


# DC and AC Impedance of Reactive Elements

Now, recall from EECS 211 the **complex impedances** of our basic circuit elements:



$$Z_R = R$$



$$Z_C = \frac{1}{j\omega C}$$



$$Z_L = j\omega L$$

For a **DC** signal ( $\omega = 0$ ), we find that:

$$Z_R = R$$

$$Z_C = \lim_{\omega \rightarrow 0} \frac{1}{j\omega C} = \infty$$

$$Z_L = j(0)L = 0$$

Thus, at **DC** we know that:

- \* a **capacitor** acts as an **open** circuit ( $I_C = 0$ ).
- \* an **inductor** acts as a **short** circuit ( $V_L = 0$ ).

Now, let's consider **two** important cases:

1. A capacitor whose capacitance  $C$  is unfathomably large.
2. An inductor whose inductance  $L$  is unfathomably large.

### 1. The Unfathomably Large Capacitor

In this case, we consider a capacitor whose capacitance is **finite**, but **very, very, very** large.

For **DC** signals ( $\omega = 0$ ), this device acts **still** acts like an **open** circuit.

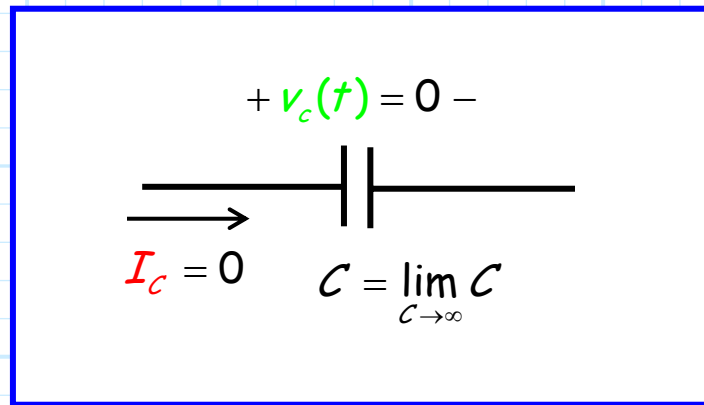
However, now consider the **AC** signal case (e.g., a small signal), where  $\omega \neq 0$ . The **impedance** of an unfathomably large capacitor is:

$$Z_C = \lim_{C \rightarrow \infty} \frac{1}{j\omega C} = 0$$

**Zero** impedance!

→ An unfathomably large capacitor acts like an **AC short**.

Quite a trick! The unfathomably large capacitance acts like an **open** to **DC** signals, but likewise acts like a **short** to **AC** (small) signals!



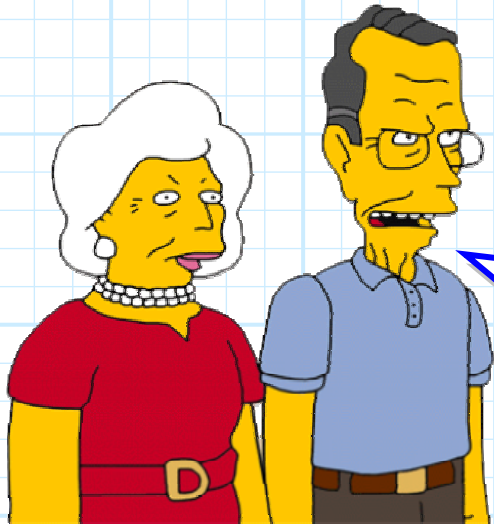
**Q:** *I fail to see the **relevance** of this analysis at this juncture. After all, **unfathomably large capacitors do not exist, and are impossible to make (being unfathomable and all).***



**A:** True enough! However, we can make **very big** (but **fathomably large**) capacitors. Big capacitors will not act as a **perfect AC short circuit**, but **will** exhibit an impedance of **very small** magnitude (e.g., a few Ohms), provided that the AC signal frequency is sufficiently large.

In this way, a **very large** capacitor acts as an **approximate AC short**, and as a **perfect DC open**.

We call these large capacitors **DC blocking capacitors**, as they allow **no DC current** to flow through them, while allowing **AC current** to flow **nearly unimpeded!**



**Q:** *But you just said this is true "provided that the AC signal frequency is **sufficiently large**." Just **how large** does the signal frequency  $\omega$  need to be?*

**A:** Say we desire the AC impedance of our capacitor to have a magnitude of **less than ten Ohms**:

$$|Z_c| < 10$$

Rearranging, we find that this will occur if the frequency  $\omega$  is:

$$10 > |Z_c|$$

$$10 > \frac{1}{\omega C}$$

$$\omega > \frac{1}{10C}$$

For **example**, a  $50 \mu\text{F}$  capacitor will exhibit an impedance whose magnitude is less than 10 Ohms for all AC signal frequencies above **320 Hz**.

Likewise, **almost** all AC signals in modern electronics will operate in a spectrum much higher than 320 Hz.

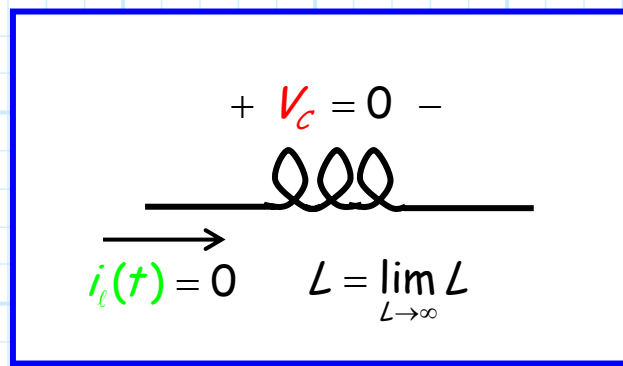
Thus, a  $50 \mu\text{F}$  blocking capacitor will **approximately** act as an AC short and (precisely) act as a DC open.

## 2. The Unfathomably Large Inductor

Similarly, we can consider an **unfathomably large inductor**. In addition to a **DC** impedance of **zero** (a DC short), we find for the **AC** case (where  $\omega \neq 0$ ):

$$Z_L = \lim_{L \rightarrow \infty} j\omega L = \infty$$

In other words, an unfathomably large inductor acts like an **AC open circuit!**



The unfathomably large inductor acts like an **short** to **DC** signals, but likewise acts like an **open** to **AC** (small) signals!

As before, an unfathomably large inductor is **impossible** to build.

However, a **very large** inductor will typically exhibit a **very large AC impedance** for all but the lowest of signal frequencies  $\omega$ .

We call these large inductors "AC chokes" (also known RF chokes), as they act as a **perfect short** to **DC** signals, yet so effectively impede AC signals (with sufficiently high frequency) that they act **approximately** as an **AC open circuit**.

For example, if we desire an **AC choke** with an impedance magnitude greater than  $100 \text{ k}\Omega$ , we find that:

$$|Z_L| > 10^5$$

$$\omega L > 10^5$$

$$\omega > \frac{10^5}{L}$$

Thus, an AC choke of  $50 \text{ mH}$  would exhibit an impedance magnitude of greater than  $100 \text{ k}\Omega$  for all signal frequencies greater than **320 kHz**.

Note that this is still a fairly low signal frequency for **many** modern electronic applications, and thus this inductor would be an adequate AC choke. Note however, that building an AC choke for **audio** signals ( $20 \text{ Hz}$  to  $20 \text{ kHz}$ ) is typically **very** difficult!