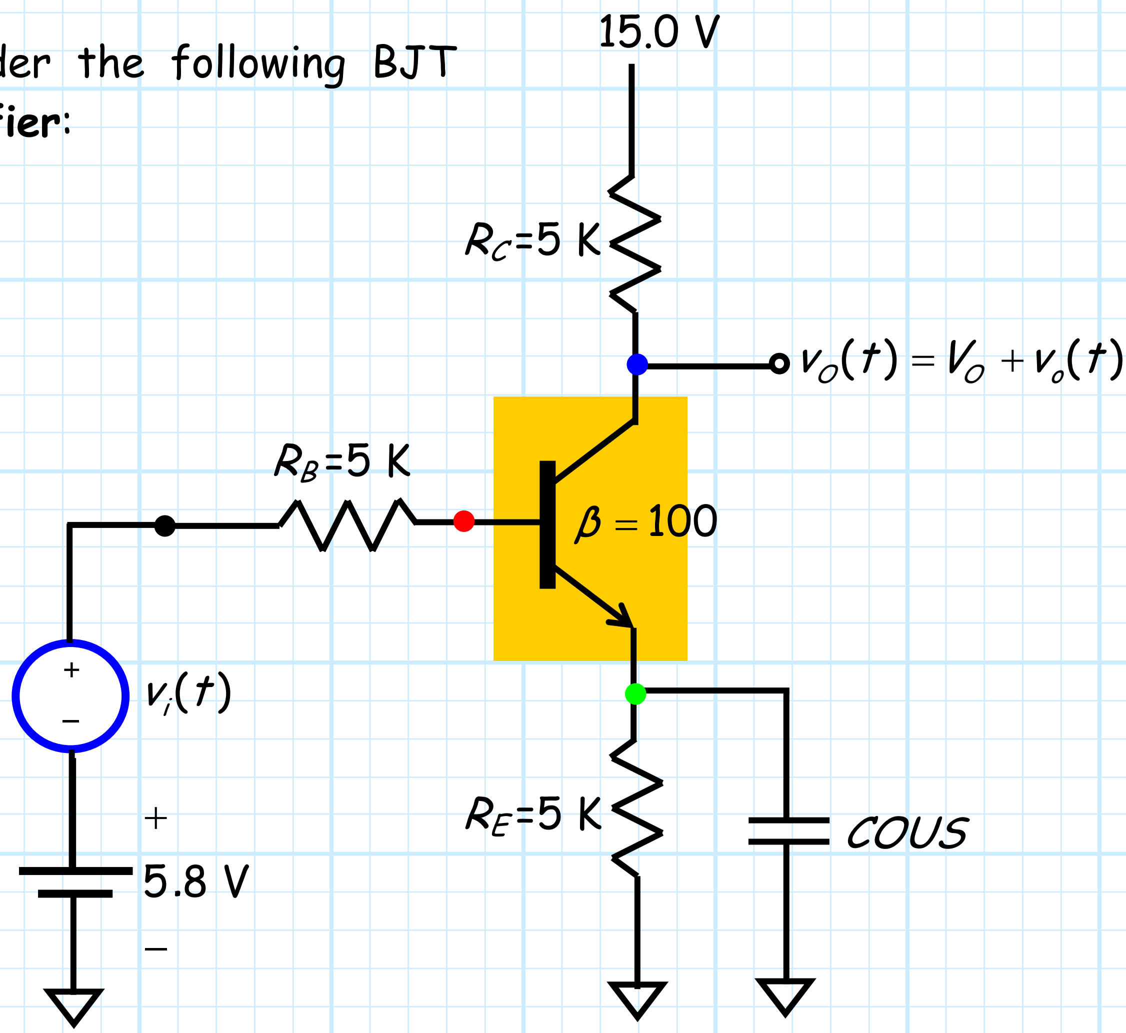
Example: A Small-Signal Analysis of a BJT Amplifier

Consider the following BJT **amplifier**:



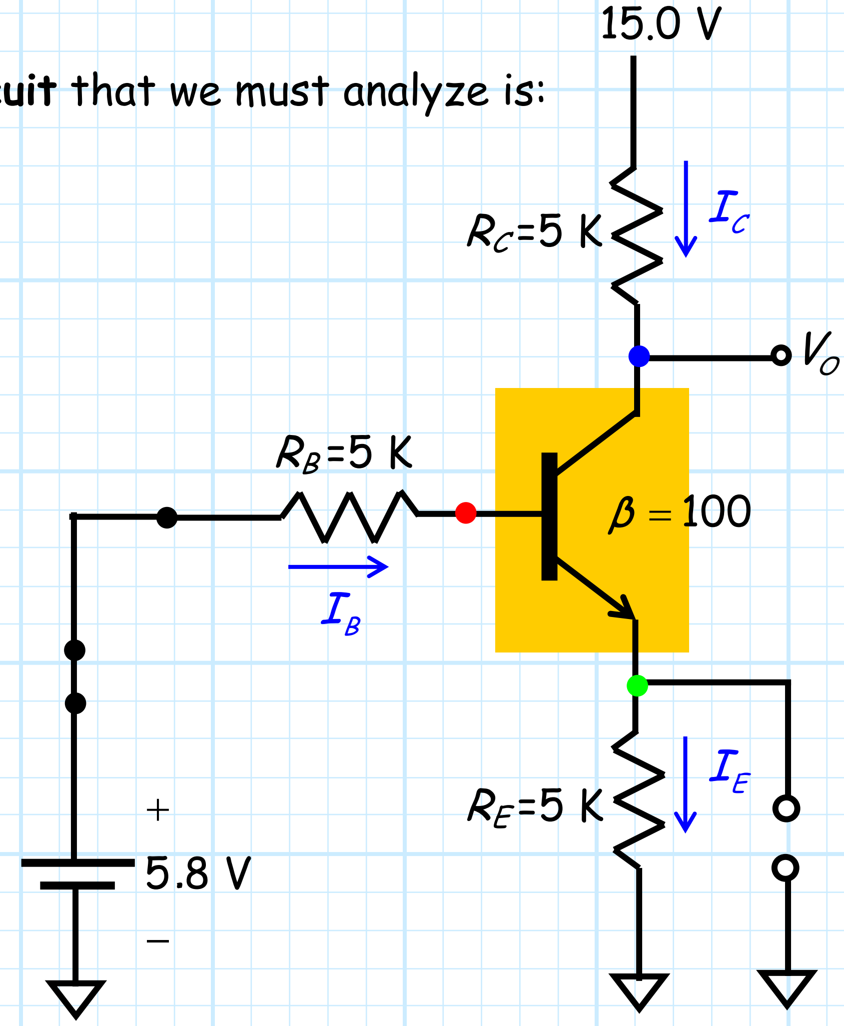
Let's determine its **small-signal**, **open-circuit voltage gain**:

$$A_{vo} = \frac{v_o(t)}{v_i(t)}$$

To do this, we must follow each of our **five** small-signal analysis **steps**!

Step 1: Complete a **D.C. Analysis**

The **DC circuit** that we must analyze is:



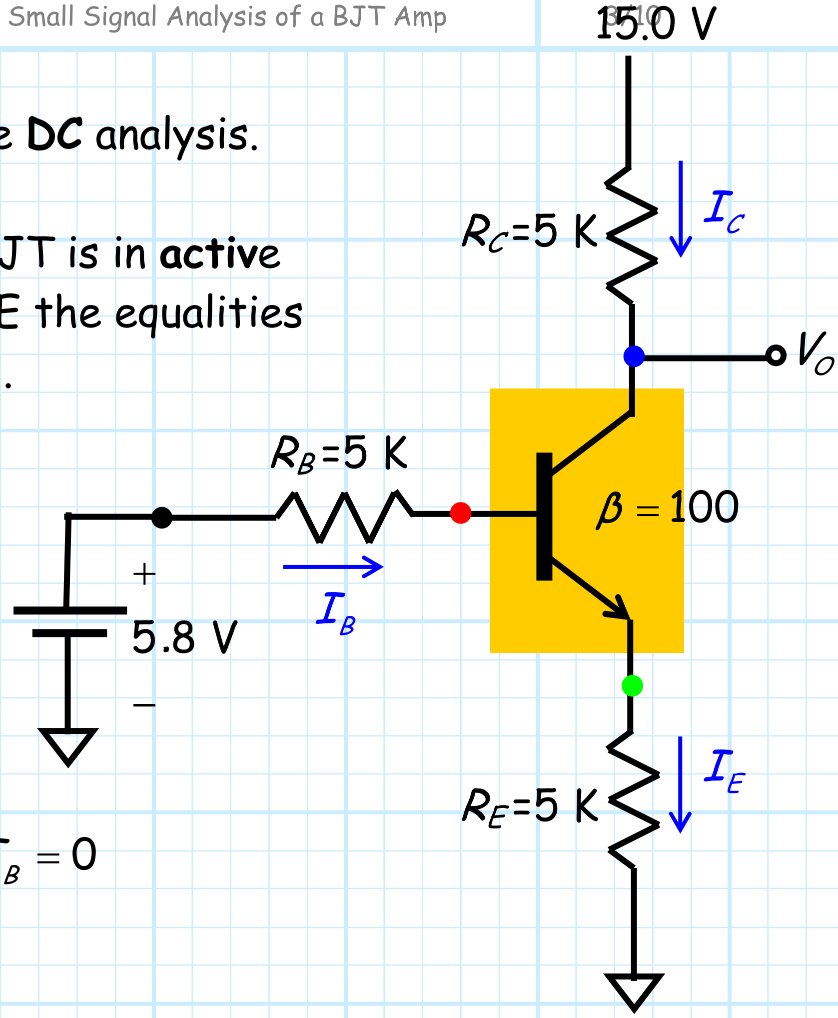
Note what we have done to the original circuit:

1) We turned **off** the **small-signal** voltage source ($v_i(t) = 0$), thus replacing it with a **short** circuit.

2) We replaced the **capacitor** with an **open** circuit—its DC impedance.

Now we proceed with the **DC** analysis.

We ASSUME that the BJT is in **active** mode, and thus ENFORCE the equalities $V_{BE} = 0.7$ V and $I_C = \beta I_B$.



We now begin to ANALYZE the circuit by writing the Base-Emitter Leg KVL:

$$5.8 - 5I_B - 0.7 - 5(\beta+1)I_B = 0$$

Therefore:

$$I_B = \frac{5.1}{5+5(101)} = 0.01 \text{ mA}$$

and thus:

$$I_C = \beta I_B = 1.0 \text{ mA}$$

$$I_E = I_B + I_C = 1.01 \text{ mA}$$

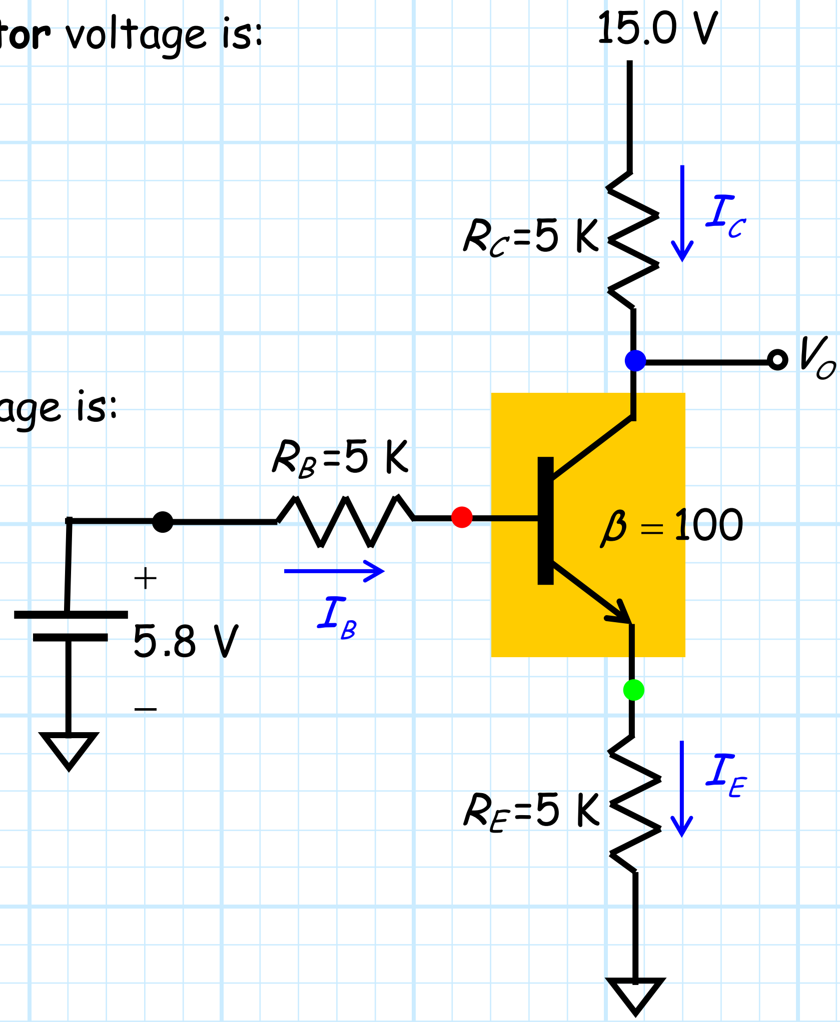
Q: *Since we know the DC bias currents, we have **all** the information we need to determine the **small-signal parameters**.*

*Why don't we proceed **directly** to step 2?*

A: Because we still need to CHECK our assumption! To do this, we must determine either V_{CE} or V_{CB}.

Note that the **Collector** voltage is:

$$\begin{aligned} V_C &= 15 - I_C R_C \\ &= 15 - (1.0)5 \\ &= 10.0 \text{ V} \end{aligned}$$



And the **Emitter** voltage is:

$$\begin{aligned} V_E &= I_E R_E \\ &= (1.01)5 \\ &= 5.05 \text{ V} \end{aligned}$$

Therefore, V_{CE} is:

$$\begin{aligned} V_{CE} &= V_C - V_E \\ &= 10.0 - 5.05 \\ &= 4.95 \text{ V} \end{aligned}$$

We now can complete our CHECK:

$$I_C = 1.0 \text{ mA} > 0 \quad ✓$$

$$V_{CE} = 4.95 \text{ V} > 0.7 \quad ✓$$

Time to move on to **step 2!**

Step 2: Calculate the **small-signal circuit parameters** for each BJT.

If we use the **Hybrid-Π** model, we need to determine g_m and r_π:

$$g_m = \frac{I_C}{V_T} = \frac{1.0\ mA}{0.025V} = 40\ \frac{mA}{V}$$

$$r_\pi = \frac{V_T}{I_B} = \frac{0.025\text{ V}}{0.01\text{ mA}} = 2.5\text{ K}$$

