from **Ohm's Law** :

$$i_1(s) = \frac{v_{in}(s)}{Z_1(s)}$$



And also from **Ohm's Law**:

$$i_2(s) = \frac{-v_{out}^{oc}(s)}{Z_2(s)}$$

Equating the last two expressions:

$$\frac{v_{in}(s)}{Z_1(s)} = \frac{-v_{out}^{oc}(s)}{Z_2(s)}$$

Rearranging, we find the **open-circuit voltage gain**:

$$A_{vo}(s) = \frac{v_{out}^{oc}(s)}{v_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

The result passes the sanity check

Note that this complex voltage gain $A_{vo}(s)$ is the **Eigen value** $G(s)$ of the linear operator relating $v_{in}(t)$ and $v_{out}(t)$:

$$v_{out}(t) = \mathcal{L}\left[v_{in}(t)\right]$$

Note also that **if** the impedances $Z_1(s)$ and $Z_2(s)$ are real valued (i.e., they're resistors!):

$$Z_1(s) = R_1 + j0 \qquad \text{and} \qquad Z_2(s) = R_2 + j0$$

Then the voltage gain simplifies to the **familiar**:

$$A_{vo}(s) = \frac{V_{out}^{oc}(s)}{V_{in}(s)} = -\frac{R_2}{R_1}$$

Or, we can use the Fourier transform

Now, recall that the variable s is a **complex frequency**:

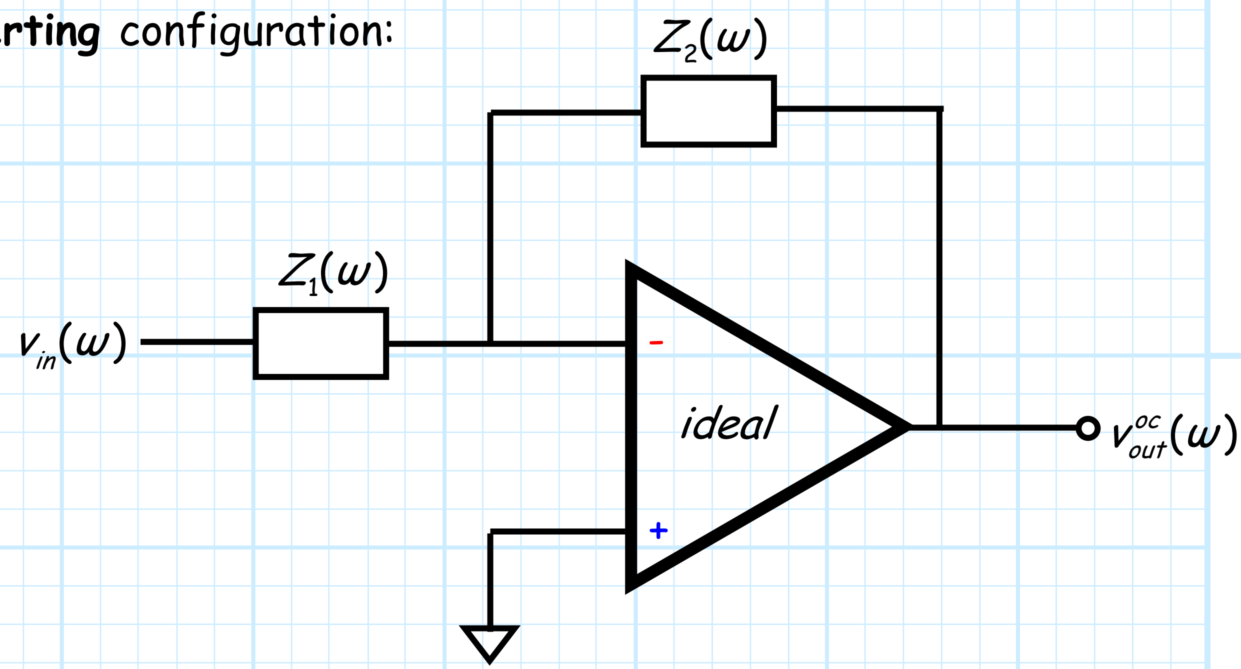
$$s = \sigma + j\omega .$$

If we set $\sigma = 0$, then $s = j\omega$, and the functions $Z(s)$ and $A_{vo}(s)$ in the Laplace domain can be written in the frequency (i.e., **Fourier**) domain!

$$A_{vo}(\omega) = A_{vo}(s)\Big|_{\sigma=0}$$

And therefore, for the **inverting** configuration:

$$A_{vo}(\omega) = \frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = -\frac{Z_2(\omega)}{Z_1(\omega)}$$

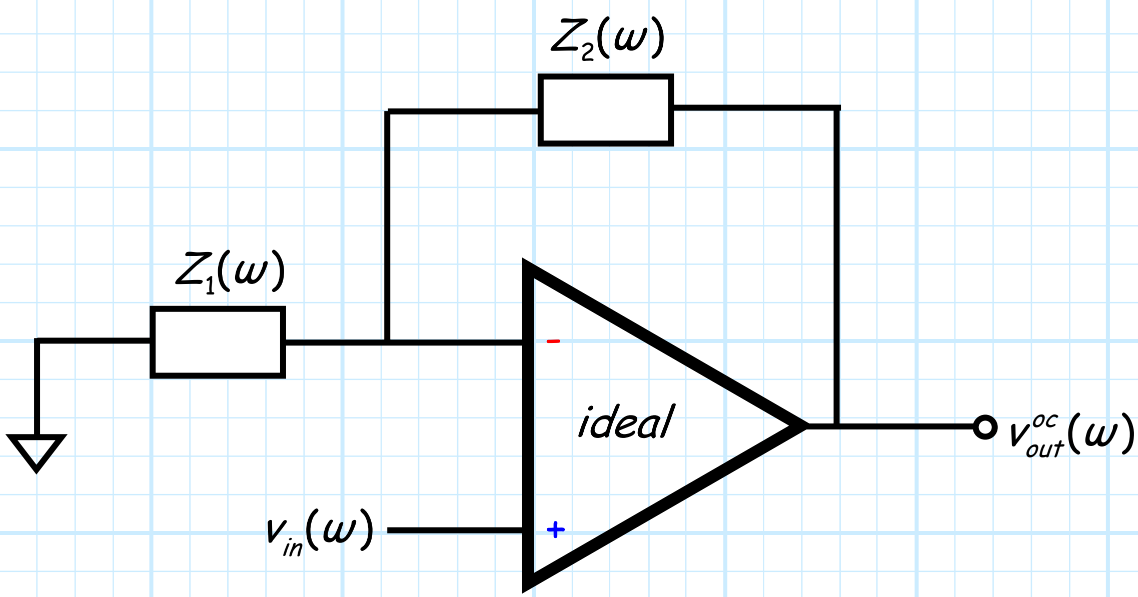


For the non-inverting

Likewise, for the non-inverting configuration, we find:

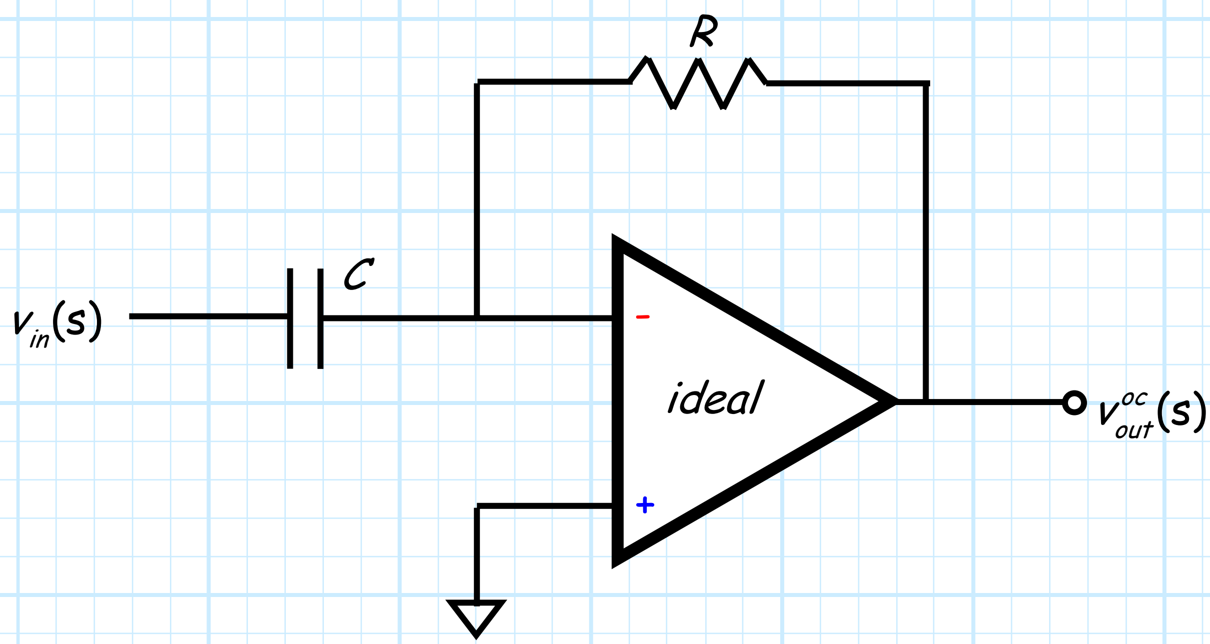
$$A_{vo}(\omega) = \frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = 1 + \frac{Z_2(\omega)}{Z_1(\omega)}$$

$$A_{vo}(s) = \frac{v_{out}^{oc}(s)}{v_{in}(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}$$



The Inverting Differentiator

The circuit shown below is the inverting **differentiator**.



Since the circuit uses the **inverting** configuration, we can conclude that the circuit transfer function is:

$$G(s) = \frac{v_{out}^{oc}(s)}{v_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Know the impedance; know the answer

For the **capacitor**, we know that its **complex impedance** is:

$$Z_1(s) = \frac{1}{sC}$$

And the complex impedance of the **resistor** is simply the real value:

$$Z_2(s) = R$$

Thus, the **eigen value** of the linear operator relating $v_{in}(t)$ to $v_{out}^{oc}(t)$ is:

$$G(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R}{1/sC} = -s\,RC$$

In other words, the (Laplace transformed) **output** signal is related to the (Laplace transformed) **input** signal as:

$$v_{out}^{oc}(s) = -s\left(RC\right)\, v_{in}(s)$$

From our knowledge of **Laplace Transforms**, we know this means that the output signal is proportional to the **derivative** of the input signal!

