BJT Small-Signal Parameters

We know that the following **small-signal** relationships are true for **BJTs**:

$$i_c = \beta i_b \qquad i_c = g_m v_{be}$$

Q: *What **other** relationship can be derived from these two??*

A: Well, **one** obvious relationship is determined by equating the two equations above:

$$i_c = \beta i_b = g_m v_{be} \qquad \therefore v_{be} = \left(\frac{\beta}{g_m}\right) i_b$$

We can thus define the small-signal parameter r_π as:

$$\frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{V_T}{I_B} \doteq r_\pi$$

Small-signal base resistance

Therefore, we can write the **new** BJT small-signal equation:

$$v_{be} = r_\pi \, i_b$$

The value r_π is commonly thought of as the small-signal **base resistance**.

We can likewise define a small-signal **emitter resistance**:

$$r_e \doteq \frac{v_{be}}{i_e}$$

We begin with the small-signal equation $i_c = \alpha \, i_e$. Combining this with $i_c = g_m v_{be}$, we find:

$$i_c = \alpha \, i_e = g_m v_{be} \qquad \therefore \quad v_{be} = \left(\frac{\alpha}{g_m} \right) i_e$$

Small-signal emitter resistance

We can thus **define** the small-signal parameter r_e as:

$$\frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E} \doteq r_e$$

Therefore, we can write **another new** BJT small-signal equation:

$$v_{be} = r_e\, i_e$$

Note that **in addition** to β, we now have **three** fundamental BJT small-signal parameters:

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E}$$

These results are not independent!

Since $I_C = \beta\, I_B$ ($I_C = \alpha\, I_E$), we find that these small signal values are **not** independent.

If we know **two** of the four values β, g_m, r_π, r_e, we can determine **all four**!

$$g_m = \frac{\alpha}{r_e} = \frac{\beta}{r_\pi} = \frac{r_\pi - r_e}{r_\pi\, r_e}$$

$$r_\pi = \frac{\beta}{g_m} = (\beta + 1) r_e = \frac{r_e}{1 - g_m\, r_e}$$

$$r_e = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1} = \frac{r_\pi}{1 + g_m\, r_\pi}$$

Make sure you can derive these!

The results on the previous page are easily determined from the equations:

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E}$$

$$I_E = I_C + I_B$$

$$I_C = \beta I_B$$

$$I_C = \alpha I_E$$

Make sure **you** can derive them!