5.4 BJT Circuits at DC

Reading **Assignment**: Review 312 BJT material

You learned about **BJTs** in 312, but if I where you I'd **review** this material!

HO: BJT Structures and Modes of Operation

HO: BJT Symbols and Conventions

HO: A Mathematical Description of BJT Behavior

HO: Steps for DC Analysis of BJT Circuits

HO: Hints for DC Analysis of BJT Circuits

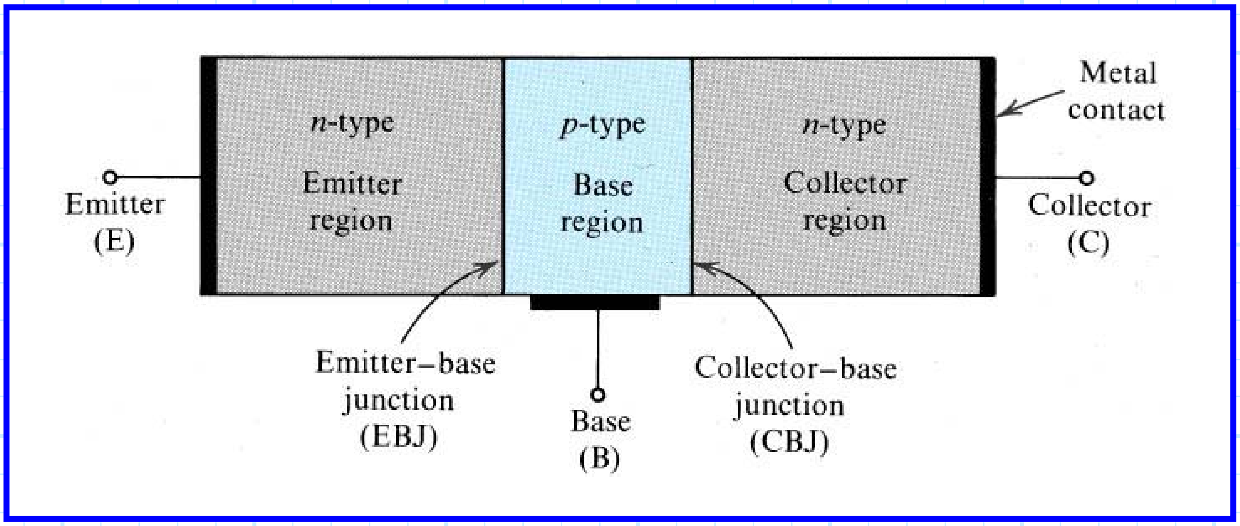
HO: Example: DC Analysis of a BJT Circuit

HO: Example: Another DC Analysis of a BJT Circuit

HO: Example: An Analysis of a *pnp* BJT Circuit

BJT Structure and Modes of Operation

First, let's start with the ***npn*** Bipolar Junction Transistor (BJT). As the **name** implies, the *npn* BJT is simply an hunk of *p*-type Silicon sandwiched between two slices of *n*-type material:



Each of the **three Silicon regions** has one terminal electrode connected to it, and thus the *npn* BJT is a **three terminal** device.

The three terminals are **named**:

1. *Collector*
2. *Base*
3. *Emitter*

Note that this *npn* BJT structure creates **two *p-n* junctions** !

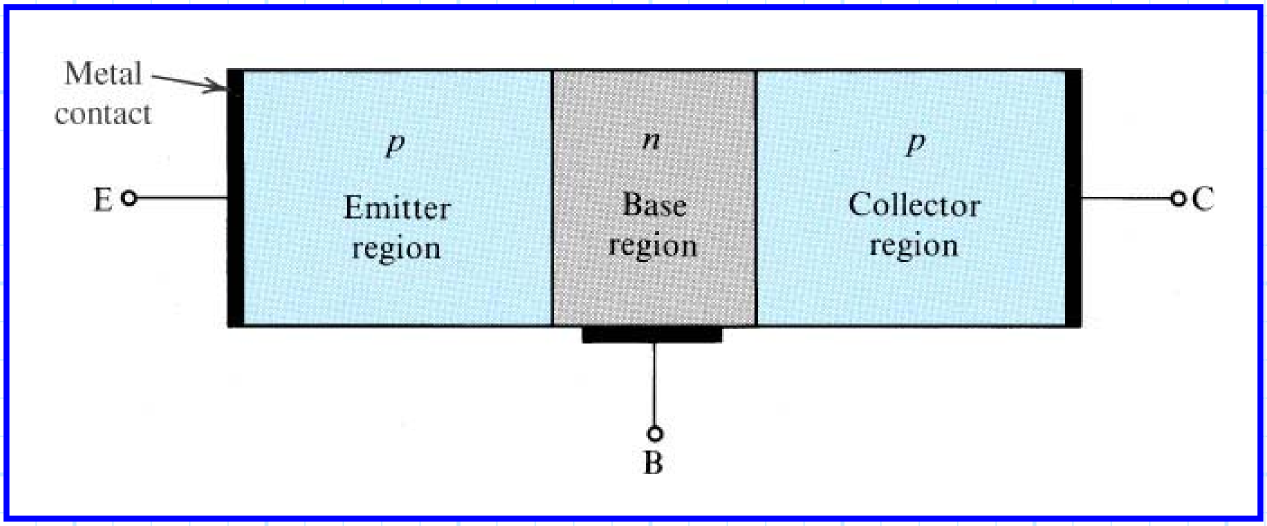
* The junction between the ***n*-type collector** and the ***p*-type base** is called the **Collector-Base Junction** (CBJ).

  Note for the **CBJ**, the **anode** is the **base**, and the **cathode** is the **collector**.

* The junction between the ***n*-type emitter** and the ***p*-type base** is called the **Emitter-Base Junction** (EBJ).

  Note for the **EBJ**, the **anode** is the base, and the **cathode** is the emitter.

Now, we find that the ***pnp*** BJT is simply the **complement** of the *npn* BJT—the *n*-type silicon becomes *p*-type, and vice versa:



Thus, the *pnp* BJT **likewise** has **three** terminals (with the same names as the *npn*), as well as **two** *p-n* junctions (the CBJ and the EBJ).

* For the *pnp* BJT, the **anode** of the **CBJ** is the **collector**, and the **cathode** of the **CBJ** is the **base**.

* Likewise, the **anode** of the **EBJ** is the **emitter**, and the **cathode** of the **EBJ** is the **base.**

Note that these results are precisely **opposite** that of ***npn*** BJT.

Now, we know that **each** *p-n* junction (for either *npn* or *pnp*) has **three** possible **modes**:

1. *forward biased*
2. *reverse biased*
3. *breakdown*

We find that **breakdown** is **not** generally a useful mode for transistor operation, and so we will **avoid** that mode.

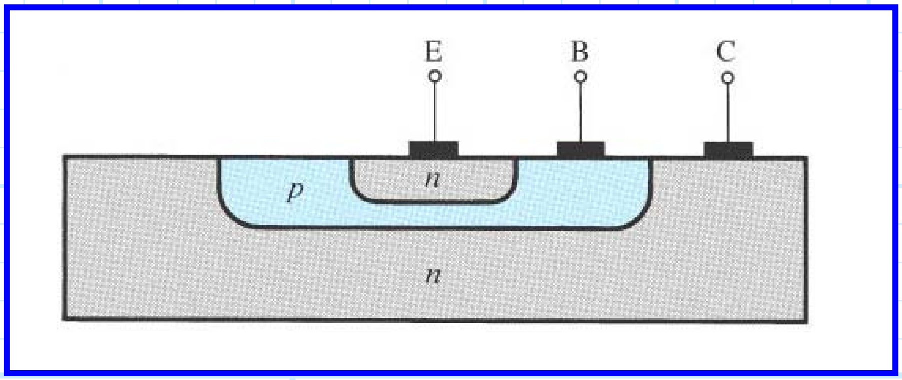
Given then that there are **two useful** *p-n* junction modes, and **two** *p-n* junctions for each BJT (i.e., CBJ and EBJ), a BJT can be in one of **four** modes!

MODE	EBJ	CBJ
1	Reverse	Reverse
2	Forward	Reverse
3	Reverse	Forward
4	Forward	Forward

Now, let's give each of these four BJT modes a **name**:

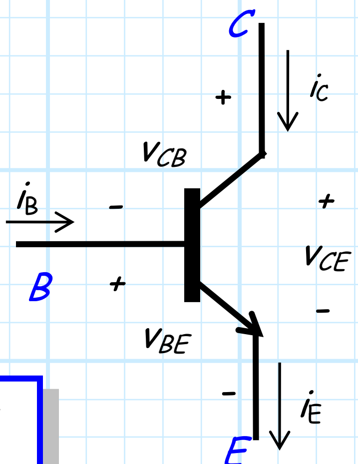
MODE	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Reverse Active	Reverse	Forward
Saturation	Forward	Forward

We will find that the **Reverse Active** mode is of **limited** usefulness, and thus the **three basic operating modes** of a BJT are Cutoff, Active, and Saturation.