Converting back to time domain

Taking the **inverse** Laplace Transform, we find:

$$v_{out}^{oc}(t) = -RC\,\frac{d\,v_{in}(t)}{d\,t}$$

For example, if the **input** is:

$$v_{in}(t) = \sin\omega t$$

then the **output** is:

$$\begin{aligned} v_{out}^{oc}(t) &= -RC\,\frac{d\,v_{in}(t)}{dt} \\ &= -RC\,\frac{d\sin\omega t}{dt} \\ &= -\omega\,RC\,\cos\omega t \end{aligned}$$

Or, with Fourier analysis

We likewise could have determined this result using **Fourier analysis** (i.e., frequency domain):

$$G(\omega) = \frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = -\frac{Z_2(\omega)}{Z_1(\omega)} = -\frac{R}{\left(1/j\omega C\right)} = -j\,\omega\,RC$$

Thus, the **magnitude** of the transfer function is:

$$\begin{aligned}|G(\omega)| &= |-j\omega\,RC| \\ &= \omega\,RC\end{aligned}$$

And since:

$$-j = e^{-j(\pi/2)} = \cos\left(-\pi/2\right) + j\sin\left(-\pi/2\right)$$

the **phase** of the transfer function is:

$$\begin{aligned}\angle G(\omega) &= -\pi/2 \quad \text{radians} \\ &= -90^\circ\end{aligned}$$

Look at the magnitude and phase

So given that:

$$\left|v_{out}^{oc}(\omega)\right| = \left|G(\omega)\right| \left|v_{in}(\omega)\right|$$

and:

$$\angle v_{out}^{oc}(\omega) = \angle G(\omega) + \angle v_{in}(\omega)$$

we find for the **input**:

$$v_{in}(t) = \sin \omega t$$

where:

$$\left|v_{in}(\omega)\right| = 1 \qquad \text{and} \qquad \angle v_{in}(\omega) = 0$$

that the **output** of the inverting differentiator is:

$$\left|v_{out}^{oc}(\omega)\right| = \left|G(\omega)\right| \left|v_{in}(\omega)\right| = \omega RC$$

and:

$$\angle v_{out}^{oc}(\omega) = \angle G(\omega) + \angle v_{in}(\omega) = -90^{\circ} + 0 = -90^{\circ}$$

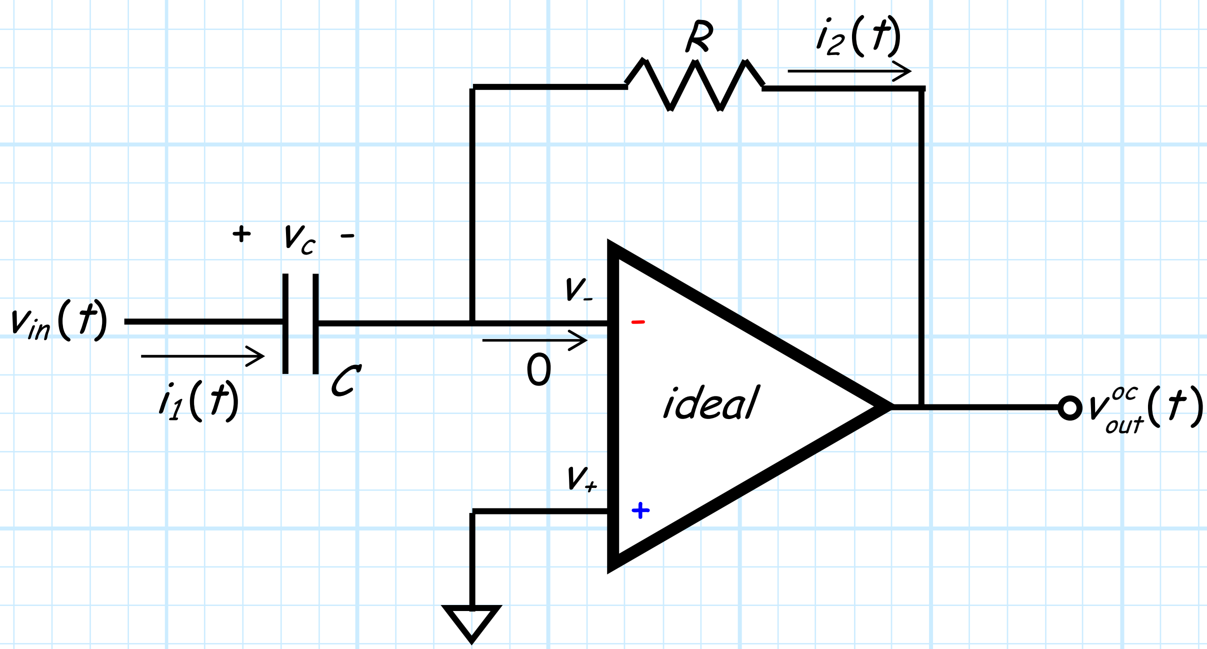
The result is the same!

Therefore, the output is:

$$\begin{aligned} v_{out}^{oc}(t) &= \omega RC \sin\left(\omega t - 90^\circ\right) \\ &= -\omega RC \cos\omega t \end{aligned}$$

Exactly the **same result** as before (using Laplace trasforms)!

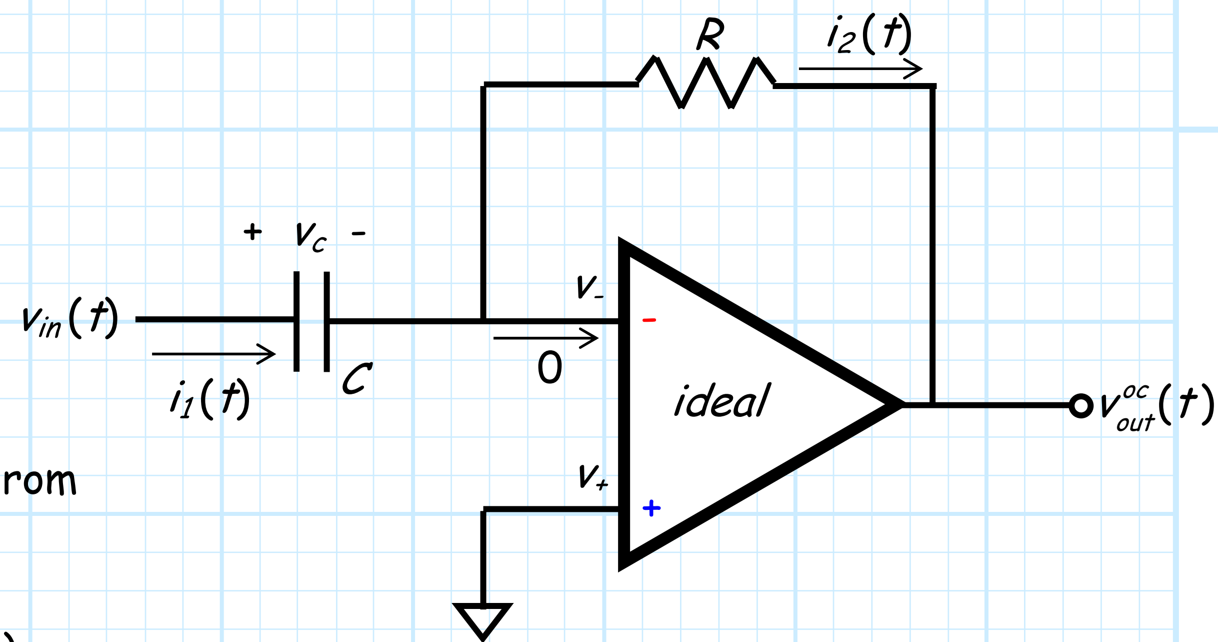
If you are still **unconvinced** that this circuit is a differentiator, consider this **time-domain** analysis.



Let's do a time-domain analysis

From our elementary **circuits knowledge**, we know that the current through a capacitor ($i_1(t)$) is:

$$i_1(t) = C\frac{d\,v_c(t)}{dt}$$



and from the circuit we see from KVL that:

$$v_c(t) = v_{in}(t) - v_-(t) = v_{in}(t)$$

therefore the **input current** is:

$$i_1(t) = C\frac{d\,v_{in}(t)}{dt}$$

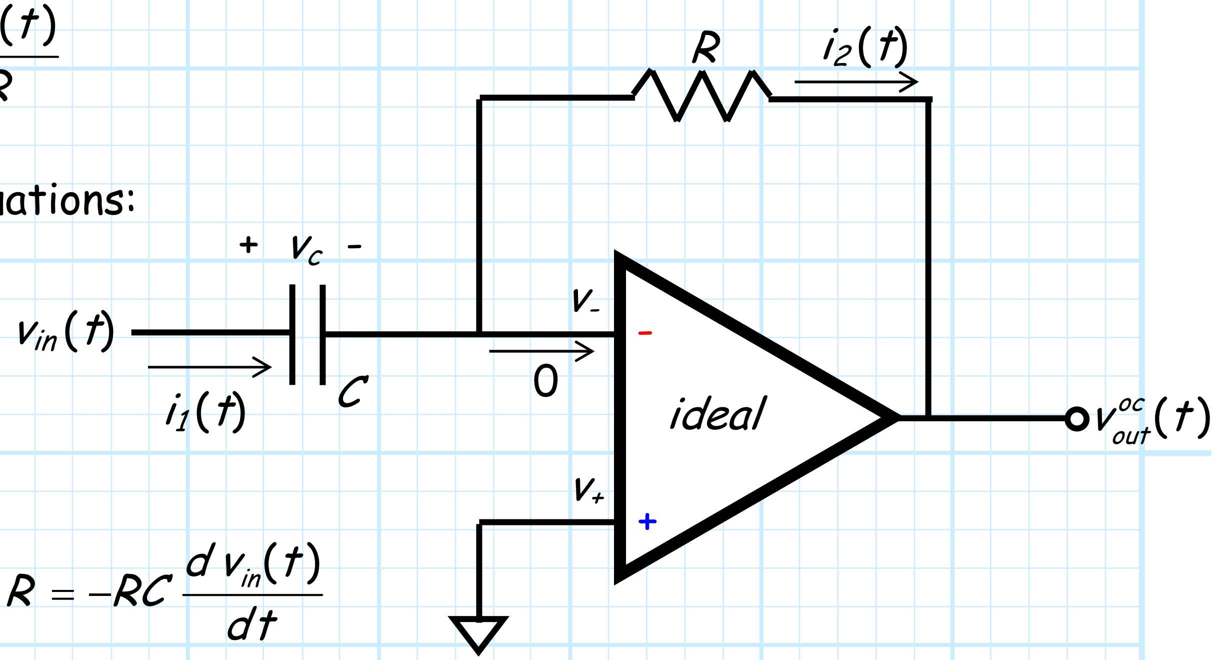
Laplace, Fourier, time-domain: the result it the same!

