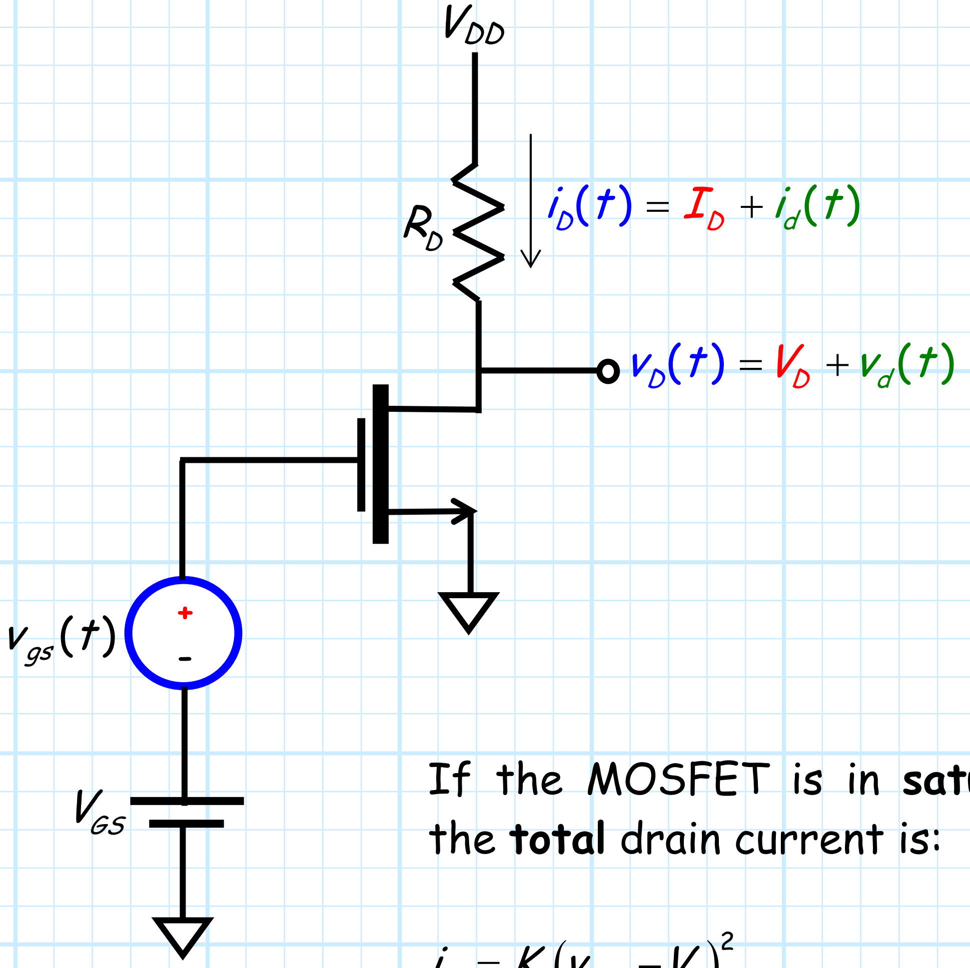
Small-Signal Response of MOSFET Circuits

Consider this circuit, which has both a **DC** and an AC **small-signal** source. As a result, each voltage and current in the circuit has **both** a **DC** and **small-signal** component.



If the MOSFET is in **saturation**, then the **total** drain current is:

$$\begin{aligned} i_D &= K\left(v_{GS} - V_t\right)^2 \\ &= K\left(V_{GS} + v_{gs} - V_t\right)^2 \\ &= K\left(V_{GS} - V_t\right)^2 + 2K\left(V_{GS} - V_t\right) v_{gs} + K\, v_{gs}^2 \end{aligned}$$

By looking at this equation, we find that the **third** term is **small** in comparison to the second **if**:

$$v_{gs} \ll 2\left(V_{GS} - V_t\right)$$

We call this equation the **small-signal** condition. For this case, we find that the drain current is:

$$\begin{aligned} i_D(t) &= I_D + i_d(t) \\ &\approx K\left(V_{GS} - V_t\right)^2 + 2K\left(V_{GS} - V_t\right)v_{gs}(t) \end{aligned}$$

Thus, it is evident that the **DC** equation is:

$$I_D = K\left(V_{GS} - V_t\right)^2$$

while the **small-signal** equation is:

$$i_d(t) = 2K\left(V_{GS} - V_t\right)v_{gs}(t)$$

Thus, we can define the MOSFET **transconductance** as:

$$g_m \doteq \frac{i_d}{v_{gs}} = 2K\left(V_{GS} - V_t\right)$$

Note this small-signal parameter g_m can likewise be **derived** from a small-signal analysis of the drain current:

$$\begin{aligned} i_d &= \left.\frac{d\, i_D}{dv_{GS}}\right|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(v_{GS}-V_t\right)\Big|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(V_{GS}-V_t\right) v_{gs} \\ &= g_m\, v_{gs} \end{aligned}$$

The MOSFET transconductance relates a small **change** in v_{GS} to a small **change** in drain current i_D. This change is completely dependent on the **DC bias** point of the MOSFET, V_{GS} and I_D.

We can likewise determine the **small-signal** voltage $v_{ds}(t)$. Writing the KVL for the drain-source leg, we find:

$$\begin{aligned} V_{DD} - R_D\, i_D &= v_{DS} \\ V_{DD} - R_D\left(I_D + i_d\right) &= V_{DS} + v_{ds} \\ V_{DD} - R_D\, I_D - R_D\, i_d &= V_{DS} + v_{ds} \end{aligned}$$

The **DC** equation is therefore:

$$V_{DD} - R_D\, I_D = V_{DS}$$

while the **small-signal** equation is:

$$-R_D\, i_d(t) = v_{ds}(t)$$

Since $i_d(t) = g_m\, v_{gs}(t)$, we find that the **small-signal** voltage $v_{ds}(t)$ is related to $v_{gs}(t)$ as:

$$\begin{aligned} v_{ds}(t) &= -R_D\, i_d(t) \\ &= -R_D\, g_m\, v_{gs}(t) \end{aligned}$$

or:

$$\frac{v_{ds}(t)}{v_{gs}(t)} = -R_D\, g_m$$

Thus, if $R_D\, g_m \gg 1$, we have small-signal **voltage gain**!