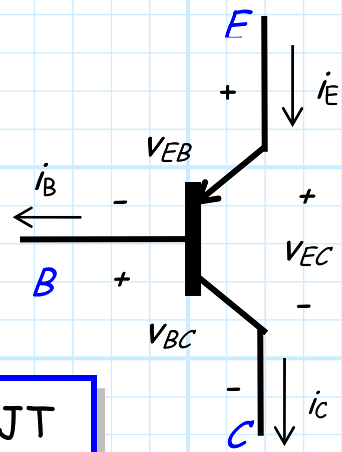


An Integrated Circuit BJT

BJT Symbols and Conventions



npn **BJT Circuit Symbol**



pnp **BJT Circuit Symbol**

From KCL **only** we find:

$$i_E = i_B + i_C$$

From KVL **only** we find:

$$v_{CE} = v_{CB} + v_{BE} \qquad (npn)$$

$$v_{EC} = v_{EB} + v_{BC} \qquad (pnp)$$

Note that:

* The circuit **symbols** are very **similar** to MOSFETs, with *npn* like N-MOS and *pnp* like P-MOS.

* Positive **current** is defined in **opposite** directions for *npn* and for *pnp* (just like N-MOS and PMOS!).

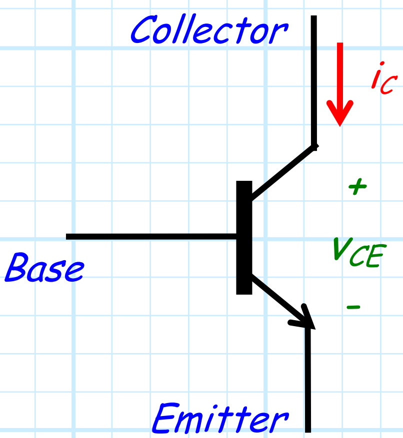
* The **voltages** are of **opposite** polarity for *npn* and *pnp*. Specifically, for *npn* we use v_{BE}, v_{CE} and v_{CB}, whereas for *pnp* we use v_{EB}, v_{EC} and v_{BC}. This convention typically results in **positive** voltage values for **both** *npn* and *pnp* (**unlike** the MOSFET convention!).

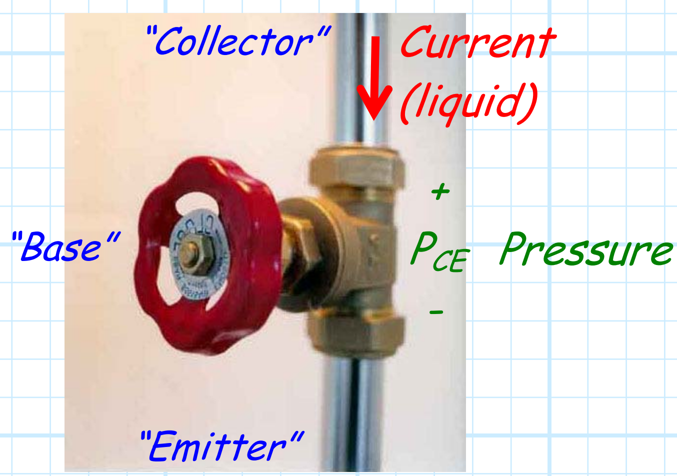
* The **base current** i_B is **not** equal to zero, therefore $i_E \neq i_C$ (unlike MOSFETS)!

A Mathematical Description of BJT Behavior

A transistor is **somewhat** like a valve used to control **liquid** current. In this **analogy** we find:

This	**…is (sort of) like…**	**this**
Electric current		*Liquid current*
Voltage		*Pressure*
Collector		*Valve Input*
Emitter		*Valve Output*
Base		*Valve Control Knob*





Cutoff is analogous to the value being **completely closed**—no current will flow through the value, **regardless** of how much pressure (v_{CE}) is applied.

Saturation is analogous to the value being **completely open**—it takes almost **no pressure** (v_{CE}) to get **a lot of current** to flow through the valve.

Active mode is analogous to having the value **partially open**—it requires **some** pressure (v_{CE}) to get current to flow. Moreover, this current can be **increased** by **further opening** the valve (increasing base current i_B) or **decreased** by **further closing** the valve (decreasing base current i_B).

We will find that BJT behavior is in many was **similar** to MOSFET behavior!

ACTIVE MODE

We found earlier that forward biasing the **emitter-base** junction (EBJ) results in **collector** (drift) current. The junction voltage for the EBJ is v_{BE} (for npn).

Thus, in active mode, the voltage **base**-to-**emitter** v_{BE} controls the **collector** current i_C. Specifically, we find that:

$$i_C = I_S \, e^{v_{BE}/V_T} \quad (npn)$$

$$i_C = I_S \, e^{v_{EB}/V_T} \quad (pnp)$$

Here we should note **two** things:

1. *The active mode equation is very **similar** to the* p-n *junction diode equation.*

No surprise here! The collector current is directly proportional to the **diffusion** current across the EBJ. That's why the equation is just like the diffusion current equation for a ***pn* junction**.

In fact, I_S is **scale current** (a device parameter), and V_T is the **thermal voltage** (25 mV)—the same values used to describe junction diodes!

2. *A BJT in ACTIVE mode is **analogous** to a MOSFET in SATURATION mode.*

Recall that for a MOSFET in SATURATION, the **drain** current i_D is "**controlled**" by the **gate**-to-**source** voltage v_{GS}.

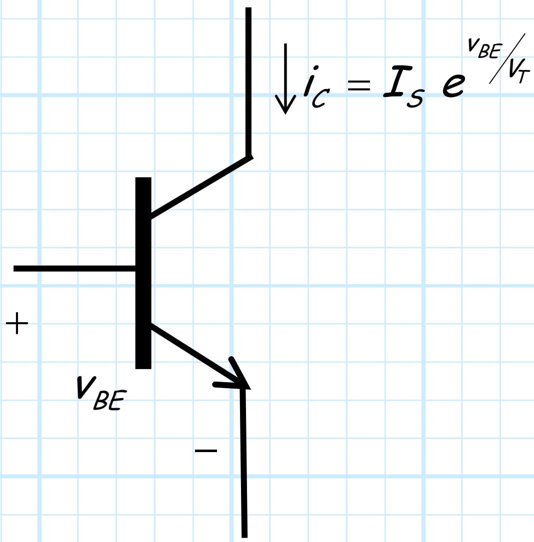
Likewise, for a BJT in ACTIVE mode, the **collector** current i_C is "**controlled**" by the **base**-to-**emitter** voltage v_{BE}.

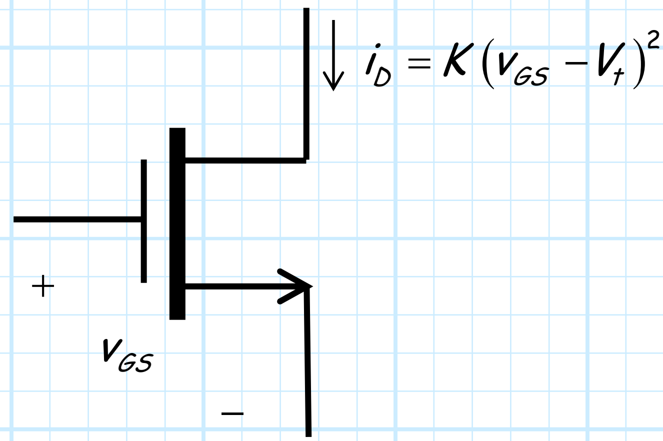
Note the **analogies**!

i_D analogous to i_C

v_{BE} analogous to v_{GS}

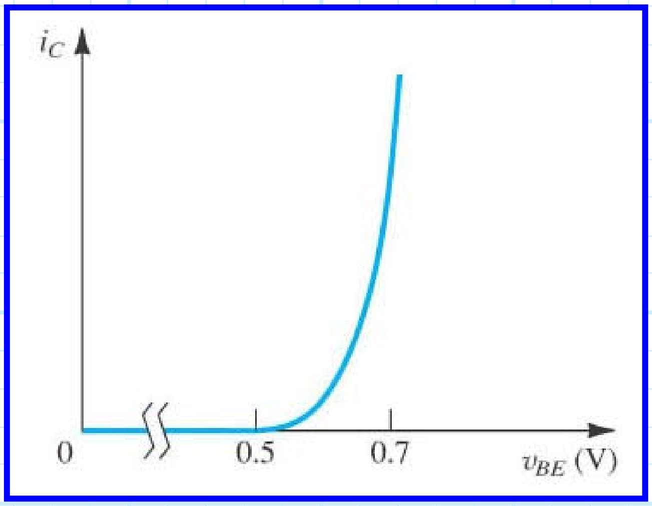
ACTIVE analogous to SATURATION

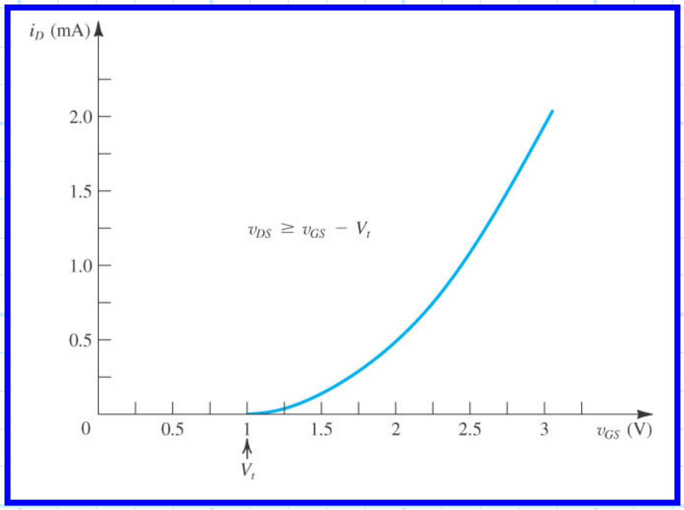




npn in ACTIVE mode

NMOS in SATURATION mode





Note also that a **necessary** (but not sufficient) condition for a *npn* BJT to be in ACTIVE mode is that $v_{BE} > 0$ (i.e., the **EBJ** is **forward biased**).