From KCL, we likewise know that:

$$i_1(t) = i_2(t)$$

and from Ohm's Law:

$$i_2(t) = \frac{v_1(t) - v_{out}^{oc}(t)}{R} = -\frac{v_{out}^{oc}(t)}{R}$$



Combining the two previous equations:

$$v_{out}^{oc}(t) = -i_1(t)\,R$$

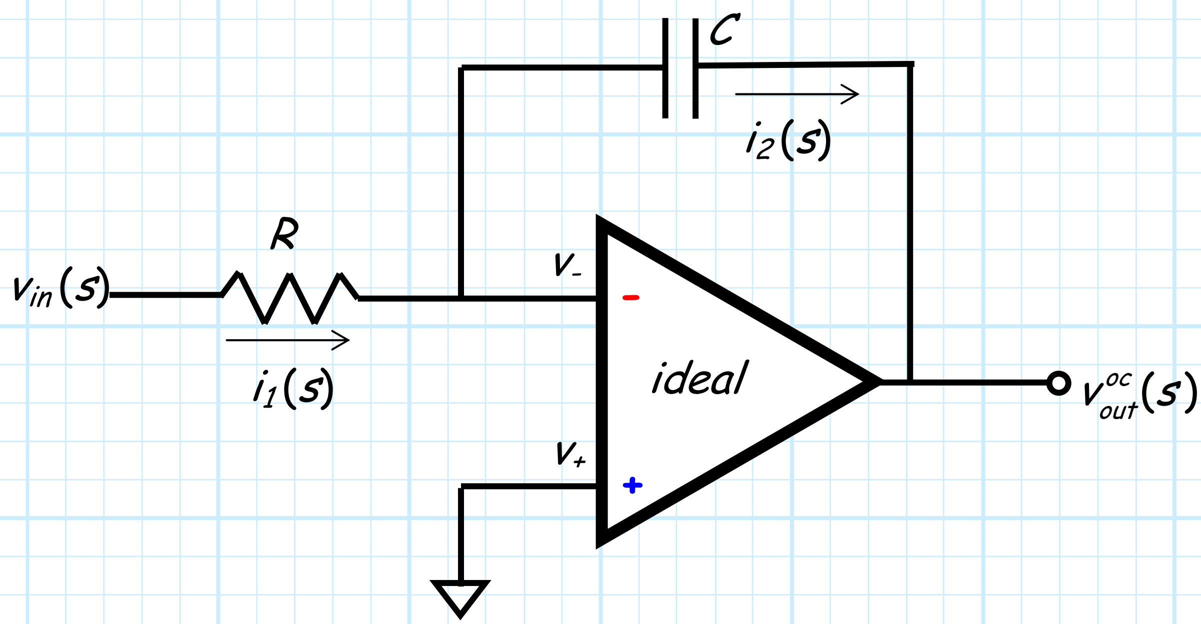
and thus:

$$v_{out}^{oc}(t) = -i_1(t)\,R = -\left(C\,\frac{d\,v_{in}(t)}{dt}\right)R = -RC\,\frac{d\,v_{in}(t)}{dt}$$

The **same** result as before!

The Inverting Integrator

The circuit shown below is the inverting **integrator**.



It's the inverting configuration!

Since the circuit uses the **inverting** configuration, we can conclude that the circuit transfer function is:

$$G(s) = \frac{v_{out}^{oc}(s)}{v_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{\left(1/s\,C\right)}{R} = \frac{-1}{s\,RC}$$

In other words, the output signal is related to the input as:

$$v_{out}^{oc}(s) = \frac{-1}{RC}\,\frac{v_{in}(s)}{s}$$

From our knowledge of **Laplace Transforms**, we know this means that the output signal is proportional to the **integral** of the input signal!

The circuit integrates the input

Taking the **inverse** Laplace Transform, we find:

$$v_{out}^{oc}(t) = \frac{-1}{RC}\int_0^t v_{in}(t')\,dt'$$

For example, if the **input** is:

$$v_{in}(t) = \sin\omega t$$

then the **output** is:

$$v_{out}^{oc}(t) = \frac{-1}{RC}\int_0^t \sin\omega t\,dt' = \frac{-1}{RC}\frac{-1}{\omega}\cos\omega t = \frac{1}{\omega RC}\cos\omega t$$

Or, in the Fourier domain

We likewise could have determined this result using **Fourier Analysis** (i.e., frequency domain):

$$G(\omega)=\frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)}=-\frac{Z_2(\omega)}{Z_1(\omega)}=-\frac{\left(1/j\omega C\right)}{R}=\frac{j}{\omega RC}$$

Thus, the **magnitude** of the transfer function is:

$$|G(\omega)|=\left|\frac{j}{\omega RC}\right|=\frac{1}{\omega RC}$$

And since:

$$j=e^{j(\pi/2)}=\cos\left(\pi/2\right)+j\sin\left(\pi/2\right)$$

the **phase** of the transfer function is:

$$\angle G(\omega)=\pi/2 \;\; \text{radians}=90^{\circ}$$

Magnitude and phase

Given that:

$$\left|v_{out}^{oc}(\omega)\right| = \left|G(\omega)\right| \left|v_{in}(\omega)\right|$$

and:

$$\angle v_{out}^{oc}(\omega) = \angle G(\omega) + \angle v_{in}(\omega)$$

we find for the **input**:

$$v_{in}(t) = \sin \omega t$$

where:

$$\left|v_{in}(\omega)\right| = 1 \qquad \text{and} \qquad \angle v_{in}(\omega) = 0$$

that the **output** of the inverting integrator is:

$$\left|v_{out}^{oc}(\omega)\right| = \left|G(\omega)\right| \left|v_{in}(\omega)\right| = \frac{1}{\omega RC}$$

and:

$$\angle v_{out}^{oc}(\omega) = \angle G(\omega) + \angle v_{in}(\omega) = 90^\circ + 0 = 90^\circ$$

See, it's an integrator

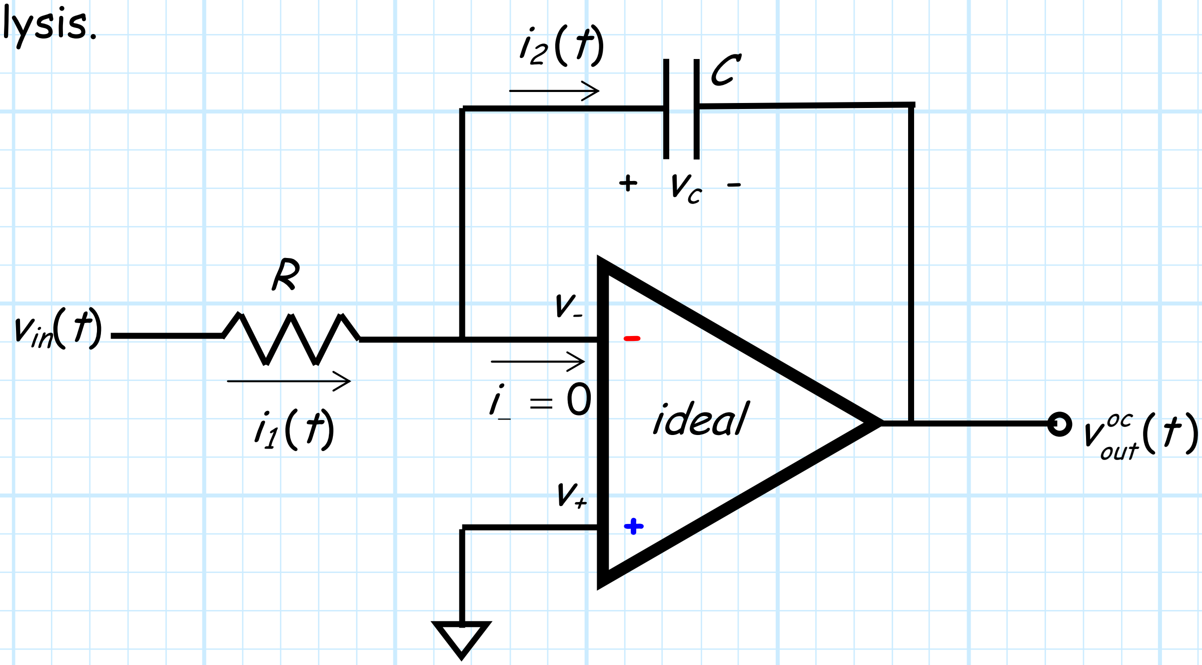
Therefore:

$$v_{out}^{oc}(t) = \frac{1}{\omega RC} \sin\left(\omega t + 90^\circ\right)$$

$$= \frac{1}{\omega RC} \cos \omega t$$

Exactly the **same result** as before!

If you are still **unconvinced** that this circuit is an integrator, consider this **time-domain** analysis.



