4.6 Small-Signal Operation and Models

Reading Assignment: *pp. 287-294*

Recall the **small-signal analysis** we performed on a BJT.

We can do a similar analysis on MOSFET amplifiers in order to determine their **small-signal gain !**

Remember there were essentially **three** steps:

1. Determine the small-signal device equation(s).

HO: Small-Signal Response of MOSFET Circuits

HO: Small-Signal Output Resistance

2. From the small-circuit device equations, determine a small-signal model.

HO: The MOSFET Small-Signal Model

3. Use the small-signal model to perform a small-signal analysis.

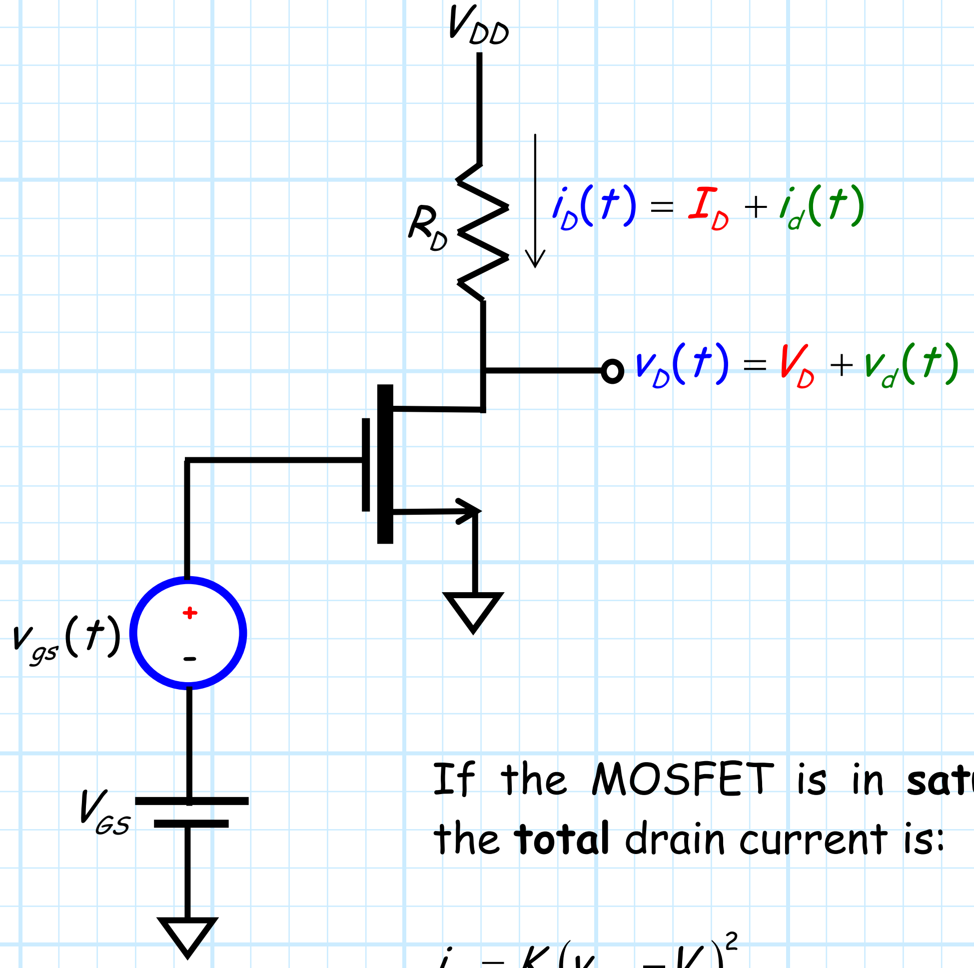
HO: Steps for Small-Signal Analysis

Example: A Small-Signal Analysis of a MOSFET Amp

Example: Another Small-Signal Analysis of a MOSFET Amp

Small-Signal Response of MOSFET Circuits

Consider this circuit, which has both a **DC** and an AC **small-signal** source. As a result, each voltage and current in the circuit has **both** a **DC** and **small-signal** component.



If the MOSFET is in **saturation**, then the **total** drain current is:

$$
\begin{aligned}
i_D &= K\left(v_{GS} - V_t\right)^2 \\
&= K\left(V_{GS} + v_{gs} - V_t\right)^2 \\
&= K\left(V_{GS} - V_t\right)^2 + 2K\left(V_{GS} - V_t\right) v_{gs} + K\, v_{gs}^2
\end{aligned}
$$

By looking at this equation, we find that the **third** term is **small** in comparison to the second **if**:

$$v_{gs} \ll 2\left(V_{GS} - V_t\right)$$

We call this equation the **small-signal** condition. For this case, we find that the drain current is:

$$\begin{aligned} i_D(t) &= I_D + i_d(t) \\ &\approx K\left(V_{GS} - V_t\right)^2 + 2K\left(V_{GS} - V_t\right)v_{gs}(t) \end{aligned}$$

Thus, it is evident that the **DC** equation is:

$$I_D = K\left(V_{GS} - V_t\right)^2$$

while the **small-signal** equation is:

$$i_d(t) = 2K\left(V_{GS} - V_t\right)v_{gs}(t)$$

Thus, we can define the MOSFET **transconductance** as:

$$g_m \doteq \frac{i_d}{v_{gs}} = 2K\left(V_{GS} - V_t\right)$$

Note this small-signal parameter g_m can likewise be **derived** from a small-signal analysis of the drain current:

$$\begin{aligned} i_d &= \left. \frac{d\, i_D}{dv_{GS}} \right|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(v_{GS} - V_t\right)\Big|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(V_{GS} - V_t\right) v_{gs} \\ &= g_m\, v_{gs} \end{aligned}$$

The MOSFET transconductance relates a small **change** in v_{GS} to a small **change** in drain current i_D. This change is completely dependent on the **DC bias** point of the MOSFET, V_{GS} and I_D.