This is **analogous** to an NMOS in SATURATION, where a **necessary** (but not sufficient) condition is that $v_{GS} > V_t$ (i.e., the channel is conducting).

Likewise, for a BJT to be in the ACTIVE mode, the **CBJ** must be in **reverse bias** (i.e, v_{BC}<0). Assuming that the forward biased EBJ results in $v_{BE} \approx 0.7\text{ V}$, we can use **KVL** to determine that the CBJ will be reverse biased only when:

$$v_{CE} > 0.7\text{ V} \quad \text{for } npn \text{ in ACTIVE}$$

$$v_{EC} > 0.7\text{ V} \quad \text{for } pnp \text{ in ACTIVE}$$

These statements above are **analogous** to the MOSFET inequality $v_{DS} > v_{GS} - V_t$ for MOSFET SAT. (more on this later!).

Now, we are tempted to make **another analogy** between base **current** i_B and gate **current** i_G, but here the analogies **end**!

Recall i_G=0 **always**, but for BJTs we find that i_B is **not equal to zero** (generally).

Instead, we found that although **most** of the charge carriers (e.g., holes or free electrons) diffusing across the EBJ end up "**drifting**" across the CBJ into the **collector**, **some** charge carriers do "exit" the **base** terminal.

Recall, however, that for every **one** charge carrier that leaves the **base** terminal, there are typically **50 to 250** (depending on the BJT) charge carriers that drift into the collector.

As a result, the **collector current** for ACTIVE mode is typically 50 to 250 times **larger** than the **base current**! I.E.:

$$50 < \frac{i_C}{i_B} < 250 \quad \text{typically, for BJT ACTIVE}$$

The **precise** value of this ratio is the **device parameter** β **(beta)** :

$$\beta \doteq \frac{i_C}{i_B} \qquad \text{for BJT ACTIVE mode}$$

Thus, we find that the **base current** can be expressed as:

$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta}\, e^{v_{BE}/V_T} \qquad (npn)$$

$$i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta}\, e^{v_{EB}/V_T} \qquad (pnp)$$

Likewise, from **KCL**, we can determine the **emitter current** for a BJT in the ACTIVE mode:

$$\begin{aligned} i_E &= i_C + i_B \\ &= \beta\, i_B + i_B \\ &= (\beta + 1)\, i_B \end{aligned}$$

Or similarly,

$$\begin{aligned} i_E &= i_C + i_B \\ &= i_C + \frac{i_C}{\beta} \\ &= \left(1 + \frac{1}{\beta}\right) i_C \\ &= \left(\frac{\beta + 1}{\beta}\right) i_C \end{aligned}$$

An **alternative** to device parameter β is the **device parameter** α, defined as:

$$\alpha = \frac{\beta}{\beta + 1}$$

Note that the value of α will be just **slightly less than one**.

We can thus **alternatively** express the current relationships as:

$$i_C = \alpha\, i_E \qquad\qquad i_B = (1 - \alpha)\, i_E$$

And therefore:

$$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha}\, e^{v_{BE}/V_T} \quad (npn)$$

$$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha}\, e^{v_{EB}/V_T} \quad (pnp)$$

Recall that the **exponential** expression for a *pn* junction turned out to be of **limited** use, as it typically led to unsolvable **transcendental equations.**

The **same** is true for **these** exponential equations! We will thus generally use the equations below to **approximate** the behavior of a BJT in the ACTIVE mode:

$$v_{BE} \approx 0.7 \quad i_C = \beta\, i_B \quad v_{CE} > 0.7 \quad (npn \text{ in ACTIVE})$$

$$v_{EB} \approx 0.7 \quad i_C = \beta\, i_B \quad v_{EC} > 0.7 \quad (pnp \text{ in ACTIVE})$$

SATURATION MODE

Recall for BJT SATURATION mode that **both** the CBJ and the EBJ are **forward biased**.

Thus, the collector current is due to **two** physical mechanisms, the **first** being charge carriers (holes or free-electrons) that

drift across the CBJ (just like ACTIVE mode), and the **second** being charge carriers that **diffuse** across the forward biased CBJ!

As a result, a **second term** appears in our mathematical description of **collector current** (when the BJT is in SATURATION):

$$i_C = I_S\, e^{v_{BE}/V_T} - \left(\frac{I_S}{\alpha_R}\right) e^{v_{BC}/V_T} \quad (npn)$$

$$i_C = I_S\, e^{v_{EB}/V_T} - \left(\frac{I_S}{\alpha_R}\right) e^{v_{CB}/V_T} \quad (pnp)$$

where α_R represents the **same** device parameter α discussed earlier (for ACTIVE mode), with the only difference that it specifies the value of α specifically for the **CBJ**.

This second term describes the current due to **diffusion** across the CBJ. Note that this current is in the **opposite** direction of the drift current (the first term), hence the **minus** sign in the second term.

Now using **KVL** (i.e., $v_{CE} = v_{CB} + v_{BE}$), we can write this collector current equation as:

$$i_C = I_S \, e^{v_{BE}/V_T} - \left(\frac{I_S}{\alpha_R}\right) e^{(v_{BE}-v_{CE})/V_T}$$

$$= I_S \, e^{v_{BE}/V_T} \left(1 - \frac{e^{-v_{CE}/V_T}}{\alpha_R}\right)$$

Thus, we can conclude:

$$i_C = I_S \, e^{v_{BE}/V_T} \left(1 - \frac{e^{-v_{CE}/V_T}}{\alpha_R}\right) \quad \text{for } npn \text{ in SAT.}$$

$$i_C = I_S \, e^{v_{EB}/V_T} \left(1 - \frac{e^{-v_{EC}/V_T}}{\alpha_R}\right) \quad \text{for } pnp \text{ in SAT.}$$

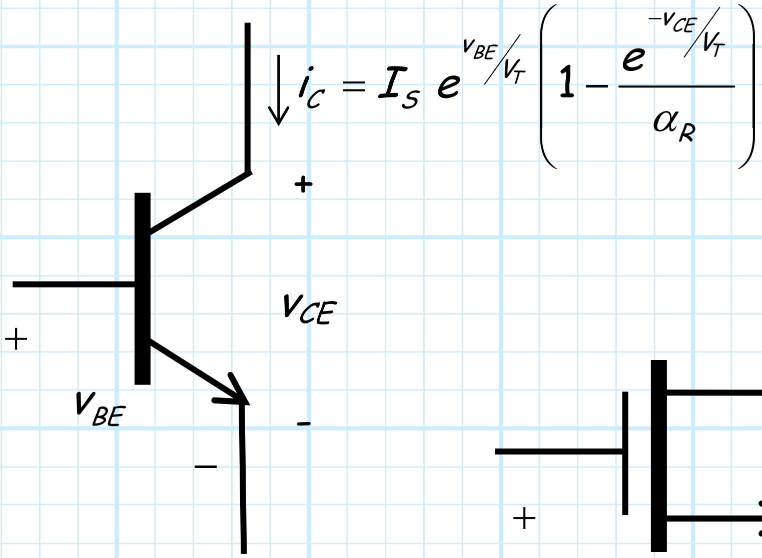
It is thus clear that for a BJT in SATURATION, the collector current i_C is dependent on **both** v_{BE} and v_{CE}.

This is precisely **analogous** to the TRIODE mode for MOSFETS!

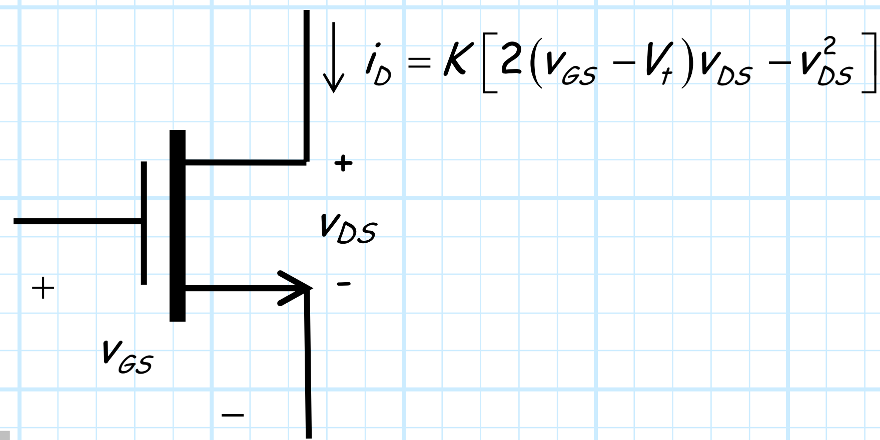
Recall for **triode** mode, drain current i_D is dependent on both v_{GS} and v_{DS}. We thus have discovered **two** new analogies:

v_{CE} analogous to v_{DS}

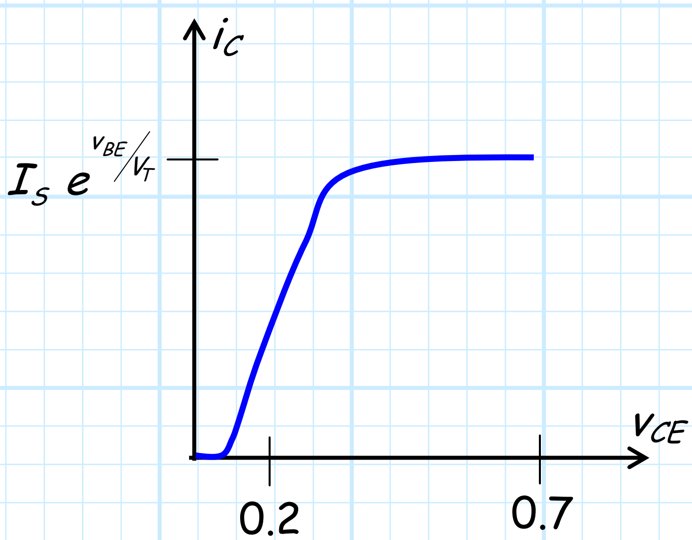
SATURATION analogous to TRIODE

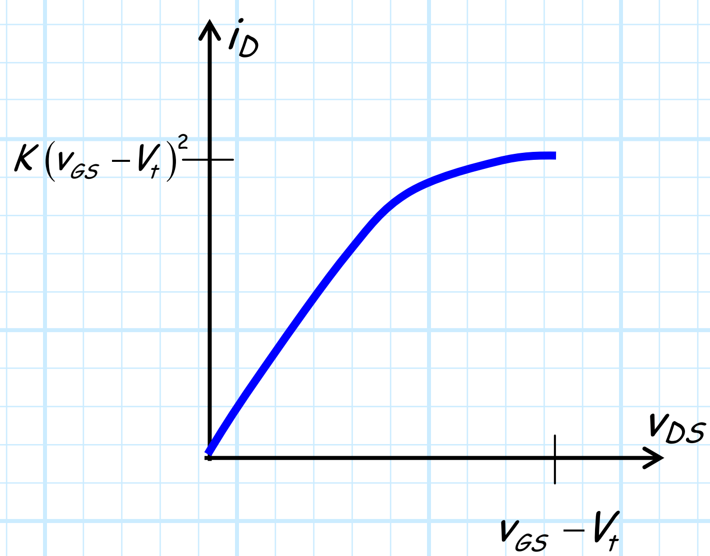


npn in SAT. mode



NMOS in TRIODE mode





Now, a BJT is in SATURATION mode if **both** the CBJ and the EBJ are **forward biased**. Assuming that $v_{BE} \approx 0.7\,V$ if the EBJ is forward biased, the CBJ voltage v_{BC} will be positive **only** if (using **KVL**):

$$\begin{aligned} v_{BC} &> 0 \\ v_{BE} - v_{CE} &> 0 \\ 0.7 - v_{CE} &> 0 \\ v_{CE} &< 0.7 \end{aligned}$$

Thus, we can conclude that a **necessary** (but not sufficient) condition for a BJT to be in SATURATION is:

$v_{CE} < 0.7$ for *npn* in SAT.

$v_{EC} < 0.7$ for *pnp* in SAT.

These inequalities are **analogous** to the MOSFET inequalities:

$v_{DS} < v_{GS} - V_t$ for NMOS in Triode

$v_{DS} > v_{GS} - V_t$ for PMOS in Triode

Now, we note for the BJT SATURATION mode that the **collector current** will **always** be **less** that that in ACTIVE mode with the same value of v_{BE}:

Sat. i_c Active i_c

$$I_S\, e^{v_{BE}/V_T}\left(1-\frac{e^{-v_{CE}/V_T}}{\alpha_R}\right) < I_S\, e^{v_{BE}/V_T} \quad \text{for all } v_{CE}$$

Thus, we can **equivalently** state that the collector current in SATURATION will be **less** than the value $\beta\, i_B$:

$$i_C < \beta\, i_B \quad \text{for BJT in SAT.}$$

This of course means that the **base** current in SAT. is **greater** than i_C/β (i.e., the base current in active) :

$$i_B > \frac{i_C}{\beta} \quad \text{for BJT in SAT.}$$

Likewise, this means that:

$$i_E < (\beta+1)\, i_B \quad \text{and} \quad i_C < \alpha\, i_E \quad \text{for BJT in SAT.}$$

But remember KCL is still valid for BJTs in SATURATION (it's **always** valid!):

$$i_E = i_B + i_C \qquad \text{(KCL)}$$

Finally, we should again note that the **exponential** equations presented for SATURATION mode are **not** particularly useful for analyzing BJT circuits (that **transcendental** equation thing again!).

Thus, we describe a BJT in SATURATION with some **approximate** equations. Since both CBJ and EBJ are forward biased, we assume that $v_{BE} \approx 0.7V$ and that $v_{BC} \approx 0.5V$, resulting in the following **approximate** description for a BJT in SATURATION:

$$v_{BE} \approx 0.7\,\text{V} \qquad v_{CE} \approx 0.2\,\text{V} \qquad i_C < \beta i_B \qquad \text{for } npn \text{ in SAT.}$$

$$v_{EB} \approx 0.7\,\text{V} \qquad v_{EC} \approx 0.2\,\text{V} \qquad i_C < \beta i_B \qquad \text{for } pnp \text{ in SAT.}$$

CUTOFF MODE

Cutoff mode for BJTs is obviously **analogous** to cutoff mode for MOSFETS.

In both cases the transistor currents are **zero**!

$$i_E = i_B = i_C = 0 \qquad \text{for BJTs in CUTOFF}$$

Note that a BJT is in cutoff if **both** EBJ and CBJ are in **reverse bias**. This is true if:

$v_{BE} < 0$ and $v_{BC} < 0$ for *npn* in CUTOFF

$v_{EB} < 0$ and $v_{CB} < 0$ for *pnp* in CUTOFF

Steps for D.C. Analysis of BJT Circuits

Q: *What **makes** a BJT operate in the cutoff or saturation or active mode??*

A: Of course, there are **no selector knobs** on a BJT for determining its operating mode. Instead, the operating mode of a BJT is determined by the remaining **circuit that surrounds it**!

Only one of the three BJT modes will result in circuit operation **consistent** with KVL, KCL, and all device equations—we have to know what in what **circuit** the BJT is placed, before we can **determine** the BJT operating mode.

Accordingly, we will need to properly **design** the circuit **surrounding** the BJT, if we wish to place it in a **specific** operating mode!

To analyze BJT circuit with D.C. sources, we **must** follow these **five steps**:

1. ASSUME an operating mode

2. ENFORCE the equality conditions of that mode.

3. ANALYZE the circuit with the enforced conditions.

4. CHECK the inequality conditions of the mode for consistency with original assumption. If consistent, the analysis is complete; if inconsistent, go to step 5.

5. MODIFY your original assumption and repeat all steps.

Let's look at each step in **detail**.

1. *ASSUME*

We can ASSUME Active, Saturation, or Cutoff!

2. *ENFORCE*

Active

For **active** region, we must ENFORCE **two equalities**.

a) Since the base-emitter junction is **forward** biased in the active region, we ENFORCE these equalities:

$$V_{BE} = 0.7 \text{ V} \quad \text{(npn)}$$

$$V_{EB} = 0.7 \text{ V} \quad \text{(pnp)}$$

b) We likewise know that in the **active** region, the base and collector currents are directly proportional, and thus we ENFORCE the equality:

$$i_C = \beta \, i_B$$

Note we can **equivalently** ENFORCE this condition with either of the the equalities:

$$i_C = \alpha i_E \quad \text{or} \quad i_E = (\beta + 1) \, i_B$$

Saturation

For **saturation** region, we must likewise ENFORCE **two equalities**.

a) Since the base-emitter junction is **forward** biased, we again ENFORCE these equalities:

$$V_{BE} = 0.7 \text{ V} \quad \text{(npn)}$$

$$V_{EB} = 0.7 \text{ V} \quad \text{(pnp)}$$

b) Likewise, since the collector base junction is **reverse** biased, we ENFORCE these equalities:

$$V_{CB} = -0.5 \text{ V} \quad \text{(npn)}$$

$$V_{BC} \simeq -0.5 \text{ V} \quad \text{(pnp)}$$

Note that from **KVL**, the above two ENFORCED equalities will require that these equalities **likewise** be true:

$$V_{CE} = 0.2 \text{ V} \quad \text{(npn)}$$

$$V_{EC} = 0.2 \text{ V} \quad \text{(pnp)}$$

Note that for saturation, you need to explicitly ENFORCE any **two** of these **three** equalities—the third will be ENFORCED **automatically** (via KVL)!!

To avoid **negative** signs (e.g., V_{CB}=-0.5), **I** typically ENFORCE the **first** and **third** equalities (e.g., V_{BE}= 0.7 and V_{CE}=0.2).

Cutoff

For a BJT in cutoff, both *pn* junctions are **reverse** biased—**no** current flows! Therefore we ENFORCE these equalities:

$$\begin{aligned} i_B &= 0 \\ i_C &= 0 \\ i_E &= 0 \end{aligned}$$

3. *ANALYZE*

Active

The task in D.C. analysis of a BJT in **active** mode is to find **one** unknown **current** and **one** additional unknown **voltage**!

a) In addition the relationship $i_C = \beta\, i_B$, we have a **second** useful relationship:

$$i_E = i_C + i_B$$

This of course is a consequence of KCL, and is true **regardless** of the BJT mode.

But think about what this means! We have **two** current equations and **three** currents (i.e., i_E, i_C, i_B)—we only need to determine **one** current and we can then immediately find the other two!

Q: *Which current do we need to find?*

A: Doesn't matter! For a BJT operating in the active region, if we know **one** current, we know them **all**!

b) In addition to $V_{BE} = 0.7$ ($V_{EB} = 0.7$), we have a **second** useful relationship:

$$V_{CE} = V_{CB} + V_{BE} \quad \text{(npn)}$$

$$V_{EC} = V_{EB} + V_{BC} \quad \text{(pnp)}$$

This of course is a consequence of KVL, and is true **regardless** of the BJT mode.

Combining these results, we find:

$$V_{CE} = V_{CB} + 0.7 \quad \text{(npn)}$$

$$V_{EC} = 0.7 + V_{BC} \quad \text{(pnp)}$$

But think about what **this** means! If we find **one** unknown voltage, we can immediately determine the **other**.

Therefore, a D.C. analysis problem for a BJT operating in the active region reduces to:

find one of these values

$$i_B,\ i_C,\ or\ i_E$$

and find one of these values

$$V_{CE} \text{ or } V_{CB} \quad (V_{EC} \text{ or } V_{BC})$$

Saturation

For the saturation mode, we know **all** the BJT **voltages**, but know nothing about BJT **currents**!

Thus, for an analysis of circuit with a BJT in saturation, we need to find any **two** of the **three** quantities:

$$i_B,\ i_C,\ i_E$$

We can then use **KCL** to find the third.

Cutoff

Cutoff is a bit of the **opposite** of saturation—we know **all** the BJT **currents** (they're all **zero**!), but we know **nothing** about BJT **voltages** !

Thus, for an analysis of circuit with a BJT in cutoff, we need to find any **two** of the **three** quantities:

$$V_{BE}\,,\ V_{CB}\,,\ V_{CE} \qquad (npn)$$

$$V_{EB}\,,\ V_{BC}\,,\ V_{EC} \qquad (pnp)$$

We can then use **KVL** to find the third.

4. *CHECK*

You do not know if your D.C. analysis is correct unless you CHECK to see if it is consistent with your original assumption!

WARNING!-Failure to CHECK the original assumption will result in a SIGNIFICANT REDUCTION in credit on exams, regardless of the accuracy of the analysis !!!

Q: *What exactly do we CHECK?*

A: We ENFORCED the mode **equalities**, we CHECK the mode **inequalities.**

Active

We must CHECK **two** separate inequalities after analyzing a circuit with a BJT that we ASSUMED to be operating in **active** mode. One inequality involves BJT **voltages**, the other BJT **currents**.

a) In the **active** region, the Collector-Base Junction is "off" (i.e., **reverse** biased). Therefore, we must CHECK our analysis results to see if they are **consistent** with:

$$V_{CB} > 0 \quad \text{(npn)}$$

$$V_{BC} > 0 \quad \text{(pnp)}$$

Since $V_{CE} = V_{CB} + 0.7$, we find that an **equivalent** inequality is:

$$V_{CE} > 0.7 \quad \text{(npn)}$$

$$V_{EC} > 0.7 \quad \text{(pnp)}$$

We need to check **only** one of these two inequalities (**not both**!).

b) In the active region, the Base-Emitter Junction is "on" (i.e., **forward** biased). Therefore, we must CHECK the results of our analysis to see if they are **consistent** with:

$$i_B > 0$$

Since the active mode constants α and β are **always** positive values, **equivalent** expressions to the one above are:

$$i_C > 0 \qquad \text{and} \qquad i_E > 0$$

In other words, we need to CHECK and see if **any** one of the currents is positive—if one is positive, they are **all** positive!

Saturation

Here we must CHECK inequalities involving BJT **currents**.

a) We know that for saturation mode, the ratio of collector current to base current will be **less than beta**! Thus we CHECK:

$$i_C < \beta\, i_B$$

b) We know that **both** *pn* junctions are **forward** biased, hence we CHECK to see if all the **currents are positive**:

$$\begin{aligned} i_B &> 0 \\ i_C &> 0 \\ i_E &> 0 \end{aligned}$$

Cutoff

For **cutoff** we must CHECK two BJT **voltages**.

a) Since the EBJ is **reverse biased**, we CHECK:

$$V_{BE} < 0 \qquad (npn)$$

$$V_{EB} < 0 \qquad (pnp)$$

b) Likewise, since the CBJ is also **reverse biased**, we CHECK:

$$V_{CB} > 0 \qquad (npn)$$

$$V_{BC} > 0 \qquad (pnp)$$

If the results of our analysis are consistent with **each** of these inequalities, then we have made the **correct** assumption! The numeric results of our analysis are then likewise correct. We can stop working!

However, if **even one** of the results of our analysis is **inconsistent** with active mode (e.g., currents are negative, or $V_{CE} < 0.7$), then we have made the **wrong** assumption! Time to move to step 5.

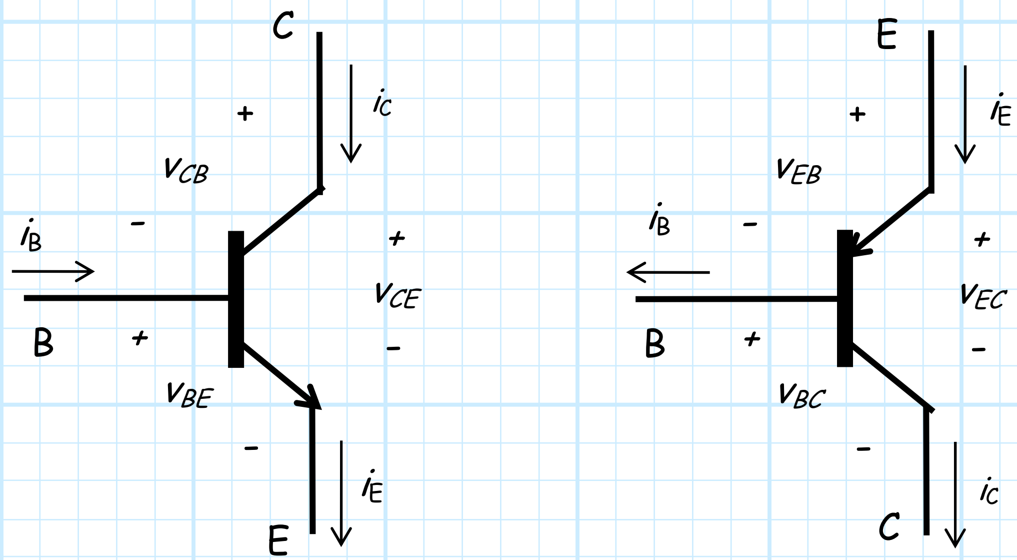
5. *MODIFY*

If one or more of the BJTs are **not** in the active mode, then it must be in either **cutoff** or **saturation**. We must change our assumption and start **completely** over!

In general, **all** of the results of our previous analysis are incorrect, and thus must be **completely** scraped!

Hints for BJT Circuit Analysis

1. Know the BJT **symbols** and **current/voltage** definitions!



2. Know what **quantities** must be determined for each **assumption** (e.g., for active mode, you must determine one BJT current and one BJT voltage).