The time-domain solution

From our elementary **circuits knowledge**, we know that the voltage across a capacitor is:

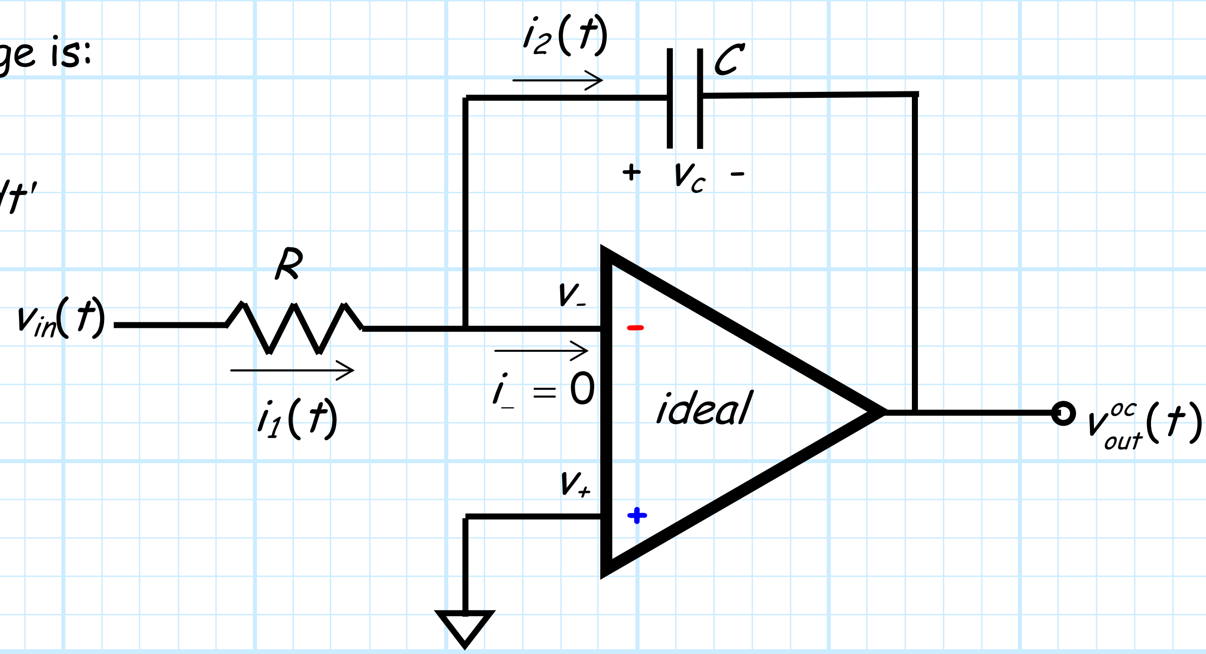
$$v_c(t) = \frac{1}{C}\int_0^t i_2(t')\,dt'$$

and from the circuit we see that:

$$v_c(t) = v_-(t) - v_{out}^{oc}(t) = -v_{out}^{oc}(t)$$

therefore the **output** voltage is:

$$v_{out}^{oc}(t) = -\frac{1}{C}\int_0^t i_2(t')\,dt'$$



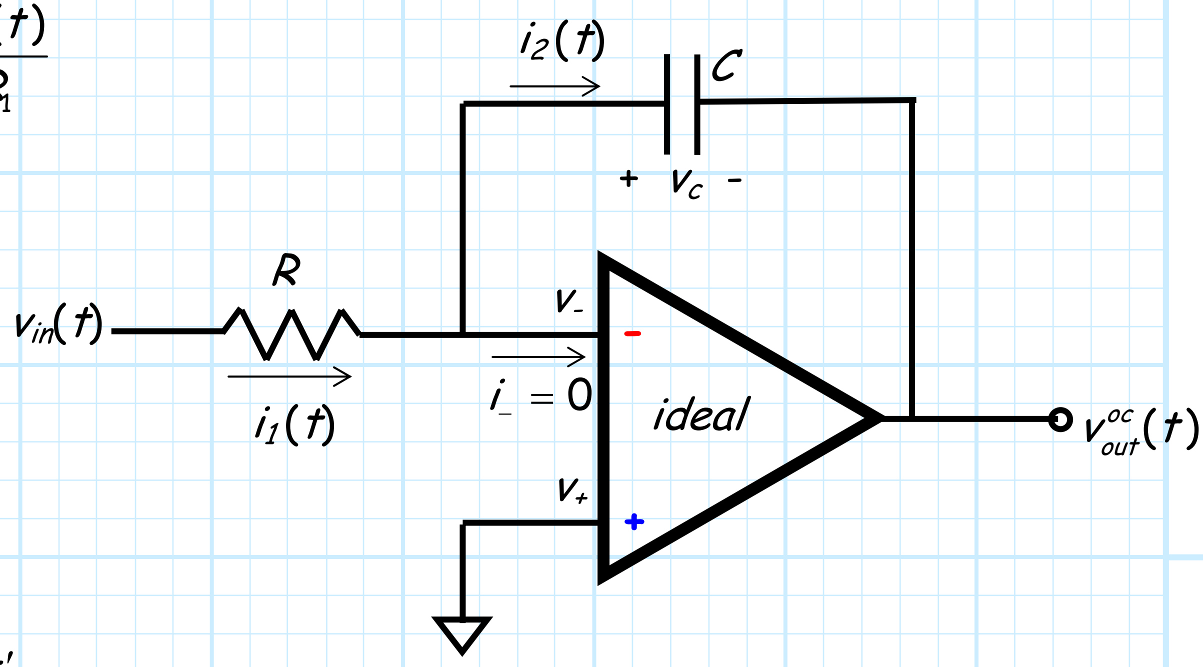
The same result no matter how we do it!

From KCL, we likewise know that:

$$i_1(t) = i_2(t)$$

and from Ohm's Law:

$$i_1(t) = \frac{v_{in}(t) - v_-(t)}{R_1} = \frac{v_{in}(t)}{R_1}$$



Therefore:

$$i_2(t) = \frac{v_{in}(t)}{R_1}$$

and thus:

$$v_{out}^{oc}(t) = \frac{-1}{C}\int_0^t i_2(t')\,dt'$$

$$= \frac{-1}{RC}\int_0^t v_{in}(t')\,dt'$$

The **same** result as before!

An Application of the Inverting Integrator

Note the time average of a signal $v(t)$ over some arbitrary time T is mathematically stated as:

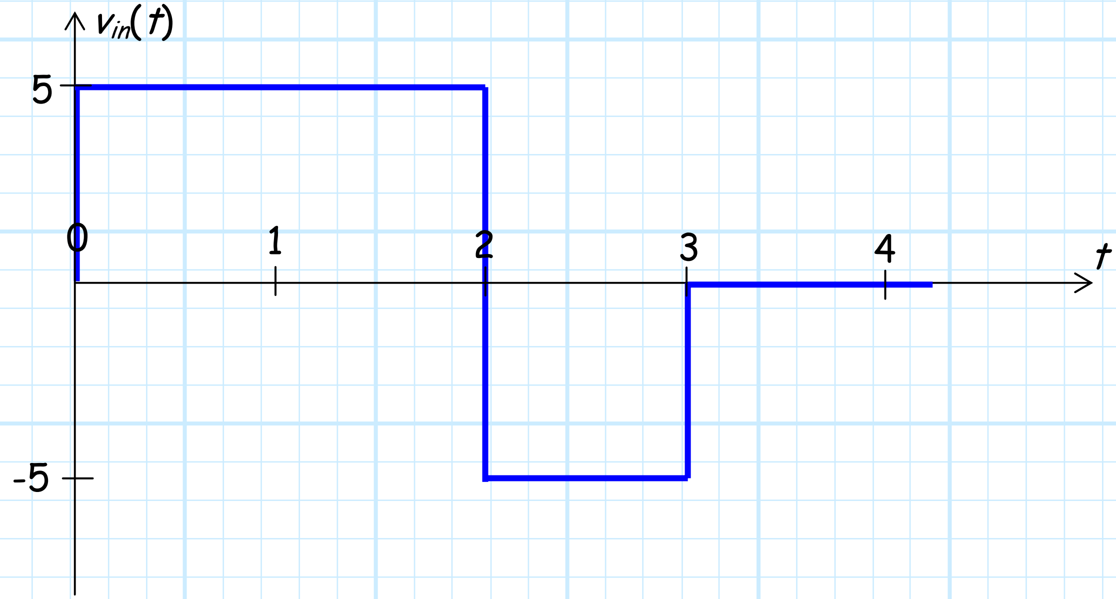
$$\text{average of } v(t) \doteq \overline{v(t)} = \frac{1}{T}\int_0^T v(t)\, dt$$

Note that this is **exactly** the form of the output of an op-amp **integrator**!

We can use the inverting integrator to determine the **time-averaged** value of some input signal $v(t)$ over some arbitrary time T.

Make sure you see this!

For **example**, say we wish to determine the time-averaged value of the input signal:



I.E.,

$$v_{in}(t) = \begin{cases} 5 & 0 < t < 2 \\ -5 & 2 < t < 3 \\ 0 & t > 3 \end{cases}$$

The **time average** of this function over a period from 0 < t < T=3 is therefore:

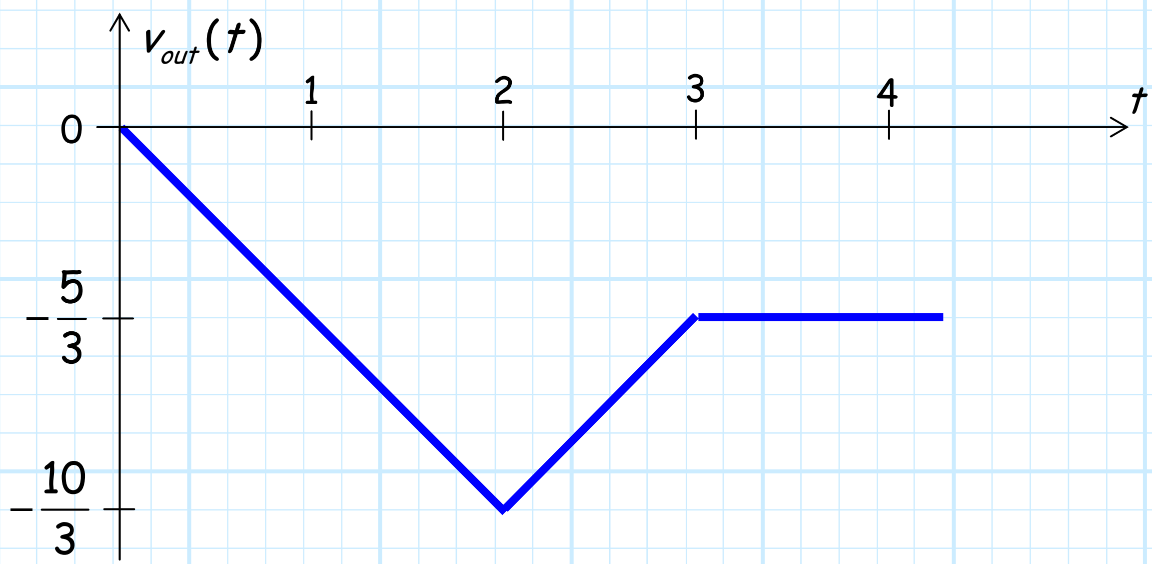
$$\overline{v_{in}(t)} = \frac{1}{3}\int_0^3 v_{in}(t)\, dt = \frac{5}{3}$$

This better make sense to you!

We could likewise determine this average using an **inverting integrator**. We select a resistor R and a capacitor C such that the product RC = 3 seconds.

The output of this integrator would be:

$$v_{out}(t) = \frac{-1}{3}\int_0^t v_{in}(t')\,dt' = \begin{cases} -\dfrac{5t}{3} & 0 < t < 2 \\ \dfrac{5t-20}{3} & 2 < t < 3 \\ -\dfrac{5}{3} & t > 3 \end{cases}$$



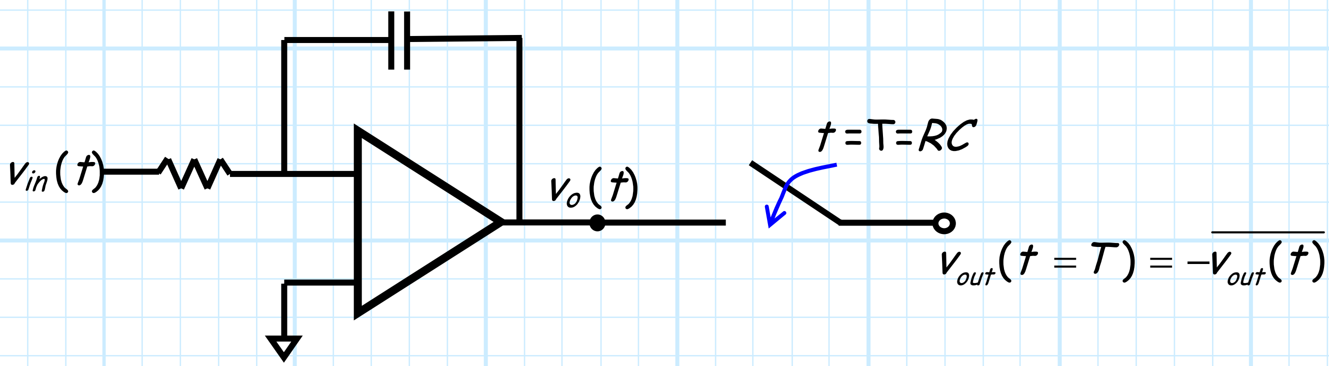
We must sample a the correct time!

Note that the value of the output voltage **at t =3** is:

$$v_{out}(t=3) = \frac{-1}{3}\int_0^3 v_{in}(t')\,dt' = -\frac{5}{3}$$

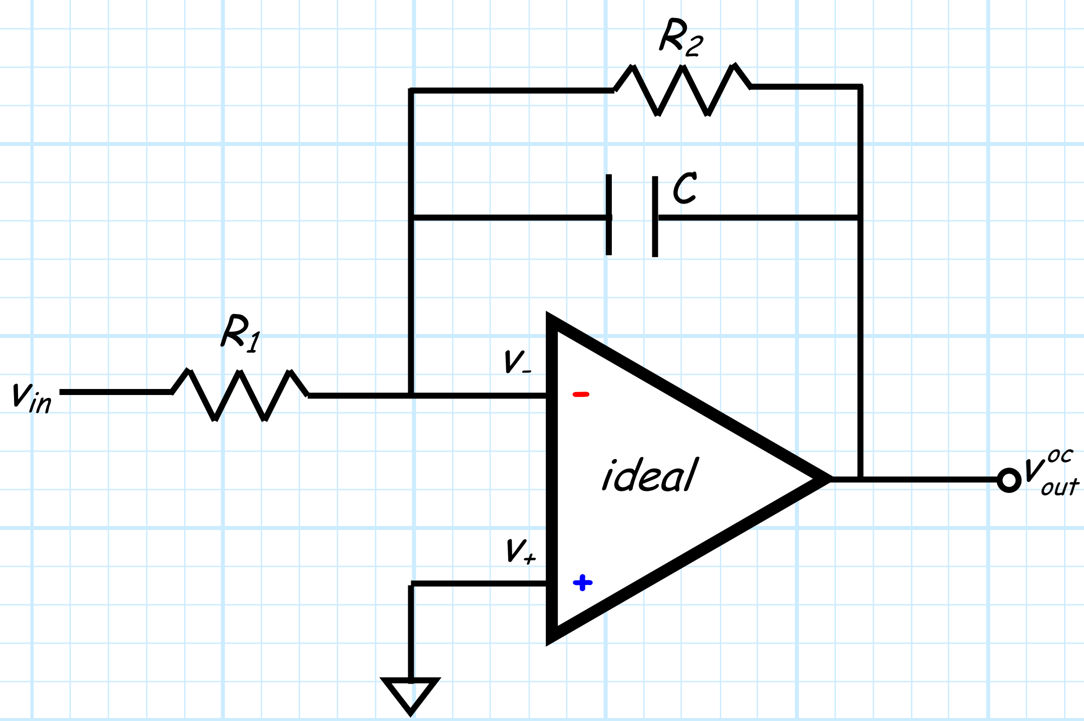
The **time-averaged** value (times -1)!

Thus, we can use the inverting integrator, along with a voltage sampler (e.g., A to D converter) to determine the **time-averaged** value of a function over some time period T.



Example: An Inverting Network

Now let's determine the complex **transfer function** of this circuit:



It's the inverting configuration!

Note this circuit uses the **inverting** configuration, so that:

$$G(\omega) = -\frac{Z_2(\omega)}{Z_1(\omega)}$$

where $Z_1 = R_1$, and:

$$Z_2 = R_2 \left\| \frac{1}{j\omega C} \right. = \frac{R_2}{1 + j\omega R_2 C}$$

Therefore, the **transfer function** of this circuit is:

$$G(\omega) = \frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = -\frac{R_2}{R_1}\frac{1}{1 + j\omega R_2 C}$$

Another low-pass filter

Thus, the transfer function **magnitude** is:

$$|G(\omega)|^2 = \left(-\frac{R_2}{R_1}\right)^2 \frac{1}{1+\left(\omega/\omega_0\right)^2}$$

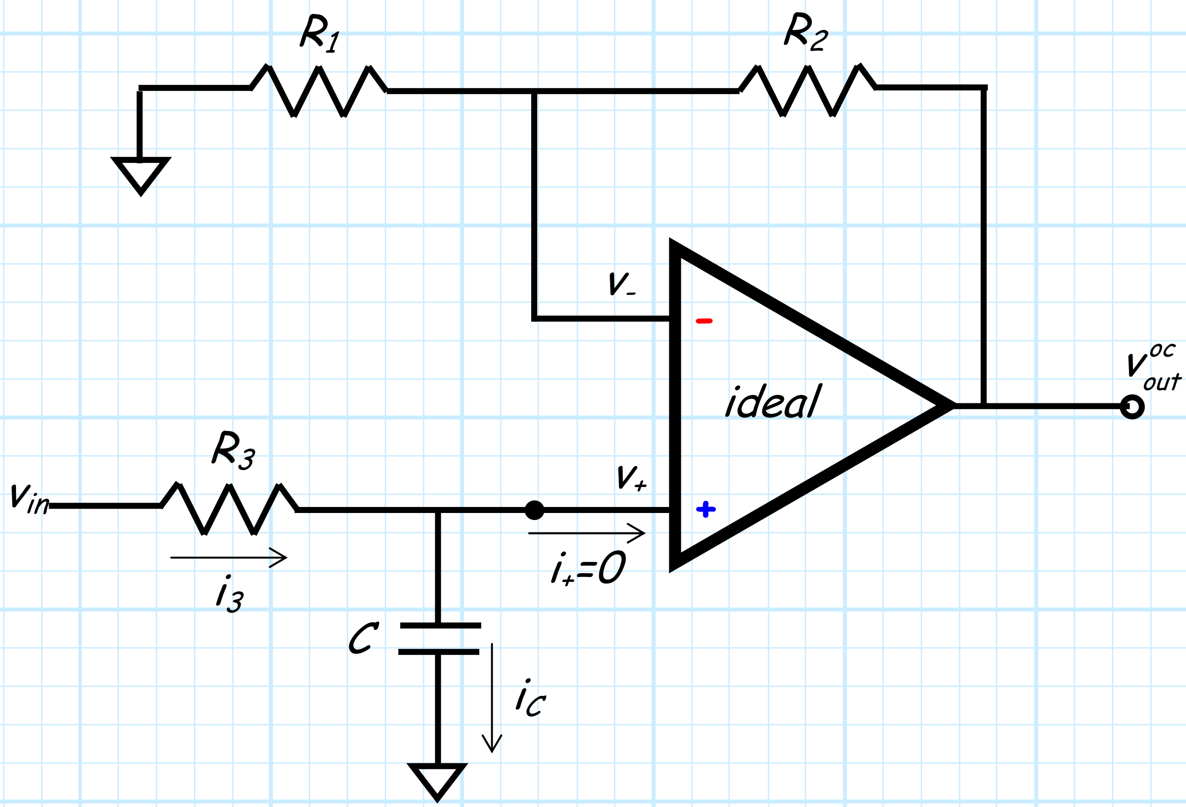
where:

$$\omega_0 = \frac{1}{R_2 C}$$

Thus, just as with the previous example, this circuit is a **low-pass filter**, with **cutoff** frequency ω_0 and pass-band **gain** $\left(R_2/R_1\right)^2$.

Example: A Non-Inverting Network

Let's determine the **transfer function** $G(\omega) = v_{out}^{oc}(\omega) / v_{in}(\omega)$ for the following circuit:



Some enjoyable circuit analysis

From **KCL**, we know:

$$i_3(\omega) = i_C(\omega) + i_+(\omega) = i_C(\omega) + 0 = i_C(\omega)$$

where:

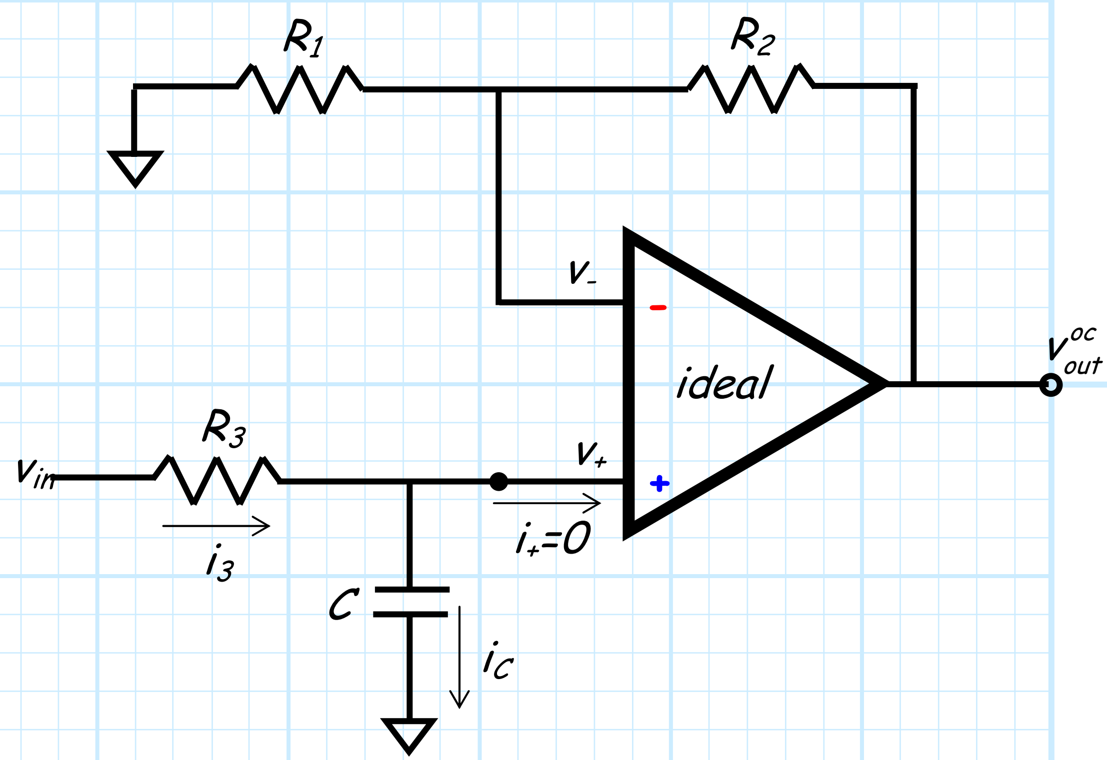
$$i_3(\omega) = \frac{v_{in}(\omega) - v_+(\omega)}{R_3} \quad \text{and} \quad i_C(\omega) = \frac{v_+(\omega) - 0}{\left(1/j\omega C\right)} = j\,\omega C\, v_+(\omega)$$

Equating, we find an expression involving $v_{in}(\omega)$ and $v_2(\omega)$ only:

$$\frac{v_{in}(\omega) - v_+(\omega)}{R_3} = j\,\omega C\, v_+(\omega)$$

and performing a little **algebra**, we find:

$$v_2(\omega) = \frac{v_{in}(\omega)}{1 + j\omega R_3 C}$$

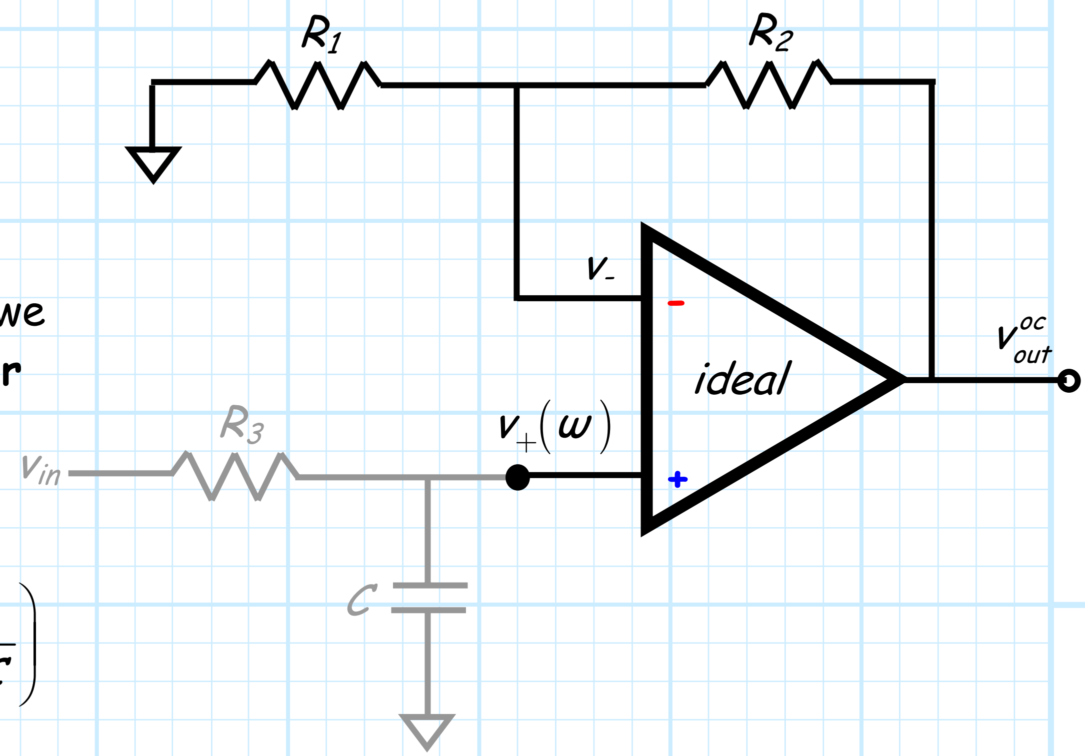


No need to go further: we have a template!

The remainder of the circuit is simply the **non-inverting amplifier** that we studied earlier.

We know that:

$$v_{out}^{oc}(\omega) = \left(1 + \frac{R_2}{R_1}\right) v_+(\omega)$$



Combining these two relationships, we can determine the **complex transfer function** for this circuit:

$$G(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + j\omega R_3 C}\right)$$

It's a low-pass filter!!!

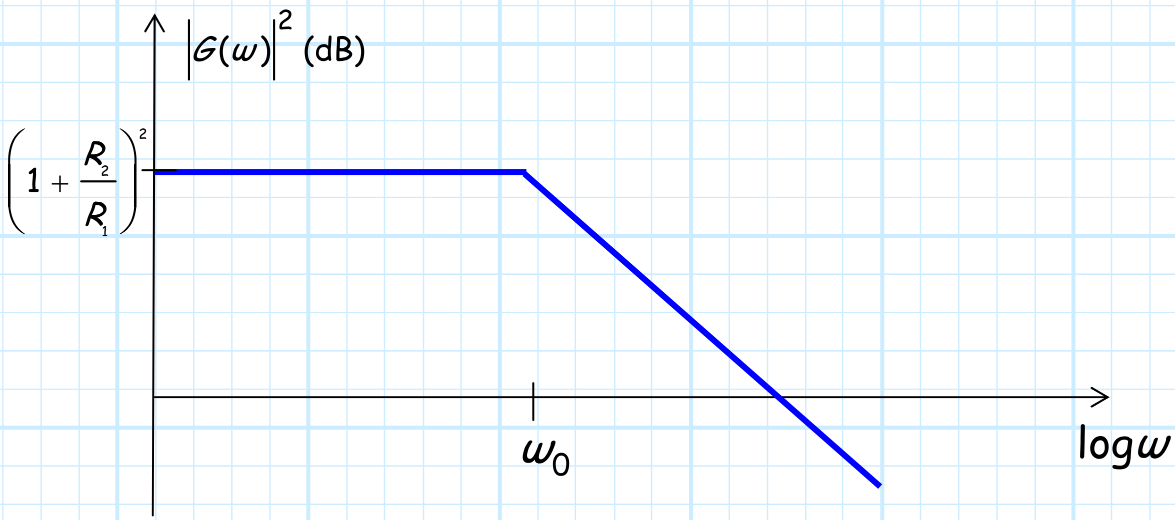
The **magnitude** of this transfer function is therefore:

$$|G(\omega)|^2 = \left(1+\frac{R_2}{R_1}\right)^2 \frac{1}{1+\left(\omega/\omega_0\right)^2}$$

where:

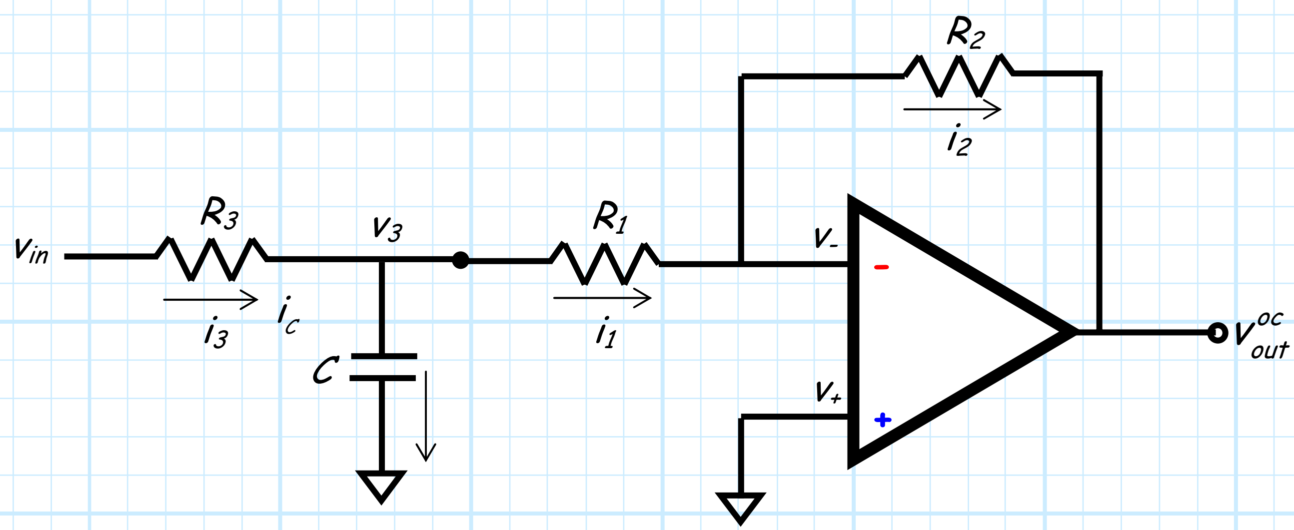
$$\omega_0 = \frac{1}{R_3 C}$$

This is a **low-pass filter**—one with **pass-band gain**!



Example: Another Inverting Network

Consider now the transfer function of **this** circuit:

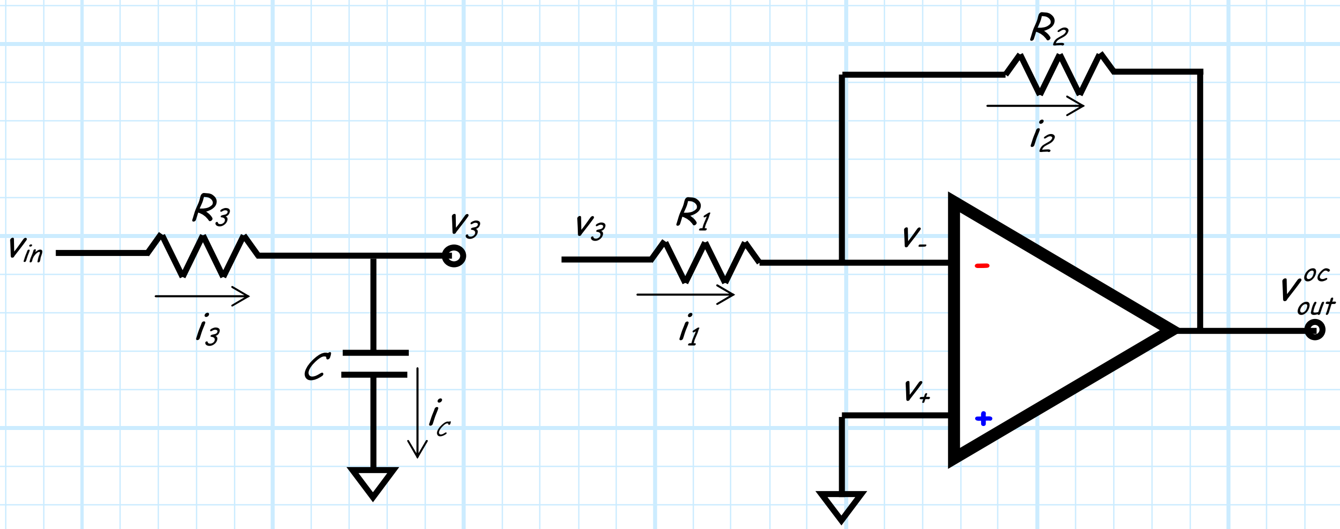


Some more enjoyable circuit analysis

To accomplish this analysis, we must first…

Wait! *You don't need to explain this to* ***me****.*

It is obvious that we can divide this is circuit into ***two*** *pieces—the first being a complex* ***voltage divider*** *and the second a* ***non-inverting*** *amplifier.*



Can we analyze the circuit this way?

The transfer function of the complex voltage divider is:

$$\frac{v_3(\omega)}{v_{in}(\omega)} = \frac{1/_{j\omega C}}{R_3 + 1/_{j\omega C}} = \frac{1}{1 + j\,\omega R_3 C}$$

and that of the inverting amplifier:

$$\frac{v_{out}^{oc}(\omega)}{v_3(\omega)} = -\frac{R_2}{R_1}$$

*And so of course **I** have correctly determined that the transfer function of this circuit is:*

$$\frac{v_{out}^{oc}(\omega)}{v_{in}(\omega)} = \frac{v_{out}^{oc}(\omega)}{v_3(\omega)}\,\frac{v_3(\omega)}{v_{in}(\omega)} = -\frac{R_2}{R_1}\,\frac{1}{1 + j\omega R_3 C}$$

No, we cannot

NO! This is **not** correct:

$$\frac{v_o(\omega)}{v_i(\omega)} \neq -\frac{R_2}{R_1}\frac{1}{1+j\omega R_3 C}$$

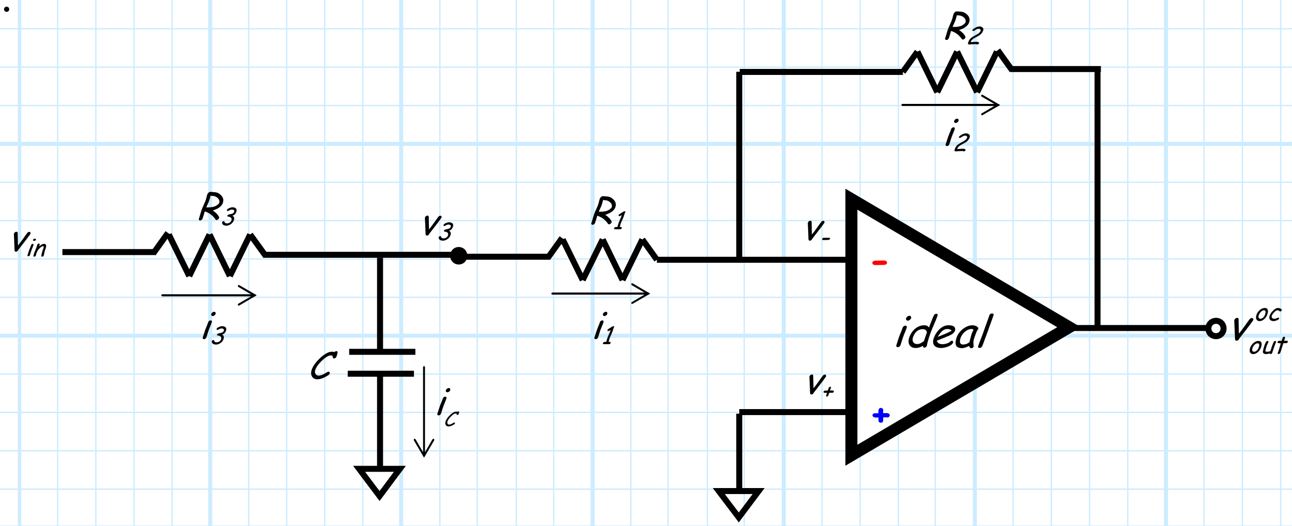
The problem with the above "analysis" is that we **cannot** apply **this** complex voltage divider equation to determine $v_3(\omega)$:

$$v_3(\omega) \neq \frac{1/j\omega C}{R_3 + 1/j\omega C}\, v_{in}(\omega)$$

The reason of course is that the output of this voltage divider is **not** open-circuited, and thus current $i_3(\omega) \neq i_C(\omega)$.

My computer suspiciously crashed while writing this (really, it did!)

We **cannot** divide this circuit into two independent pieces, we must analyze it as **one** circuit.

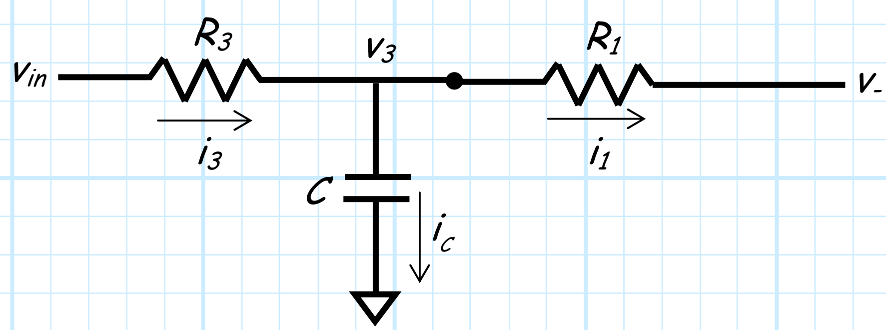


*Of course what I **meant** to say was that we should determine the **impedance** Z_1 of input network, and **then** use the inverting configuration equation* $T(\omega) = -Z_2/Z_1$.

An even worse idea than Vista

NO! This idea is as bad as the last one!

We **cannot** specify an impedance for the input network:



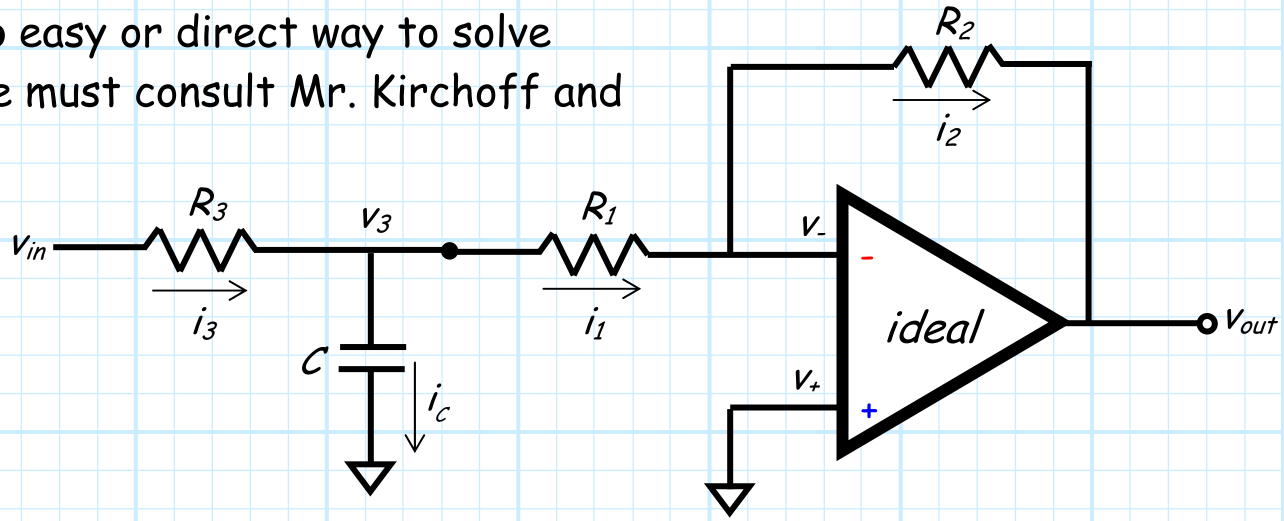
After all, would we define this impedance as:

$$Z_1 = \frac{v_{in} - v_-}{i_3} \quad \text{or} \quad Z_1 = \frac{v_{in} - v_-}{i_1} \quad ???$$

Windows Vista™

Don't look for templates: trust what you know

So, there is **no** easy or direct way to solve this circuit, we must consult Mr. Kirchoff and his laws!