We can likewise determine the **small-signal** voltage $v_{ds}(t)$. Writing the KVL for the drain-source leg, we find:

$$V_{DD} - R_D\, i_D = v_{DS}$$

$$V_{DD} - R_D\left(I_D + i_d\right) = V_{DS} + v_{ds}$$

$$V_{DD} - R_D\, I_D - R_D\, i_d = V_{DS} + v_{ds}$$

The **DC** equation is therefore:

$$V_{DD} - R_D\; I_D = V_{DS}$$

while the **small-signal** equation is:

$$-R_D\;\, i_d(t) = v_{ds}(t)$$

Since $i_d(t) = g_m\, v_{gs}(t)$, we find that the **small-signal** voltage $v_{ds}(t)$ is related to $v_{gs}(t)$ as:

$$\begin{aligned} v_{ds}(t) &= -R_D\, i_d(t) \\ &= -R_D\, g_m\, v_{gs}(t) \end{aligned}$$

or:

$$\frac{v_{ds}(t)}{v_{gs}(t)} = -R_D\, g_m$$

Thus, if $R_D\, g_m \gg 1$, we have small-signal **voltage gain**!

MOSFET Output Resistance

Recall that due to **channel-length modulation**, the MOSFET drain current is **slightly** dependent on v_{DS}, and thus is more accurately described as:

$$i_D = K\left(v_{GS} - V_t\right)^2\left(1 + \lambda v_{DS}\right)$$

In order to determine the relationship between the small-signal voltage v_{gs} and small-signal current i_d we can apply a **small-signal analysis** of this equation:

$$\begin{aligned} i_d &= \left.\frac{d\, i_D}{d\, v_{GS}}\right|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(v_{GS} - V_t\right)\Big|_{v_{GS}=V_{GS}} v_{gs} \\ &= 2K\left(V_{GS} - V_t\right) v_{gs} \\ &= g_m v_{gs} \end{aligned}$$

Note that we evaluated the derivative at the DC **bias** point V_{GS}. The result, as we expected, was the **transconductance** g_m.

We can likewise determine the relationship between small-signal voltage v_{ds} and the small-signal current i_d:

$$
\begin{aligned}
i_d &= \left.\frac{d\,i_D}{dv_{DS}}\right|_{v_{GS}=V_{GS}} v_{ds} \\
&= \lambda K \left(v_{GS} - V_t\right)^2 \Big|_{v_{GS}=V_{GS}} v_{ds} \\
&= \lambda K \left(V_{GS} - V_t\right)^2 v_{ds} \\
&= \frac{v_{ds}}{r_o}
\end{aligned}
$$

where r_o is defined as the MOSFET **output resistance**:

$$
\begin{aligned}
r_o &= \frac{1}{\lambda K \left(V_{GS} - V_t\right)^2} \\
&= \frac{1}{\lambda\, I_D}
\end{aligned}
$$

The small signal drain current i_d of a MOSFET (biased at a DC operating point I_D, V_{GS}) is therefore:

$$
i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}
$$

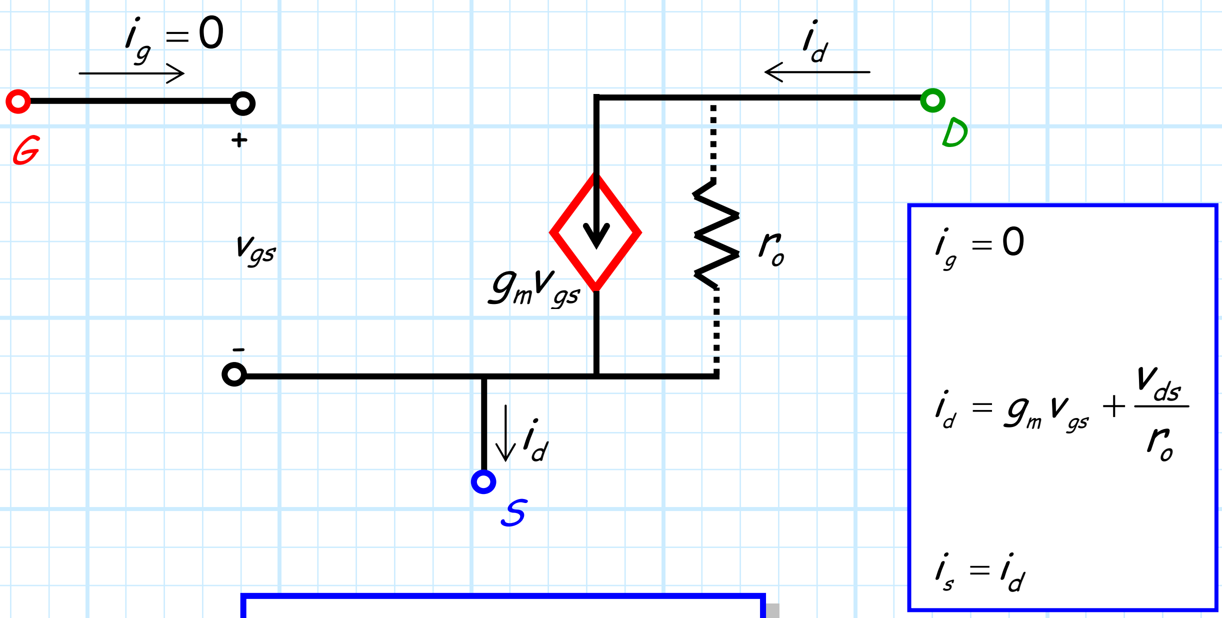
where:

$$
g_m = 2K\left(V_{GS} - V_t\right)
$$

$$
r_o = \frac{1}{\lambda K \left(V_{GS} - V_t\right)^2}
$$

The MOSFET Small-Signal Model

To determine the small-signal performance of a given MOSFET amplifier circuit, we can replace the MOSFET with its **small-signal model**:



NMOS Small-Signal Model

Note that this circuit provides **precisely** the same circuit equations as did our small-signal MOSFET analysis.

MOSFET Small-Signal Analysis Steps

Complete **each** of these steps if you **choose** to correctly complete a MOSFET Amplifier **small-signal** analysis.

Step 1: Complete a **D.C. Analysis**

Turn **off** all **small-signal** sources, and then complete a circuit analysis with the remaining **D.C. sources** only.

* Complete this DC analysis exactly, precisely, the same way you performed the DC analysis in section 4.3. That is, you assume (the **saturation** mode), enforce, analyze, and **check (do not forget to check!)**.

* Note that you enforce and check exactly, precisely the same the same equalities and inequalities as discussed in section 4.3.

* Remember, if we "turn off" a **voltage** source (e.g., $v_i(t) = 0$), it becomes a **short** circuit.

* However, if we "turn off" a **current** source (e.g., $i_i(t) = 0$), it becomes an **open** circuit!

* Small-signal amplifiers frequently employ large **capacitors**. Remember, the impedance of a capacitor at **DC** is infinity—a DC **open** circuit.

The goal of this DC analysis is to determine:

1) The DC voltage V_{GS} for **each** MOSFET.

2) The DC voltage V_{DS} for **each** MOSFET (you need this value for the CHECK).

You do not **necessarily** need to determine any other DC currents or voltages within the amplifier circuit!

Once you have found these values, you can **CHECK** your active assumption, and then move on to **step 2**.

Step 2: Calculate the **small-signal circuit parameters** for each MOSFET.

Recall that we now understand **2** MOSFET small-signal parameters:

$$g_m = 2K(V_{GS} - V_t) \qquad r_o = \frac{1}{\lambda K (V_{GS} - V_t)^2}$$

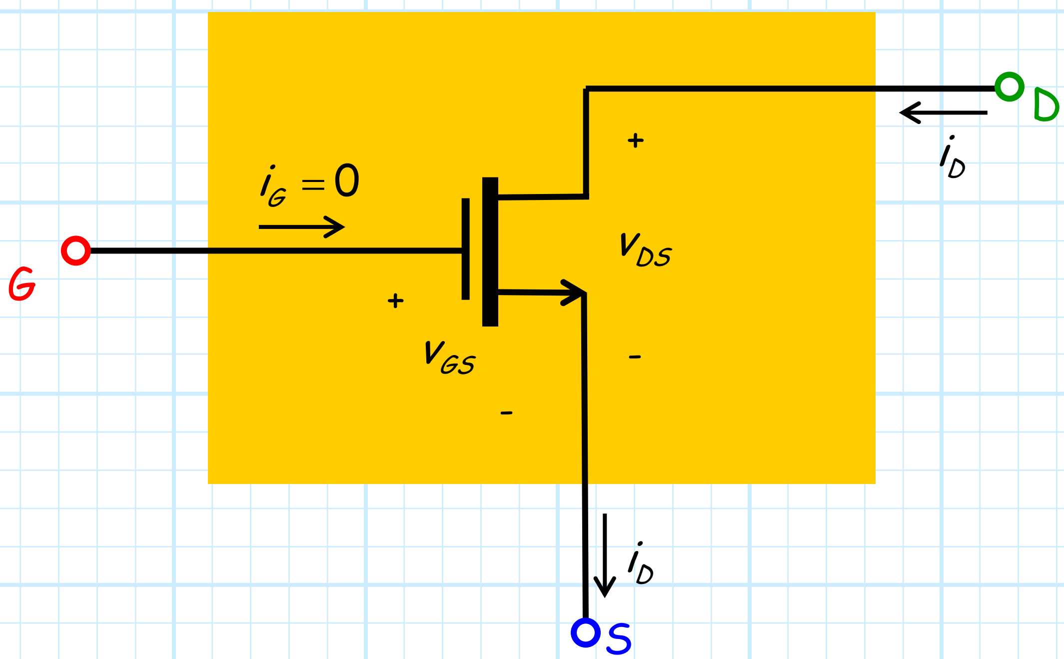
Step 3: Carefully replace all MOSFETs with their **small-signal circuit model.**

This step often gives students **fits**!

