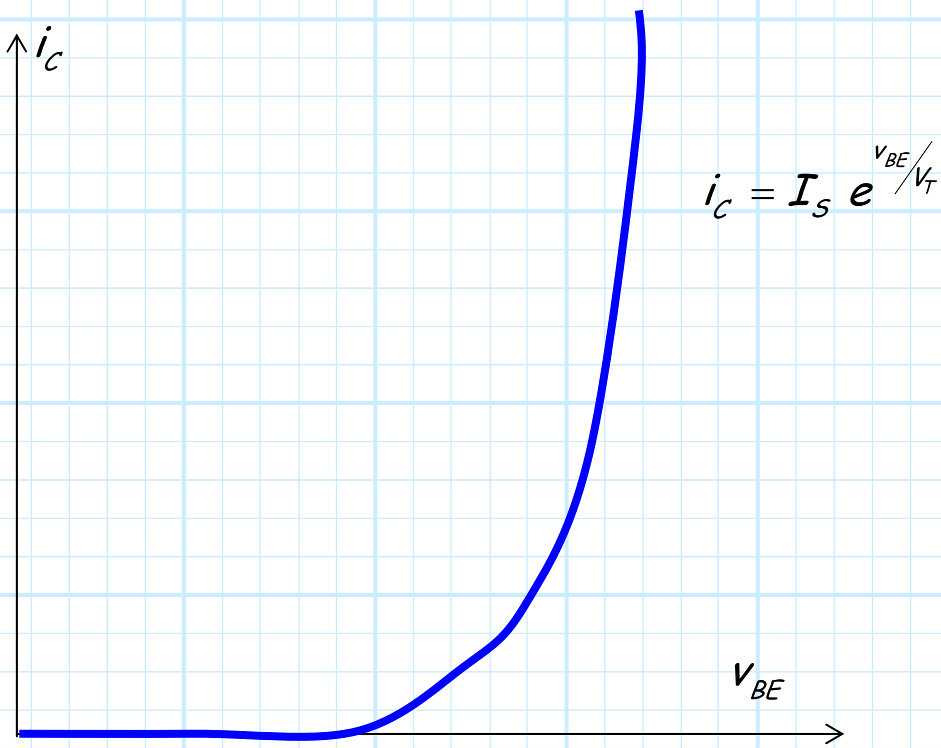
A Small-Signal Analysis of a BJT

The **collector current** i_C of a BJT is related to its **base-emitter voltage** v_{BE} as:



One messy result

Say the current and voltage have both **D.C.** (I_C, V_{BE}) and **small-signal** (i_c, v_{be}) components:

$$i_C(t) = I_C + i_c(t)$$

and

$$v_{BE}(t) = V_{BE} + v_{be}(t)$$

Therefore, the **total** collector current is:

$$i_C(t) = I_S \, e^{v_{BE}(t)/V_T}$$

$$I_C + i_c(t) = I_S \, e^{\frac{V_{BE}+v_{be}(t)}{V_T}}$$

Apply the Small-Signal Approximation

Q: *Yikes! The exponential term is very messy. Is there some way to* ***approximate*** *it?*

A: Yes! The collector current i_c is a **function** of base emitter voltage v_{BE}.

Let's perform a **small-signal analysis** to determine an **approximate** relationship between i_c and v_{BE}.

Note that the value of $v_{BE}(t) = V_{BE} + v_{be}(t)$ is **always** very close to the D.C. voltage for all time t (since $v_{be}(t)$ is very **small**).

We therefore will use this D.C. voltage as the **evaluation point** (i.e., bias point) for our small-signal analysis.

How fast it grows!

We first determine the value of the collector current i_C when the base emitter voltage v_{BE} is equal to the **DC value** V_{BE} :

$$i_C\big|_{v_{BE}=V_{BE}} = \mathcal{I}_s\, e^{\frac{v_{BE}}{V_T}}\Big|_{v_{BE}=V_{BE}} = \mathcal{I}_s\, e^{\frac{V_{BE}}{V_T}} = \mathcal{I}_C$$

Of course, the result is the **D.C. collector current** $\mathcal{I}_C$.