3. Write **separate** equations for the BJT (device) and the remainder of the circuit (KVL, KCL, Ohm's Law).

4. Write the KVL equation for the circuit's "**Base-Emitter Leg**". In other words, write a KVL that includes v_{BE}.

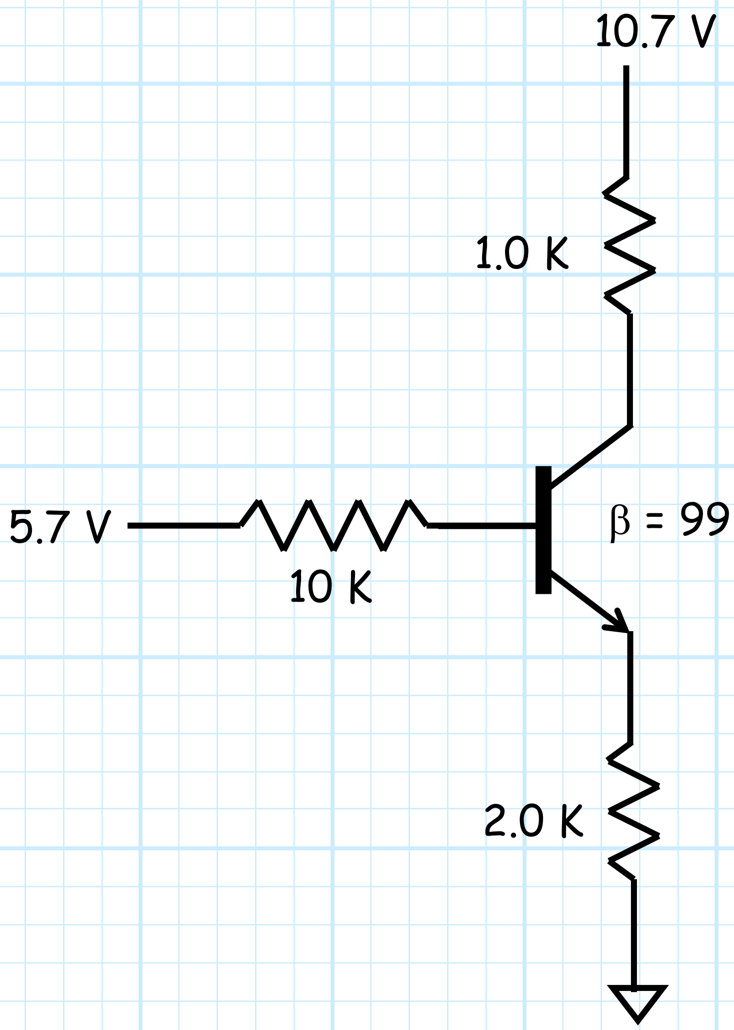
5. Forget about what the problem is asking for! Just start by determining **any** and **all** the circuit quantities that you can. If you end up solving the **entire** circuit, the answer will in there somewhere!

6. If you get stuck, try working the problem **backward**! For example, to find a resistor value, you must find the voltage across it and the current through it.

7. Make sure you are using **all** the information provided in the problem!

Example: D.C. Analysis of a BJT Circuit

Consider **again** this circuit from lecture:



Q: *What is I_B, I_C, I_E and also V_{CE}, V_{CB}, V_{BE} ??*

A: I don't know ! But, we can find out—IF we complete **each** of the five steps **required** for BJT DC analysis.

Step 1 – **ASSUME** an operating mode.

Let's ASSUME the BJT is in the ACTIVE region !

Remember, this is just a **guess**; we have no way of knowing for sure what mode the BJT is in at this point.

Step 2 - **ENFORCE** the conditions of the assumed mode.

For **active** region, these are:

$$V_{BE} = 0.7 \ V \quad \text{and} \quad I_C = \beta I_B = 99 \ I_B$$

Step 3 – **ANALYZE** the circuit.

This is the **BIG** step !

Q: *Where do we even start ?*

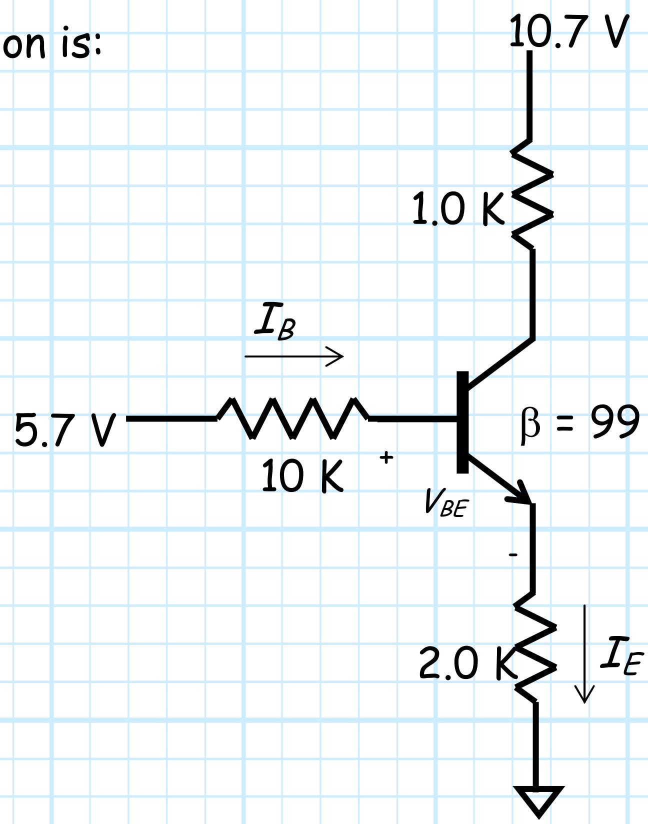
A: Recall what the hint sheet says:

"Write KVL equations for the base-emitter "leg"

I think we should try that !

The **base-emitter KVL** equation is:

$$5.7 - 10\,I_B - V_{BE} - 2\,I_E = 0$$



This is the **circuit** equation; note that it contains 3 unknowns (i_B, i_C, and V_{BE}).

Now let's add the relevant **device** equations:

$$V_{BE} = 0.7\ V$$

$$I_E = (\beta + 1)\,I_B = 100\,I_B$$

Look what we now have ! **3** equations and **3** unknowns (this is a **good** thing).

Inserting the device equations into the B-E KVL:

$$5.7 - 10\,I_B - 0.7 - 2(99+1)\,I_B = 0$$

Therefore:

$5.0 - 210\,I_B = 0$ ➡ 1 equations and 1 unknown !

Solving, we get:

$$I_B = \frac{5.0}{210} = \underline{23.8\ \mu A}$$

Q: *Whew ! That was an **awful** lot of work for just one current, and we still have **two more** currents to find.*

A: No we don't ! Since we determined **one** current for a BJT in **active** mode, we've determined them **all** !

I.E.,

$$I_C = \beta\ I_B = \underline{2.356\ \text{mA}}$$

$$I_E = (\beta + 1)\ I_B = \underline{2.380\ \text{mA}}$$

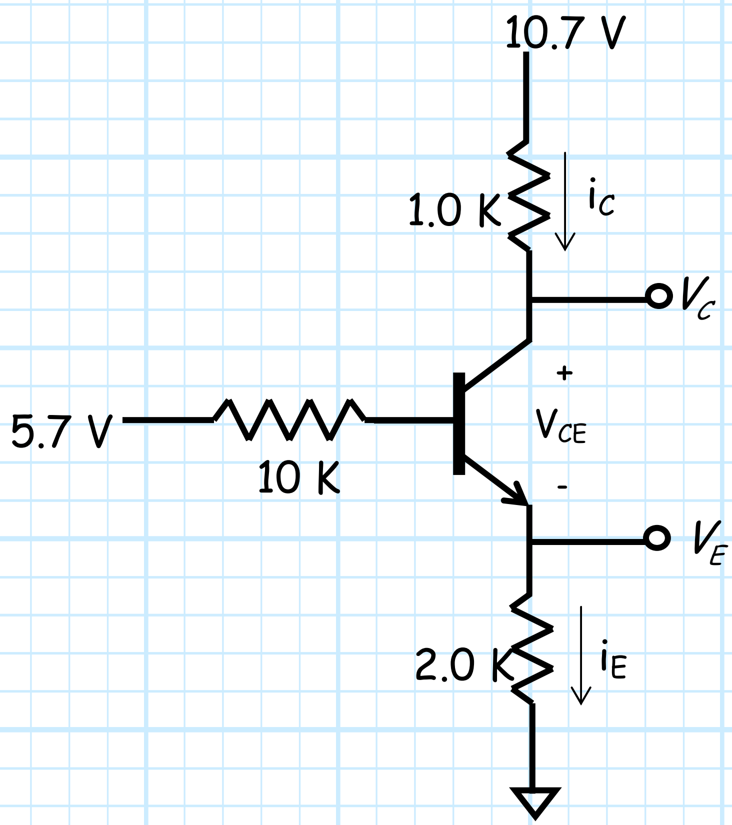
(Note that $I_C + I_B = I_E$)

Now for the **voltages** !

Since we know the **currents**, we can find the voltages **using KVL.**

For example, let's **determine** V_{CE}. We can do this **either** by finding the voltage at the **collector** V_C (wrt ground) and voltage at the **emitter** V_E (wrt ground) and then subtracting ($V_{CE} = V_C - V_E$).

OR, we can determine V_{CE} **directly** from **the C-E KVL equation**.



$$
\begin{aligned}
V_C &= 10.7 - I_C(1) \\
&= 10.7 - 2.36 \\
&= 8.34 \quad \text{V}
\end{aligned}
$$

and:

$$
\begin{aligned}
V_E &= 0 + I_E(2) \\
&= 0 + 4.76 \\
&= 4.76 \quad \text{V}
\end{aligned}
$$

Therefore,

$V_{CE} = V_C - V_E = \underline{3.58 \text{ V}}$

Note we could have **likewise** written the C-E KVL:

$$10.7 - I_C(1) - V_{CE} - I_E(2) = 0$$

Therefore,

$$V_{CE} = 10.7 - I_C(1) - I_E(2) = 3.58 \quad \text{V}$$

Q: *So, I guess we write the collector-base KVL to find V_{CB} ?*

A: You can, but a **wiser** choice would be to simply apply **KVL** to the **transistor** !

I.E., $V_{CE} = V_{CB} + V_{BE}$!!

Therefore $V_{CB} = V_{CE} - V_{BE} = \underline{2.88 \text{ V}}$

Q: *This has been **hard**. I'm glad we're finished !*

A: Finished ! We still have **2 more** steps to go!

