BJT Small-Signal Parameters

We know that the following small-signal relationships are true for BJTs:

\[ i_c = \beta i_b \quad i_c = g_m v_{be} \]

**Q:** What other relationship can be derived from these two??

**A:** Well, one obvious relationship is determined by equating the two equations above:

\[ i_c = \beta i_b = g_m v_{be} \quad \therefore v_{be} = \left( \frac{\beta}{g_m} \right) i_b \]

We can thus define the small-signal parameter \( r_\pi \) as:

\[ \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{V_T}{I_B} = r_\pi \]
Small-signal base resistance

Therefore, we can write the **new** BJT small-signal equation:

\[ v_{be} = r_\pi i_b \]

The value \( r_\pi \) is commonly thought of as the small-signal base resistance.

We can likewise define a small-signal **emitter resistance**:

\[ r_e \triangleq \frac{v_{be}}{i_e} \]

We begin with the small-signal equation \( i_c = \alpha i_e \). Combining this with \( i_c = g_m v_{be} \), we find:

\[ i_c = \alpha i_e = g_m v_{be} \quad \therefore \quad v_{be} = \left( \frac{\alpha}{g_m} \right) i_e \]
Small-signal emitter resistance

We can thus define the small-signal parameter $r_e$ as:

$$\frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E} \doteq r_e$$

Therefore, we can write another new BJT small-signal equation:

$$v_{be} = r_e i_e$$

Note that in addition to $\beta$, we now have three fundamental BJT small-signal parameters:

$$g_m = \frac{I_C}{V_T} \quad r_{\pi} = \frac{V_T}{I_B} \quad r_e = \frac{V_T}{I_E}$$
These results are not independent!

Since $I_c = \beta I_B$ ($I_c = \alpha I_E$), we find that these small signal values are not independent.

If we know two of the four values $\beta, g_m, r_\pi, r_e$, we can determine all four!

\[
\begin{align*}
g_m &= \frac{\alpha}{r_e} = \frac{\beta}{r_\pi} = \frac{r_\pi - r_e}{r_\pi r_e} \\
r_\pi &= \frac{\beta}{g_m} = (\beta + 1) r_e = \frac{r_e}{1 - g_m r_e} \\
r_e &= \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1} = \frac{r_\pi}{1 + g_m r_\pi}
\end{align*}
\]
**Make sure you can derive these!**

The results on the previous page are easily determined from the equations:

\[
g_m = \frac{I_C}{V_T} \quad r_\pi = \frac{V_T}{I_B} \quad r_e = \frac{V_T}{I_E}
\]

\[
I_E = I_C + I_B
\]

\[
I_C = \beta I_B
\]

\[
I_C = \alpha I_E
\]

Make sure you can derive them!