We now determine the **change** in collector current due to a **change** in base-emitter voltage (i.e., a first **derivative**), **evaluated** at the D.C. voltage V_{BE} :

$$\begin{aligned}\left.\frac{d\, i_C}{d\, v_{BE}}\right|_{v_{BE}=V_{BE}} &= \left.\frac{d\left(\mathcal{I}_S \exp\left[v_{BE}/V_T\right]\right)}{d\, v_{BE}}\right|_{v_{BE}=V_{BE}} \\ &= \left.\frac{\mathcal{I}_S}{V_T}\, e^{v_{BE}/V_T}\right|_{v_{BE}=V_{BE}} \\ &= \frac{\mathcal{I}_S}{V_T}\, e^{V_{BE}/V_T} \qquad \left[A/V\right]\end{aligned}$$

A simple approximation

Thus, when the base-emitter voltage is equal to the D.C. "bias" voltage V_{BE}, the collector current i_C will equal the D.C. "bias" current I_C.

Likewise, this collector current will increase (decrease) by an amount of $(I_S/V_T)e^{V_{BE}/V_T}$ mA for every 1mV increase (decrease) in v_{BE}.

Thus, we can easily **approximate** the collector current when the base-emitter voltage is equal to values such as:

$$v_{BE} = V_{BE} + 1\text{ mV}$$
$$v_{BE} = V_{BE} + 3\text{ mV}$$
$$v_{BE} = V_{BE} - 2\text{ mV}$$
$$v_{BE} = V_{BE} - 0.5\text{ mV}$$

Respectively, the answers are:

$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(1) \quad \text{mA}$$

$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(3) \quad \text{mA}$$

$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(\text{-}2) \quad \text{mA}$$

$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(\text{-}0.5) \quad \text{mA}$$

where we have assumed that scale current I_S is expressed in mA, and thermal voltage V_T is expressed in mV.

The small signal approximation

Recall that the **small-signal voltage** $v_{be}(t)$ represents a small **change** in $v_{BE}(t)$ from its nominal (i.e., bias) voltage V_{BE}.

For example, we might find that the value of $v_{be}(t)$ at four different times t are:

$$\begin{aligned} v_{be}(t_1) &= 1 \text{ mV} \\ v_{be}(t_2) &= 3 \text{ mV} \\ v_{be}(t_3) &= -2 \text{ mV} \\ v_{be}(t_4) &= -0.5 \text{ mV} \end{aligned}$$

Thus, we can approximate the collector current using the **small-signal approximation** as:

$$i_C(t) = I_C + \left(I_S / V_T\right) e^{V_{BE}/V_T} \, v_{be}(t)$$

where of course $I_C = I_S e^{V_{BE}/V_T}$.

This is a very useful result, as we can now **explicitly** determine an expression for the **small-signal current** $i_c(t)$!

The small-signal collector current

Recall $i_C(t) = I_C + i_c(t)$, therefore:

$$i_C(t) = I_C + i_c(t) = I_C + \left(I_S / V_T\right) e^{V_{BE}/V_T} \; v_{be}(t)$$

Subtracting the D.C. current from each side, we are left with an expression for the **small-signal current** $i_c(t)$, in terms of the **small-signal voltage** $v_{be}(t)$:

$$i_c(t) = \left(I_S / V_T\right) e^{V_{BE}/V_T} \; v_{be}(t)$$

We can **simplify** this expression by noting that $I_C = I_S e^{V_{BE}/V_T}$, resulting in:

$$\begin{aligned}\left(I_S / V_T\right) e^{V_{BE}/V_T} &= \frac{I_S \, e^{V_{BE}/V_T}}{V_T} \\ &= \frac{I_C}{V_T}\end{aligned}$$

and thus:

$$i_c(t) = \frac{I_C}{V_T} v_{be}(t)$$

Transconductance: A small signal parameter

We define the value I_C / V_T as the **transconductance** g_m:

$$g_m = \frac{I_C}{V_T} \qquad \left[A/V \right]$$

and thus the **small-signal equation** simply becomes:

$$i_c(t) = g_m \, v_{be}(t)$$

How transistors got their name

Let's now consider for a moment the transconductance g_m.

The term is short for transfer conductance: conductance because its units are amps/volt, and transfer because it relates the **collector** current to the voltage from **base to emitter**—the collector voltage is **not relevant** (if in **active** mode)!