We know that $i_1 = i_2$, where:

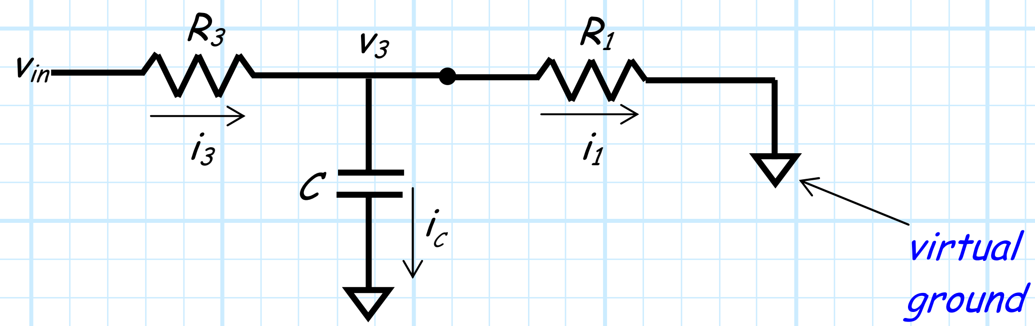
$$i_1 = \frac{v_3 - v_-}{R_1} = \frac{v_3}{R_1} \qquad \text{and} \qquad i_2 = \frac{v_+ - v_{out}}{R_2} = \frac{-v_{out}}{R_2}$$

Combining these equations, we get the **expected** result:

$$v_{out} = -\frac{R_2}{R_1} v_3$$

Don't forget virtual ground!

We must therefore determine v_3 in terms of v_i:



Note R_1 and C are connected in **parallel**!

Thus, from **voltage division**, we find:

$$v_3 = \frac{R_1 \left\| \frac{1}{j\omega C} \right.}{R_3 + \left(R_1 \left\| \frac{1}{j\omega C} \right. \right)} v_{in}$$

where:

$$R_1 \left\| \frac{1}{j\omega C} \right. = \frac{R_1 \left(\frac{1}{j\omega C} \right)}{R_1 + \frac{1}{j\omega C}} = \frac{R_1}{1 + j\omega R_1 C}$$

The Eigen value at last!

Performing some **algebra**, we find:

$$v_3 = \left(\frac{R_1}{(R_1 + R_3) + j\omega R_1 R_3 C} \right) v_{in}$$

and since:

$$v_{out} = \frac{-R_2}{R_1} v_3$$

we **finally** discover that:

$$G(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \left(\frac{-R_2}{(R_1 + R_3) + j\,\omega R_1 R_3 C} \right)$$

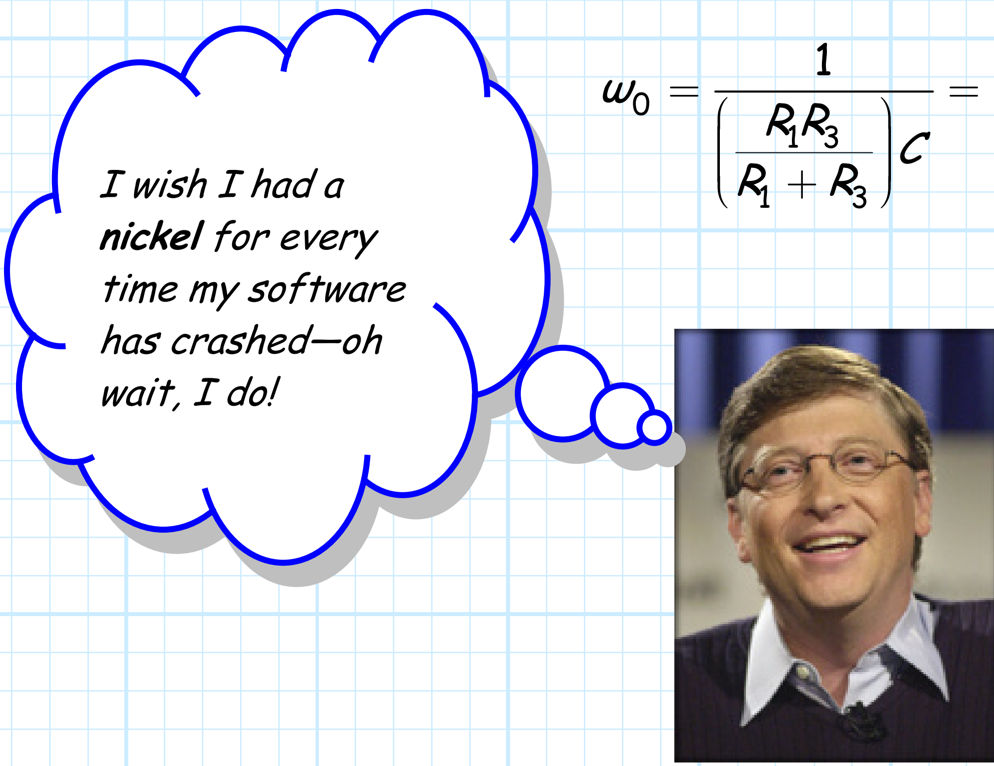
This again is a low-pass filter

We can rearrange this transfer function to find that this circuit is likewise a **low-pass filter** with **pass-band gain**:

$$G(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \frac{-R_2}{R_1 + R_3}\left(\frac{1}{1 + j\left(\omega/\omega_o\right)}\right)$$

where the **cutoff frequency** ω_0 is:

$$\omega_0 = \frac{1}{\left(\frac{R_1 R_3}{R_1 + R_3}\right)C} = \frac{1}{\left(R_1 \| R_3\right)C}$$



Example: A Complex Processing Circuit using the Inverting Configuration

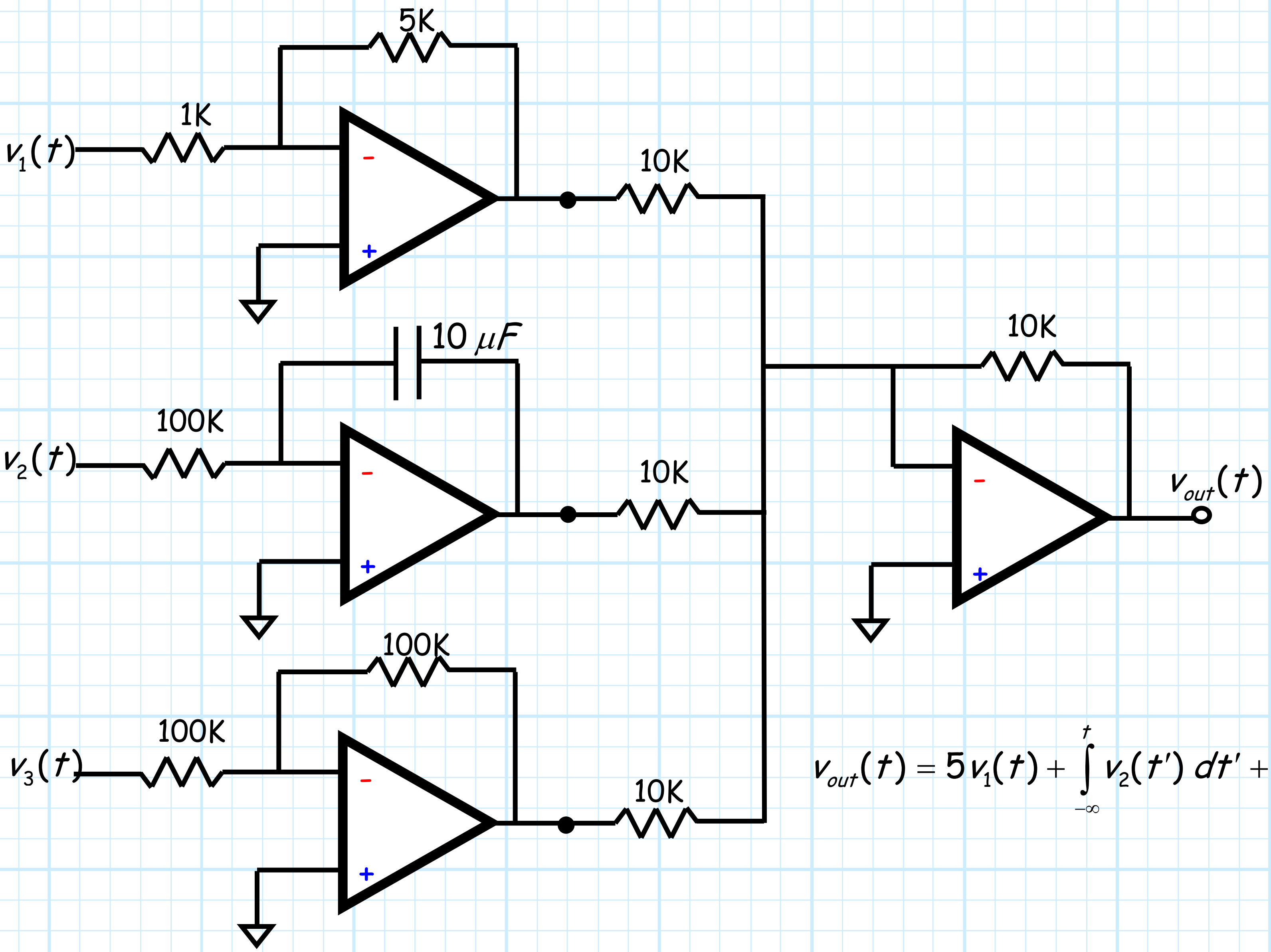
Note that we can combine inverting amplifiers to form a more **complex** processing system.

For **example**, say we wish to take **three** input signals $v_1(t)$, $v_2(t)$, and $v_3(t)$, and process them such that the open-circuit output voltage is:

$$v_{out}(t) = 5\,v_1(t) + \int_{-\infty}^{t} v_2(t')\,dt' + \frac{d\,v_3(t)}{dt}$$

Assuming that we use **ideal** (or near ideal) op-amps, with an **output resistance equal to zero** (or at least very small), we can realize the above signal processor with the following circuit:

This circuit performs this operation!



$$v_{out}(t) = 5v_1(t) + \int_{-\infty}^{t} v_2(t')\, dt' + \frac{d\, v_3(t)}{dt}$$