However, it is actually a **very simple** and straight-forward step. It does require four important things from the student—**patience**, **precision**, **persistence** and **professionalism**!

First, note that a **MOSFET** is:

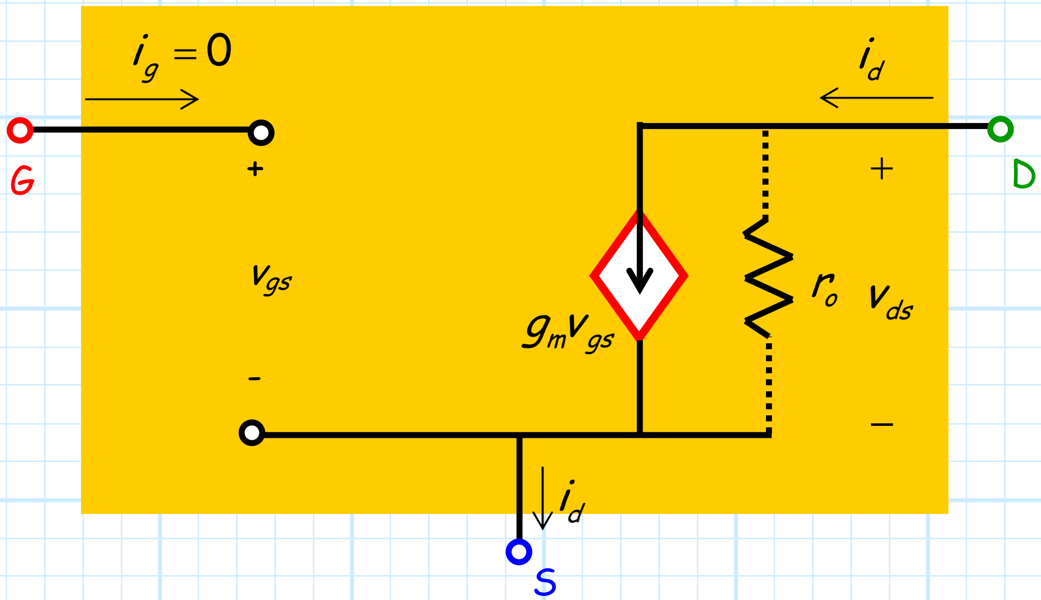
> A device with **three** terminals, called the gate, drain, and source. Its behavior is described in terms of current i_D and voltages v_{GS}, v_{DS}.



Now, **contrast** the MOSFET with its small-signal circuit model. A MOSFET small-signal circuit model is:

A device with **three** terminals, called the gate, drain, and source. Its behavior is described in terms of current i_d and voltages v_{gs}, v_{ds}.

Exactly the **same**—what a coincidence!



Therefore, replacing a MOSFET with its small-signal circuit model is very simple—you simply change the stuff **within** the orange box!

Note the parts of the circuit **external** to the orange box do not change! In other words:

1) **every** device attached to the MOSFET **terminals** (i.e, gate, drain, source) is attached in **precisely** the same way to the terminals of the **circuit model**.

2) **every** external voltage or current (e.g., v_i, v_o, i_R) is defined in **precisely** the same way both before and after the MOSFET is replaced with its circuit model is (e.g., if the output voltage is the drain voltage in the MOSFET circuit, then the output voltage is **still** the drain voltage in the small-signal circuit!).

Step 4: Set all **D.C. sources** to zero.

Remember:

A zero voltage DC source is a **short**.

A zero current DC source is an **open**.

The schematic in now in front of you is called the **small-signal circuit**. Note that it is **missing** two things—**DC sources** and MOSFET **transistors**!

* Note that steps three and four are **reversible**. You could turn off the DC sources **first**, and then replace all MOSFETs with their small-signal models—the resulting small-signal circuit will be the **same**!

* You will find that the small-signal circuit schematic can often be greatly **simplified**. Once the DC voltage sources are turned **off**, you will find that the terminals of many devices are connected to **ground**.

* Remember, all terminals connected to ground are **also** connected to each other! For **example**, if the source terminal is connected to ground, and one terminal of a resistor is connected to ground, then that resistor terminal is connected to the source!

* As a result, you often find that resistors in different parts of the circuit are actually connected in **parallel**, and thus can be **combined** to simplify the circuit schematic!

* Finally, note that the AC impedance of a **very large capacitor** (i.e., $|Z_C| = 1/\omega C$) is small for all but the lowest frequencies ω. If this impedance is smaller than the other circuit elements (e.g., < 10Ω), we can view the impedance as **approximately zero**, and thus replace the **large** capacitor with a (AC) **short**!

Organizing and **simplifying** the small-signal circuit will pay **big** rewards in the next step, when we **analyze** the small-signal circuit.

Step 5: Analyze **small-signal circuit**.

We now can **analyze** the small-signal **circuit** to find all small-signal **voltages** and **currents**.

* For small-signal **amplifiers**, we typically attempt to find the small-signal output voltage v_o in terms of the small-signal input voltage v_i. From this result, we can find the **voltage gain** of the amplifier.

* Note that this analysis requires **only** the knowledge you acquired in **EECS 211**! The small-signal circuit will consist **entirely** of resistors and (small-signal) voltage/current sources. These are **precisely** the same resistors and sources that you learned about in EECS 211. You analyze them in **precisely** the same way.

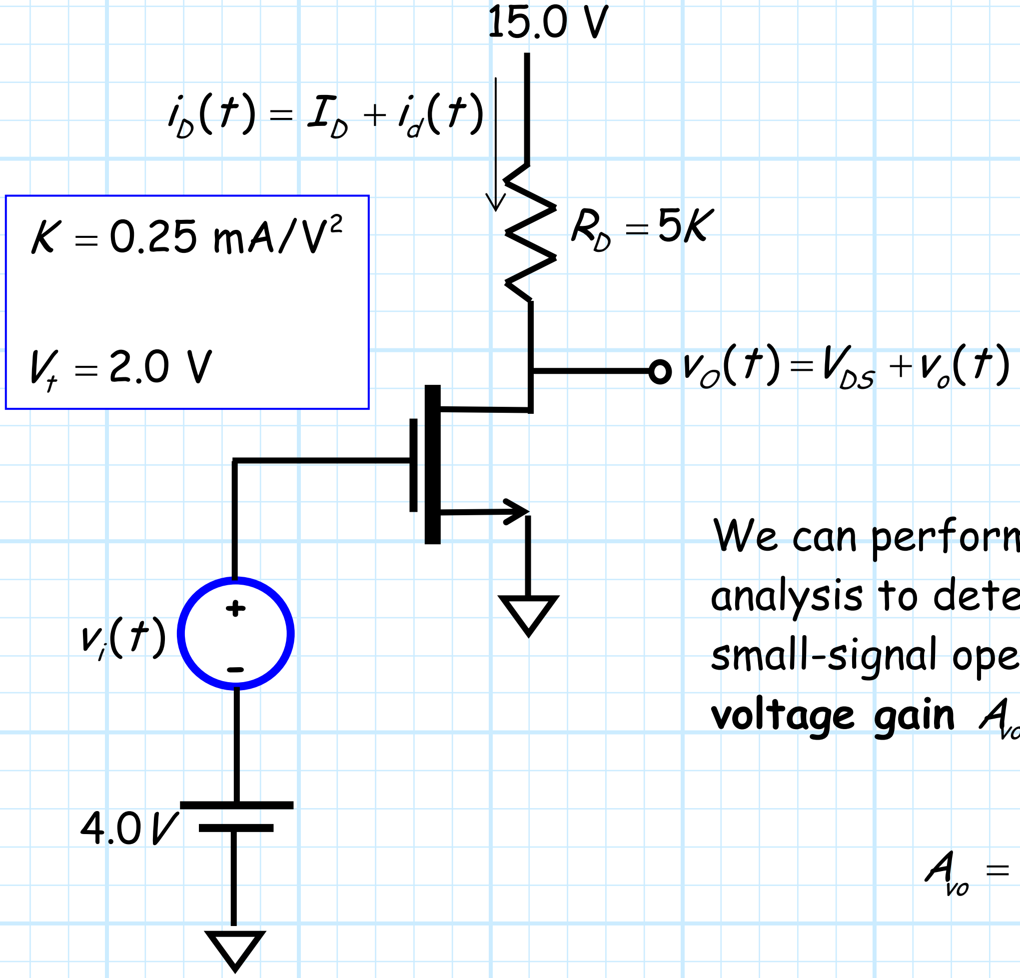
* Do **not** attempt to insert any MOSFET knowledge into your small-signal circuit analysis—there are **no** MOSFETs in a small-signal circuit!!!!!

* Remember, the MOSFET circuit model contains **all** of our MOSFET small-signal knowledge, we **do** not—indeed **must** not—add any more information to the analysis.

You must **trust** completely the MOSFET small-signal circuit model. It **will** give you the correct answer!

Example: A Small-Signal Analysis of a MOSFET Amplifier

Let's again consider this simple **NMOS Amplifier**:

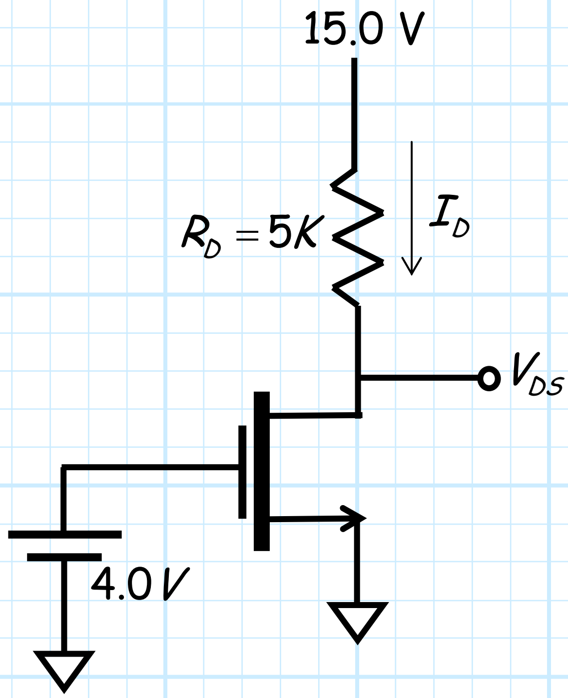


We can perform a small-signal analysis to determine the small-signal open-circuit **voltage gain** A_{vo}:

$$A_{vo} = \frac{v_o(t)}{v_i(t)}$$

Step 1: DC Analysis

Turning **off** the small signal source leaves a DC circuit of:



We **ASSUME** saturation, so that we **ENFORCE**:

$$I_D = K\left(V_{GS} - V_t\right)^2$$

It is evident that:

$$V_{GS} = 4.0 \text{ V}$$

Therefore the DC drain current is:

$$\begin{aligned} I_D &= K\left(V_{GS} - V_t\right)^2 \\ &= 0.25(4-2)^2 \\ &= 1.0 \text{ mA} \end{aligned}$$

Thus, the DC voltage V_{DS} can be determined from *KVL* as:

$$\begin{aligned} V_{DS} &= 15.0 - I_D R_D \\ &= 15.0 - 1(5) \\ &= 10.0 \text{ V} \end{aligned}$$

We **CHECK** our results and find:

$$V_{GS} = 4.0 > V_t = 2.0$$ ✓

and:

$$V_{DS} = 10.0 > V_{GS} - V_t = 2.0 \checkmark$$

Step 2: Determine the small-signal parameters

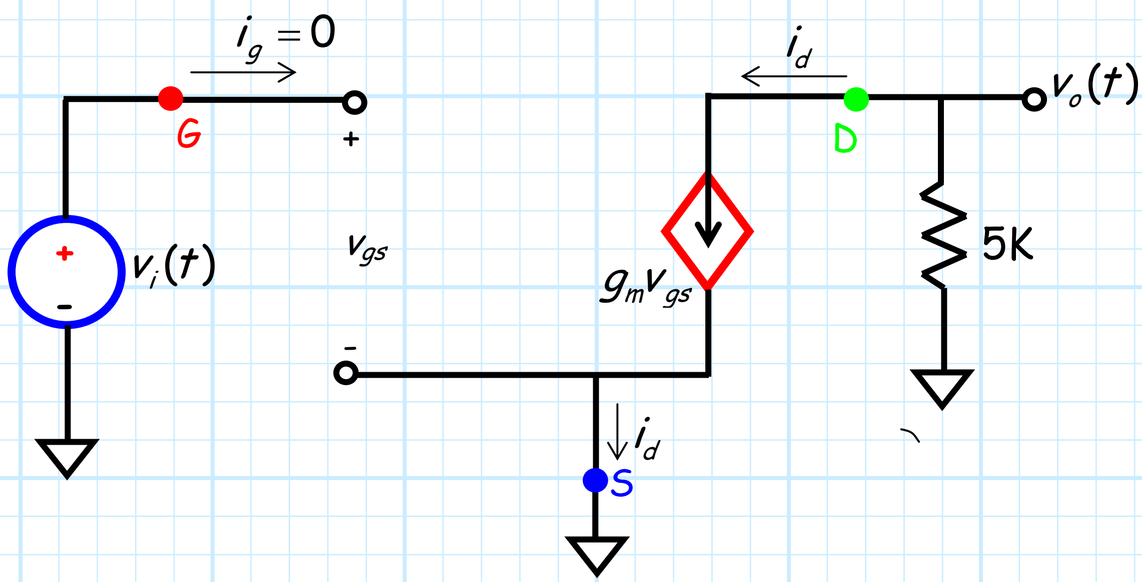
We find that the **transconductance** is:

$$\begin{aligned} g_m &= 2K\left(V_{GS} - V_t\right) \\ &= 2(0.25)(4.0 - 2.0) \\ &= 1 \ \text{mA}/\text{V} \end{aligned}$$

Note that **no** value of λ was given, so we will assume $\lambda = 0$, and thus **output resistance** $r_o = \infty$.

Steps 3 and 4: Determine the small-signal circuit

We now turn off the **two** DC voltage source, and replace the MOSFET with its **small signal model**. The result is our **small-signal circuit**:



Step 5: Analyze the small-signal circuit

The analysis of this small-signal circuit is fairly **straightforward**. First, we note from KVL that:

$$v_{gs} = v_i$$

and that:

$$\begin{aligned} i_d &= g_m v_{gs} \\ &= 1.0\, v_{gs} \\ &= v_{gs} \end{aligned}$$

and that from Ohm's Law:

$$v_o = -5\, i_d$$

Combining these equations, we find that:

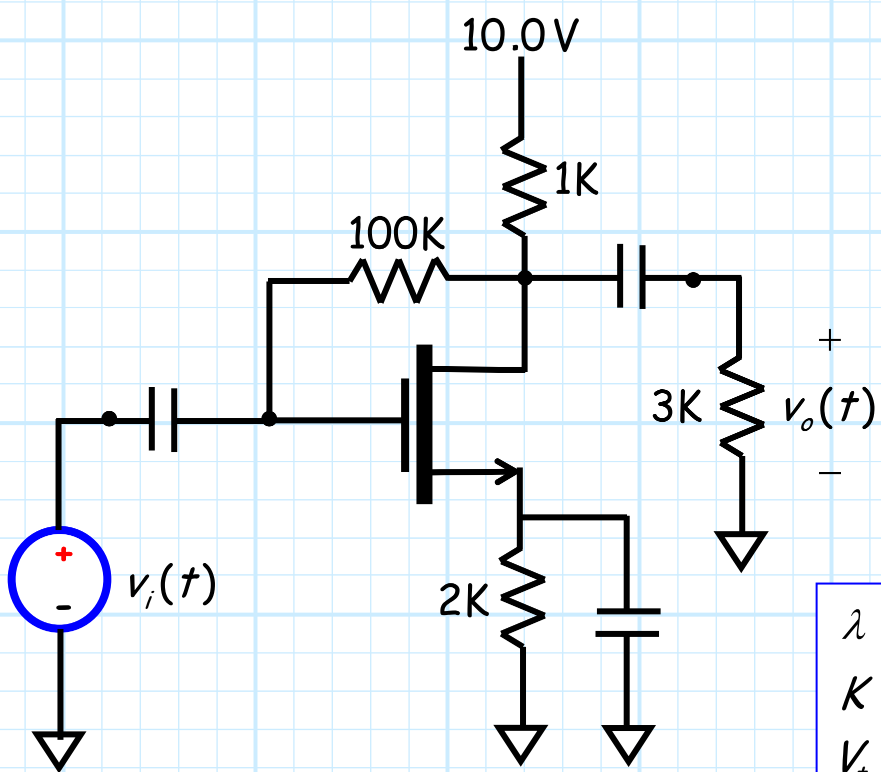
$$v_o = -5\, v_i$$

And thus the **small-signal** open-circuit voltage gain of this amplifier is:

$$A_{vo} = \frac{v_o(t)}{v_i(t)} = -5.0$$

Example: Another Small-Signal Analysis of a MOSFET Amplifier

Let's determine the small-signal **voltage gain** $A_v = v_o/v_i$ (note **not** the open-circuit gain!) of the following amplifier:



$\lambda = 0.005 \text{ V}^{-1}$

$K = 0.4 \text{ mA/V}^2$

$V_t = 2.0 \text{ V}$

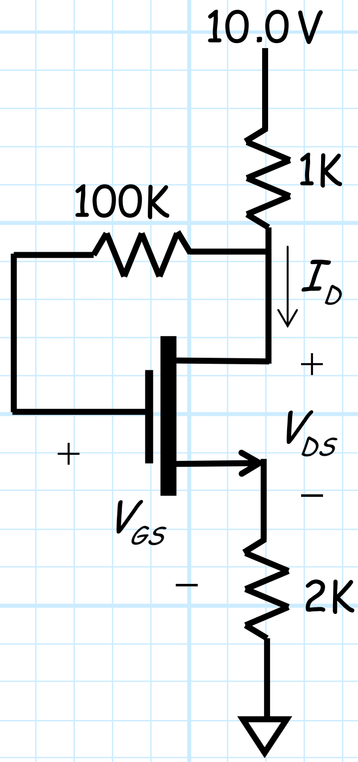
C's are COUS

Step 1: DC Analysis

Capacitors are **open** circuits at DC, therefore the DC circuit is:

We **ASSUME** the MOSFET is in saturation, thus we **ENFORCE**:

$$I_D = K\left(V_{GS} - V_t\right)^2$$



Since $I_G = 0$, we find that $V_G = V_D$, and thus $V_{GS} = V_{DS}$. From KVL, we find:

$$10.0 - (1)I_D - V_{DS} - (2)I_D = 0$$

Or since $V_{GS} = V_{DS}$:

$$V_{GS} = 10.0 - 3I_D$$

Combining this with $I_D = K\left(V_{GS} - V_t\right)^2$, we get a **quadratic equation** of V_{GS}:

$$V_{GS} = 10.0 - 3K\left(V_{GS} - V_t\right)^2$$

The **solutions** to this equation are:

$$V_{GS} = 4.2 \text{ V} \quad \text{and} \quad V_{GS} = -1.0 \text{ V}$$

Don't panic! Only **one** of these solutions satisfy our saturation assumption: $V_{GS} = 4.2 > 2.0 = V_t$.

Step 2: Determine Small-Signal Parameters

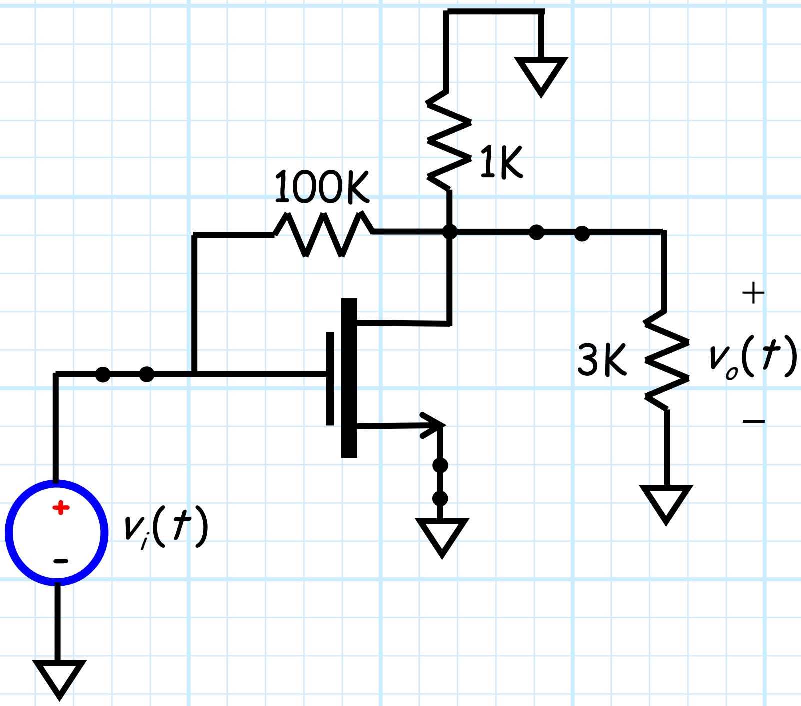
$$
\begin{aligned}
g_m &= 2K\left(V_{GS} - V_t\right) \\
&= 2(0.4)(4.2 - 2.0) \\
&= 1.76 \ \text{mA/V}
\end{aligned}
$$

$$
\begin{aligned}
r_o &= \frac{1}{\lambda K\left(V_{GS} - V_t\right)^2} \\
&= \frac{1}{0.005(0.4)(4.2 - 2.0)^2} \\
&= 103 \ \text{K}\Omega
\end{aligned}
$$

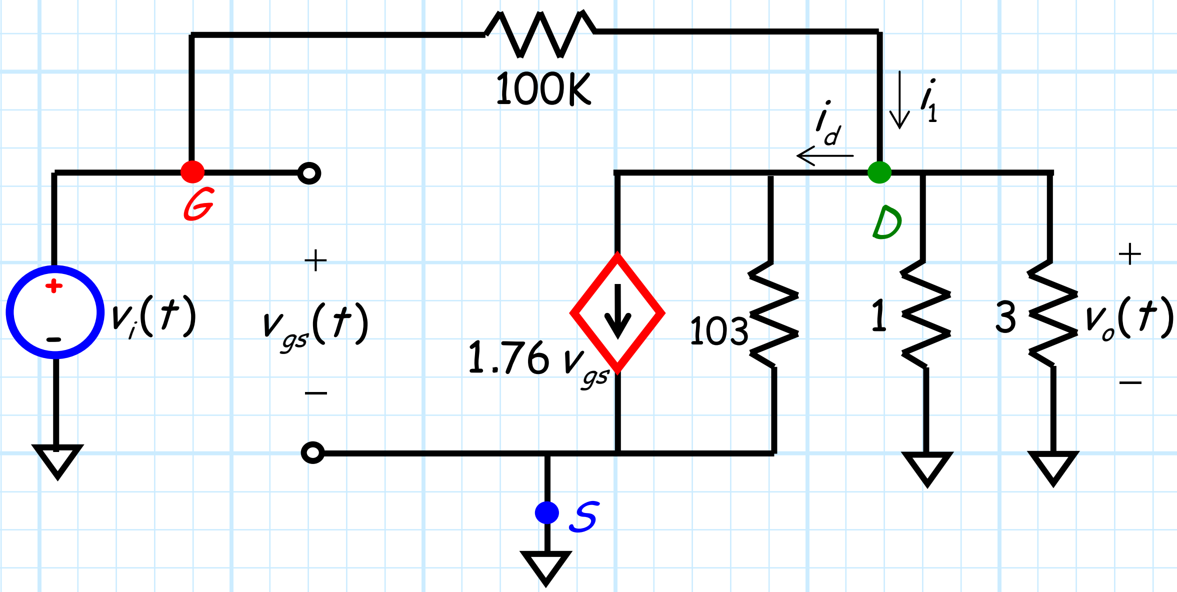
Steps 3 and 4: Determine the small-signal circuit

a) Turn **off** the DC voltage source.

b) Replace the large capacitors with **short** circuits.



c) Replace the MOSFET with its **small-signal model**.



We find first that $v_{gs} = v_i$. We likewise see from KCL that current i_1 is:

$$\begin{aligned} i_1 &= 1.76\, v_{gs} + \frac{v_o}{1} + \frac{v_o}{3} + \frac{v_o}{103} \\ &= 1.76\, v_i + 1.334\, v_o \end{aligned}$$

From Ohm's Law, we likewise find that i_1 is:

$$i_1 = \frac{v_i - v_o}{100}$$

Combining these two equations, we find:

$$v_i - v_o = 176\, v_i + 133.4\, v_o$$

And from this we find that the **small-signal voltage gain** is:

$$A_v = \frac{v_o}{v_i} = \frac{-175}{134.4} = -1.31 \quad \text{not much gain!}$$