Step 4 – CHECK to see if your results are **consistent** with your assumption.

For **active** mode:

$$V_{CE} = 3.58 \text{ V} > 0.7 \text{ V} \checkmark$$

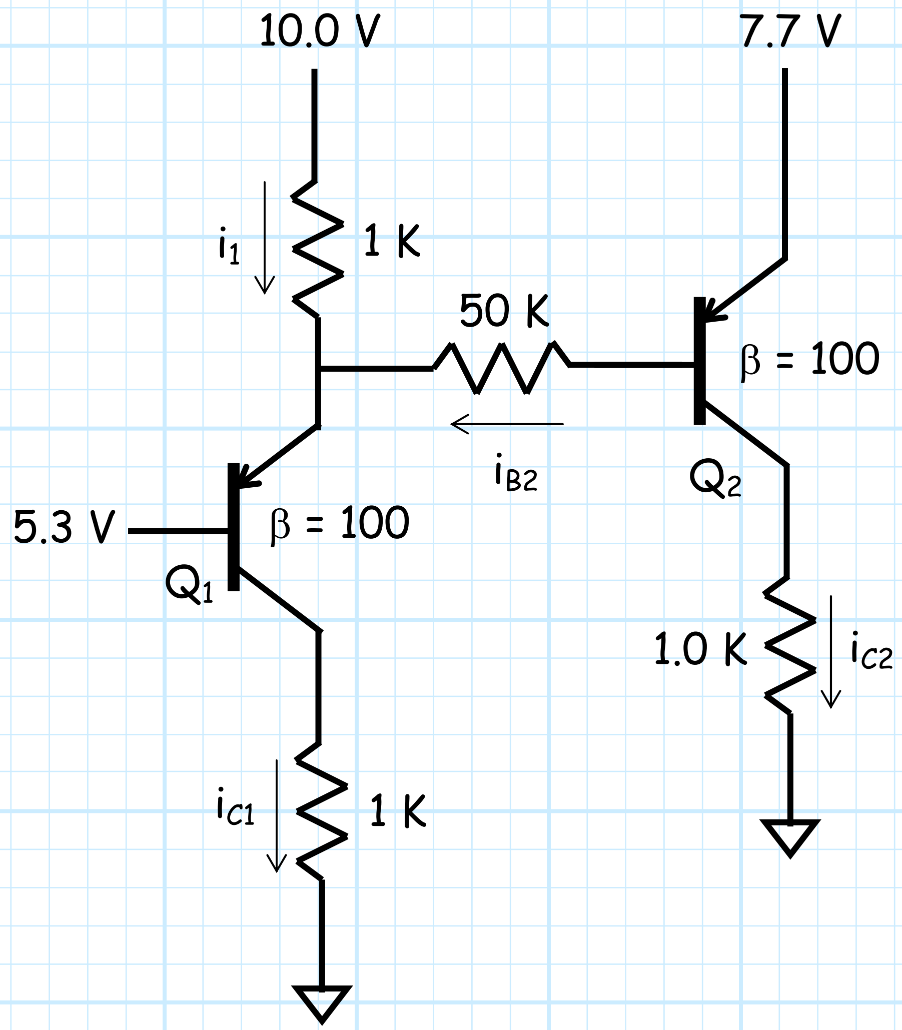
$$I_B = 23.8 \ \mu\text{A} > 0.0 \checkmark$$

Are assumption was **correct**, and therefore so are our **answers** !

No need to go on to Step 5 .

Example: Another DC Analysis of a BJT Circuit

Find the **collector voltages** of the two BJTs in the circuit below.



ASSUME both BJTs are in **active** mode, therefore ENFORCE

$$V_{EB}^{1} = V_{EB}^{1} = 0.7 \text{ V} , \quad i_{C1} = 100\, i_{B1} , \text{ and } \quad i_{C2} = 100\, i_{B2}$$

Q: *Now, how do we ANALYZE the circuit ??*

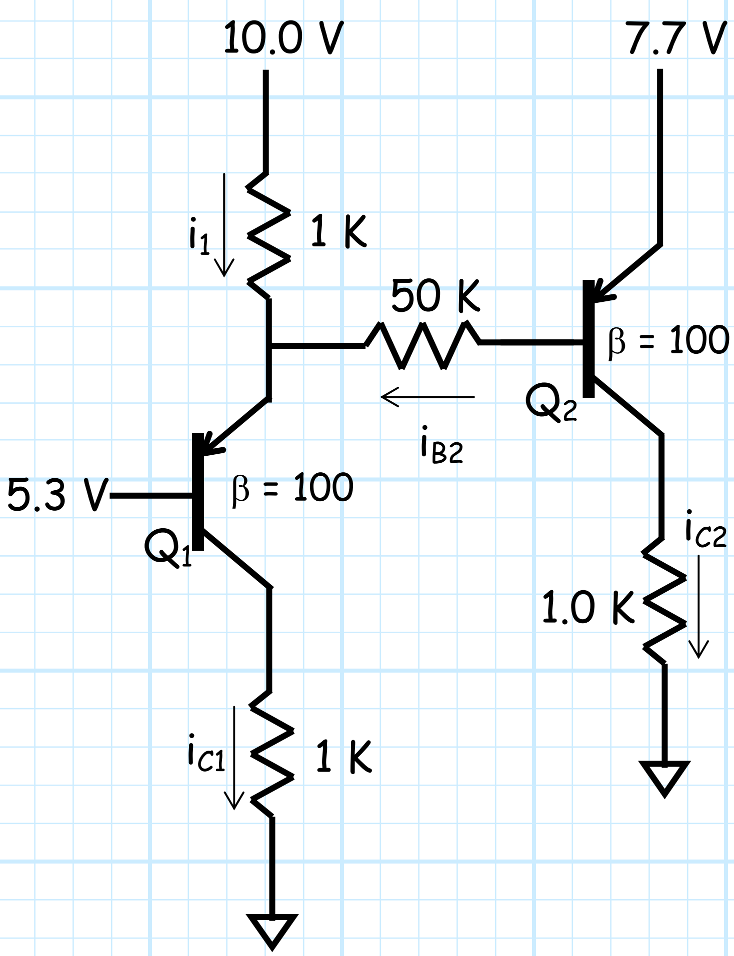
A: This seems to be a problem ! We cannot **easily** solve the emitter base KVL, as i_1 is NOT EQUAL to i_{E1} (make sure you understand this !). Instead, we find:

$$i_{E1} = i_1 + i_{B2}$$

So, what do we do ?

First, ask the question: **What do we know ??**

Look closely at the circuit, it is apparent that $\mathbf{V_{B1} = 5.3\ V}$ and $\mathbf{V_{E2} = 7.7\ V}$.



Hey! We therefore also know $\mathbf{V_{E1}}$ and $\mathbf{V_{B2}}$:

$$V_{E1} = V_{B1} + V_{EB}^{1} = 5.3 + 0.7 = 6.0\ V$$

$$V_{B2} = V_{E2} - V_{EB}^{2} = 7.7 - 0.7 = 7.0\ V$$

Wow ! From these values we get:

$$i_1 = \frac{10 - V_{E1}}{1} = \frac{10-6}{1} = 4\ mA$$

and

$$i_{B2} = \frac{V_{B2} - V_{E1}}{50} = \frac{7-6}{50} = 0.02\ mA$$

This is easy! Since we know i_1 and i_{B2}, we can **find** $\mathbf{i_{E1}}$:

$$i_{E1} = i_1 + i_{B2} = 4.0 + 0.02 = 4.02 \text{ mA}$$

Since we know **one** current for each BJT, we know **all** currents for each BJT:

$$i_{C1} = \alpha\, i_{E1} = \frac{\beta}{\beta+1}\, i_{E1} = \frac{100}{101}\, 4.02 = 3.98 \text{ mA}$$

$$i_{C2} = \beta\, i_{B2} = 100(0.02) = 2 \text{ mA}$$

Finally, we can determine the voltages $\mathbf{V_{C1}}$ and $\mathbf{V_{C2}}$.

$$V_{C1} = 0.0 + 1\, i_{C1} = 0.0 + 1(3.98) = \underline{3.98 \text{ V}}$$

$$V_{C2} = 0.0 + 1\, i_{C2} = 0.0 + 1(2.0) = \underline{2.0 \text{ V}}$$

Now, let's CHECK to see if our assumptions were correct:

$i_{C2} = 2\text{mA} > 0$ ✓ $i_{C1} = 3.98 \text{ mA} > 0$ ✓

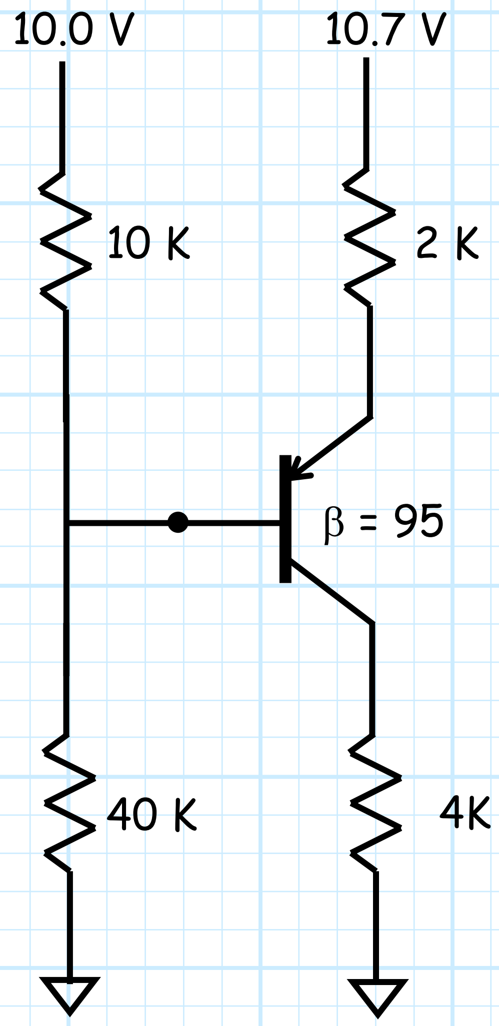
$$V_{EC}^{1} = V_{E1} - V_{C1} = 6.0 - 3.98 = 2.02 \text{ V} > 0.7 \text{ V}$$ ✓

$$V_{BC}^{2} = V_{B1} - V_{C1} = 7.0 - 2.0 = 5.0 \text{ V} > 0$$ ✓

Assumptions are **correct** !

Example: An Analysis of a *pnp* BJT Circuit

Determine the **collector current** and **collector voltage** of the BJT in the circuit below.



1. ASSUME the BJT is in **active** mode.

2. ENFORCE the conditions:

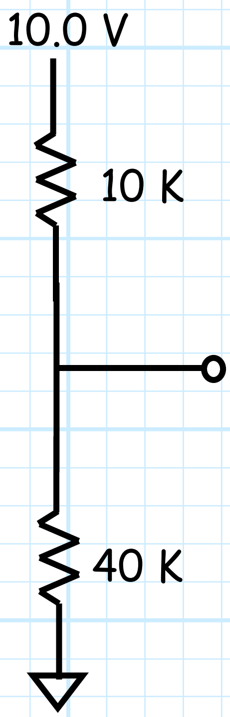
$$V_{EB} = 0.7 \text{ V} \quad \text{and} \quad i_C = \beta\, i_B$$

3. ANALYZE the circuit.

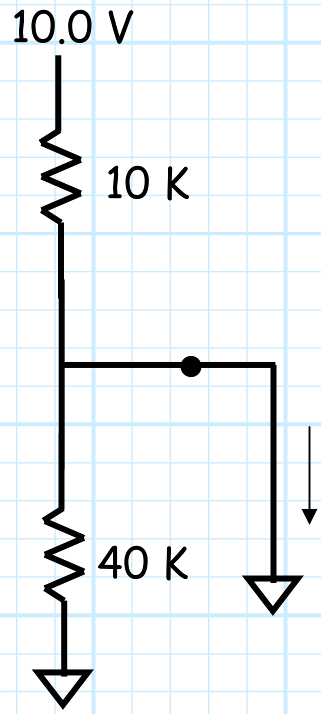
Q: *Yikes ! How do we write the base-emitter KVL ?*

A: This is a perfect opportunity to apply the **Thevenin's** equivalent circuit!

Thevenin's equivalent circuit:

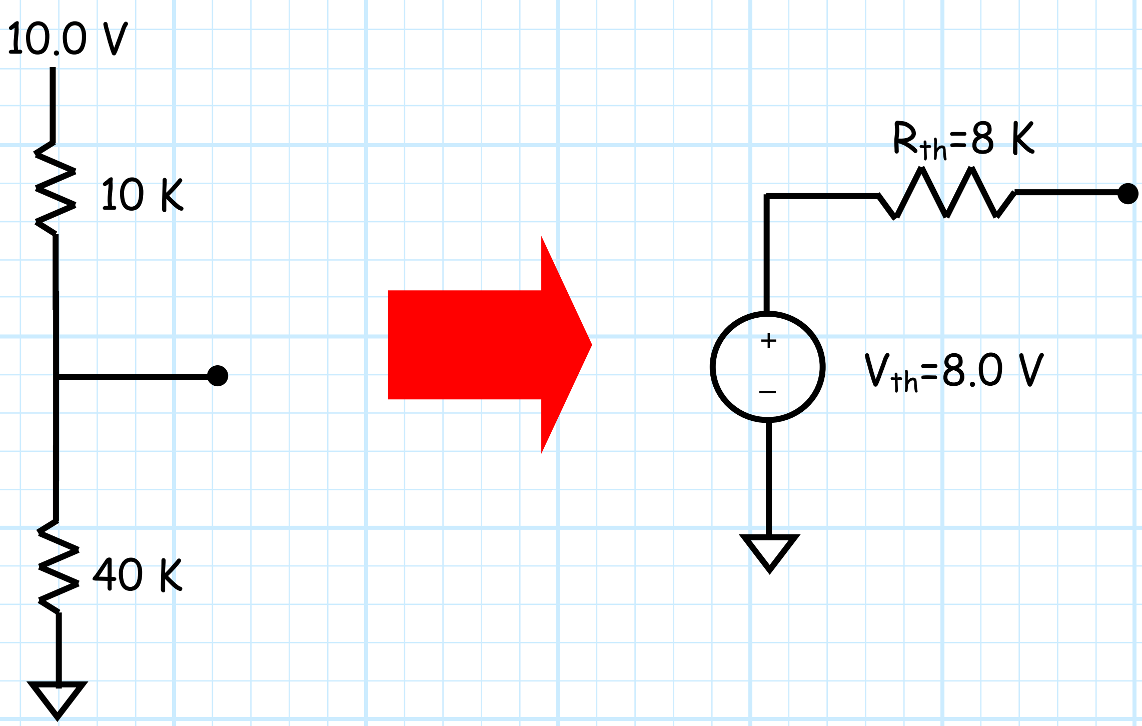


$$V_{oc} = 10\,\frac{40}{(40+10)} = 8.0\text{ V}$$



$$I_{sc} = \frac{10}{10} = 1\text{ mA}$$

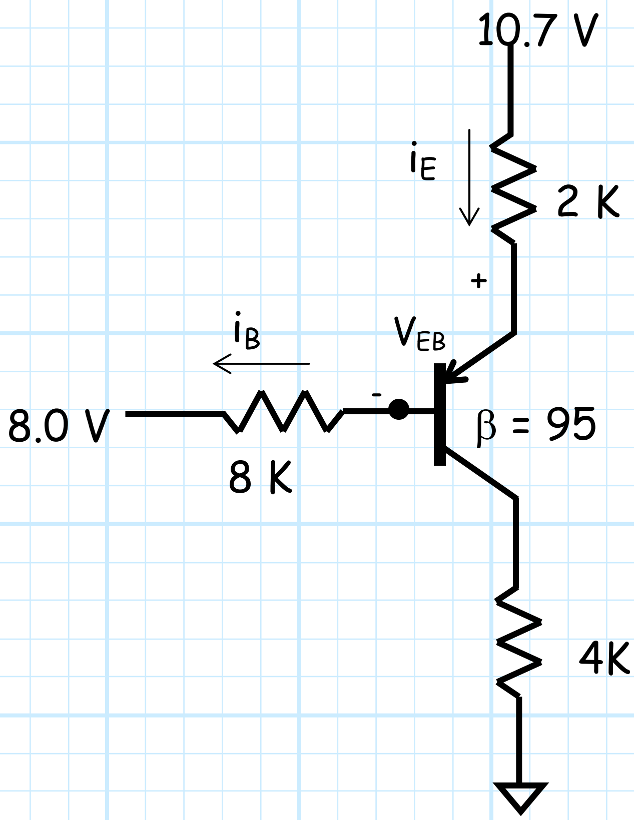
Where $V_{th} = V_{oc} = 8.0$ V and $R_{th} = V_{oc}/I_{sc} = 8/1 = 8$ K



Original Circuit

Equivalent Circuit

Therefore, we can write the BJT circuit as:



NOW we can easily write the emitter-base leg KVL:

$$10.7 - 2i_E - v_{EB} - 8i_B = 8.0$$

Along with our enforced conditions, we now have **three** equations and **three** unknowns !

Combining, we find:

$$10.7 - 2(96)i_B - 0.7 - 8\,i_B = 8.0$$

Therefore,

$$i_B = \frac{10.7 - 0.7 - 8.0}{2(96) + 8} = \frac{2}{200} = 0.01 \text{ mA}$$

and **collector current** i_C is:

$$i_C = \beta\, i_B = 95(0.01) = \underline{0.95 \text{ mA}}$$

Likewise, the **collector voltage** (wrt ground) V_C is:

$$V_C = 0.0 + 4\,i_C = \underline{3.8 \text{ V}}$$

But wait ! We're **not** done yet ! We must **CHECK** our assumption.

First, $i_B = 0.01\ mA > 0$ ✓

But, what is V_{EC} ??

Writing the emitter-collector KVL:

$$10.7 - 2\, i_E - V_{CE} - 4\, i_C = 0$$

Therefore,

$V_{EC} = 10.7 - 2(96)(0.01) - 4(0.95) = 4.98\ V > 0.7\ V$ ✓

Our assumption was **correct** !