Note we can rewrite the small-signal equation as:

$$\frac{v_{be}(t)}{i_c(t)} = \frac{1}{g_m}$$

The value ($1/g_m$) can thus be considered as transfer resistance, the value describing a **transfer resistor**.

Transfer **Resistor**—we can shorten this term to **Transistor** (this is how these devices were named)!

Summarizing

We can **summarize** our results as:

$$I_C = I_S\, e^{V_{BE}/V_T}$$ D.C. Equation

$$i_c(t) = g_m\, v_{be}(t)$$ Small-Signal Equation

$$i_C(t) = I_C + g_m\, v_{be}(t)$$ Small-Signal Approximation

Note that we know have **two** expressions for the **total** (D.C. plus small-signal) collector current. The **exact** expression:

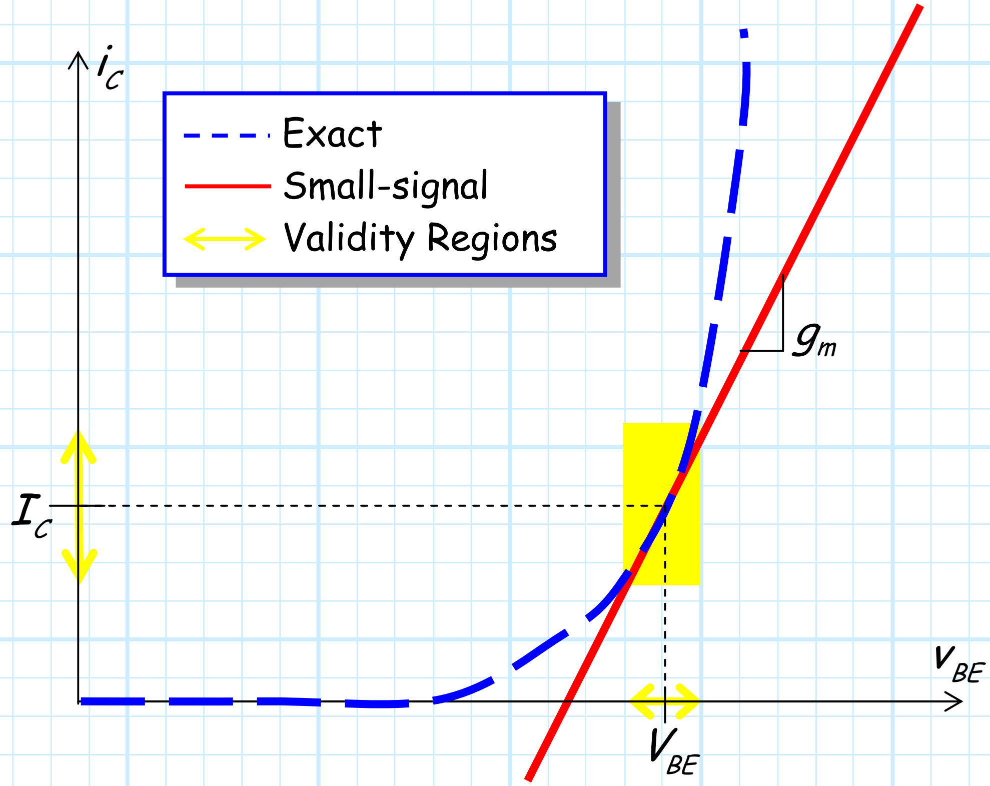
$$i_C(t) = I_S\, e^{\frac{V_{BE}+v_{be}(t)}{V_T}}$$

and the **small-signal approximation**:

$$i_C(t) = I_C + g_m\, v_{be}(t)$$

Accurate over a small region

Let's **plot** these two expressions and see how they compare:



It is evident that the small-signal approximation is accurate (it provides **nearly** the exact values) **only** for values of i_C **near** the D.C bias value I_C, and **only** for values of v_C **near** the D.C bias value V_C.

The point (V_{BE}, I_C) is alternately known as the **D.C. bias point**, the **transistor operating point**, or the **Q-point**.

Change the DC bias, change the transconductance

Note if we **change the D.C. bias** of a transistor circuit, the transistor operating point will change.

The small-signal model will **likewise** change, so that it provides accurate results in the region of this new operating point: