MOSFET Output Resistance

Recall that due to **channel-length modulation**, the MOSFET drain current is **slightly** dependent on v_{DS}, and thus is more accurately described as:

$$i_D = K\left(v_{GS} - V_t\right)^2\left(1 + \lambda v_{DS}\right)$$

In order to determine the relationship between the small-signal voltage v_{gs} and small-signal current i_d we can apply a **small-signal analysis** of this equation:

$$\begin{aligned} i_d &= \left.\frac{d\, i_D}{d\, v_{GS}}\right|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(v_{GS} - V_t\right)\Big|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(V_{GS} - V_t\right) v_{gs} \\ &= g_m\, v_{gs} \end{aligned}$$

Note that we evaluated the derivative at the DC **bias** point V_{GS}. The result, as we expected, was the **transconductance** g_m.

We can likewise determine the relationship between small-signal voltage v_{ds} and the small-signal current i_d :

$$
\begin{aligned}
i_d &= \left.\frac{d\, i_D}{dv_{DS}}\right|_{v_{GS}=V_{GS}} v_{ds} \\
&= \lambda K \left(v_{GS} - V_t\right)^2 \Big|_{v_{GS}=V_{GS}} v_{ds} \\
&= \lambda K \left(V_{GS} - V_t\right)^2 v_{ds} \\
&= \frac{v_{ds}}{r_o}
\end{aligned}
$$

where r_o is defined as the MOSFET **output resistance**:

$$
\begin{aligned}
r_o &= \frac{1}{\lambda K \left(V_{GS} - V_t\right)^2} \\
&= \frac{1}{\lambda\, I_D}
\end{aligned}
$$

The small signal drain current i_d of a MOSFET (biased at a DC operating point I_D, V_{GS}) is therefore:

$$
i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}
$$

where:

$$
g_m = 2K \left(V_{GS} - V_t\right)
$$

$$
r_o = \frac{1}{\lambda K \left(V_{GS} - V_t\right)^2}
$$