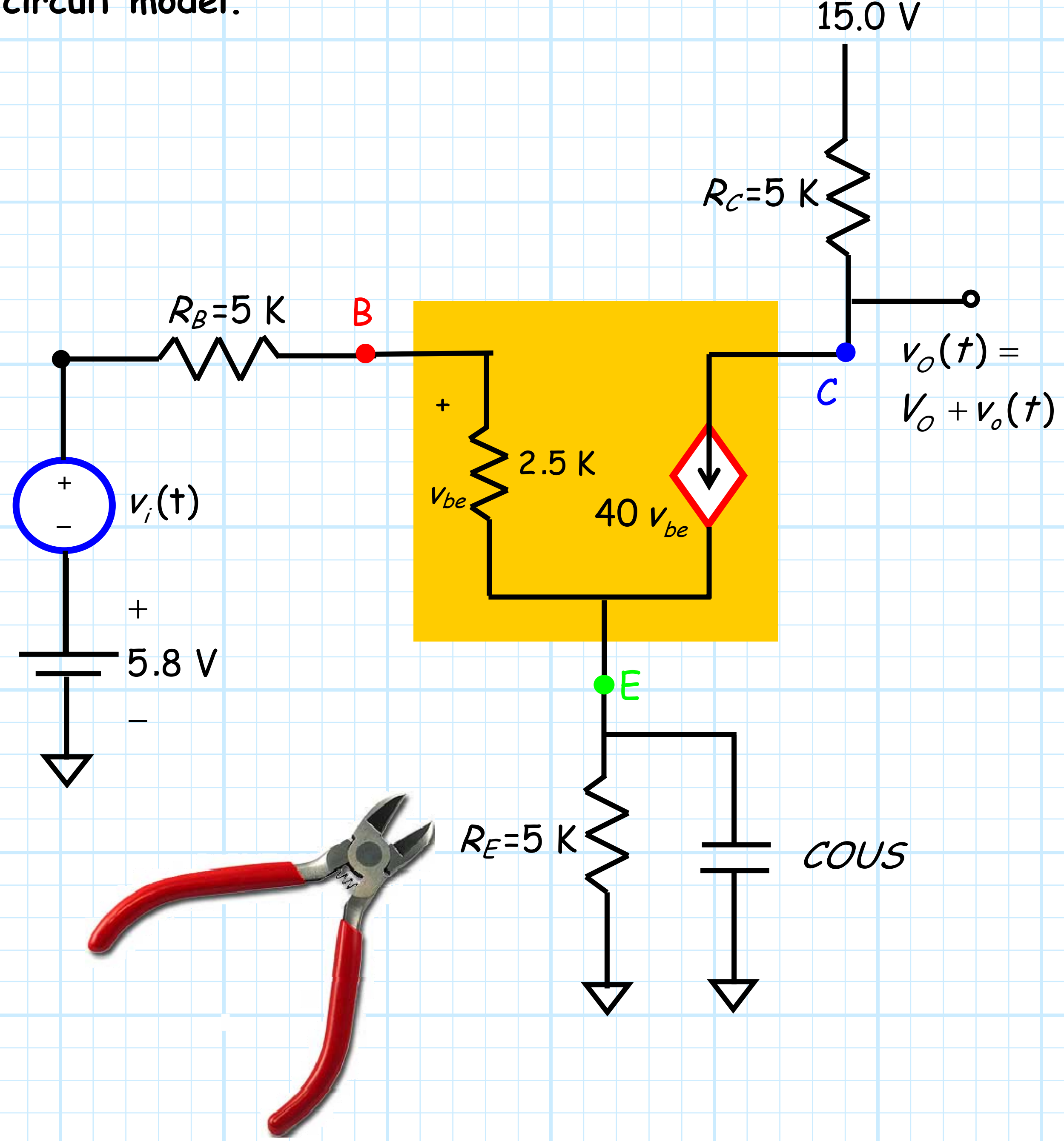
If we were to use the **T-model** we would likewise need to determine the emitter resistance:

$$r_e = \frac{V_T}{I_B} = \frac{0.025\text{ V}}{1.01\text{ mA}} = 24.7\ \Omega$$

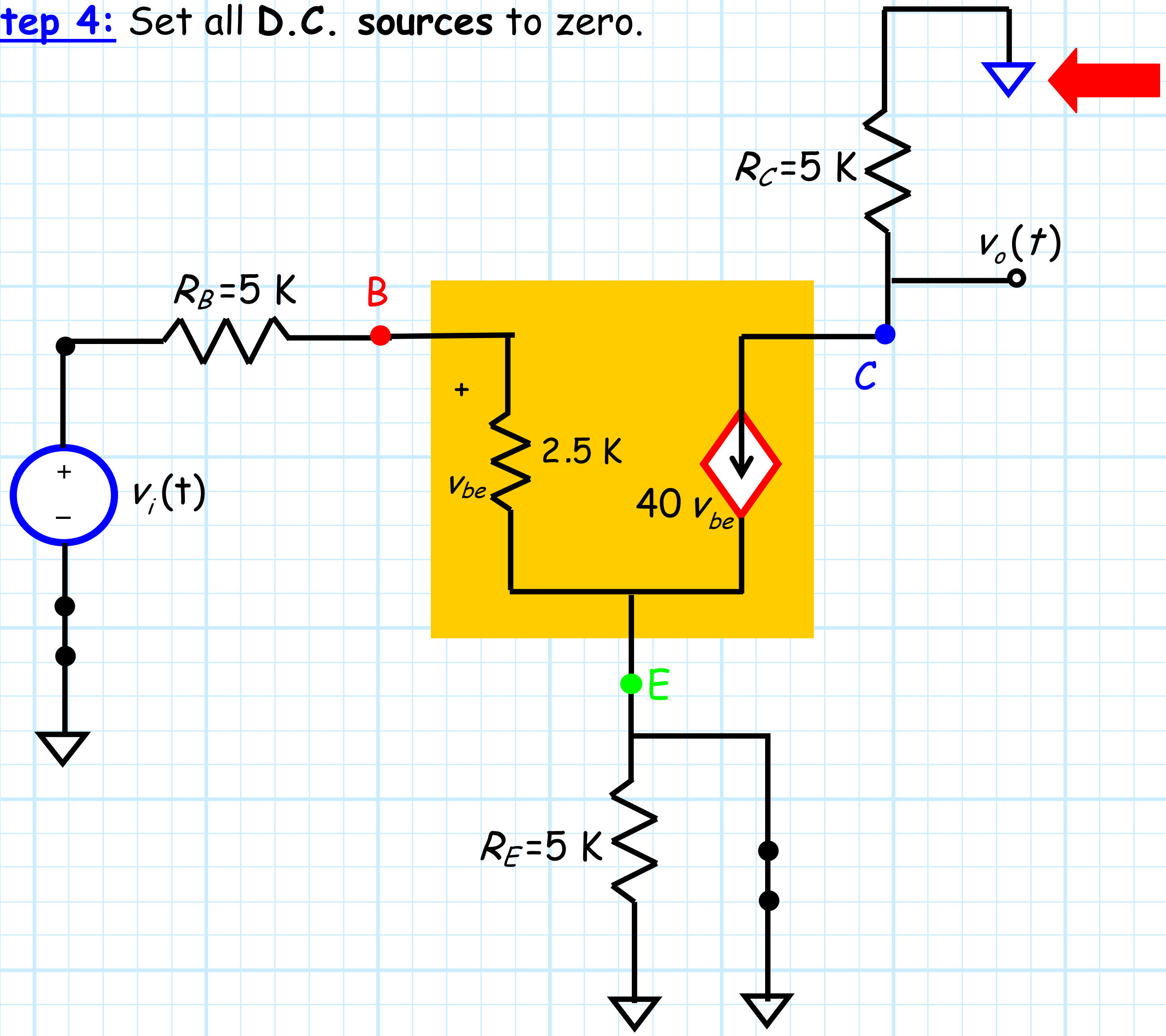
The **Early voltage** V_A of this BJT is unknown, so we will **neglect** the Early effect in our analysis.

As such, we assume that the output resistance is **infinite** ($r_o = \infty$).

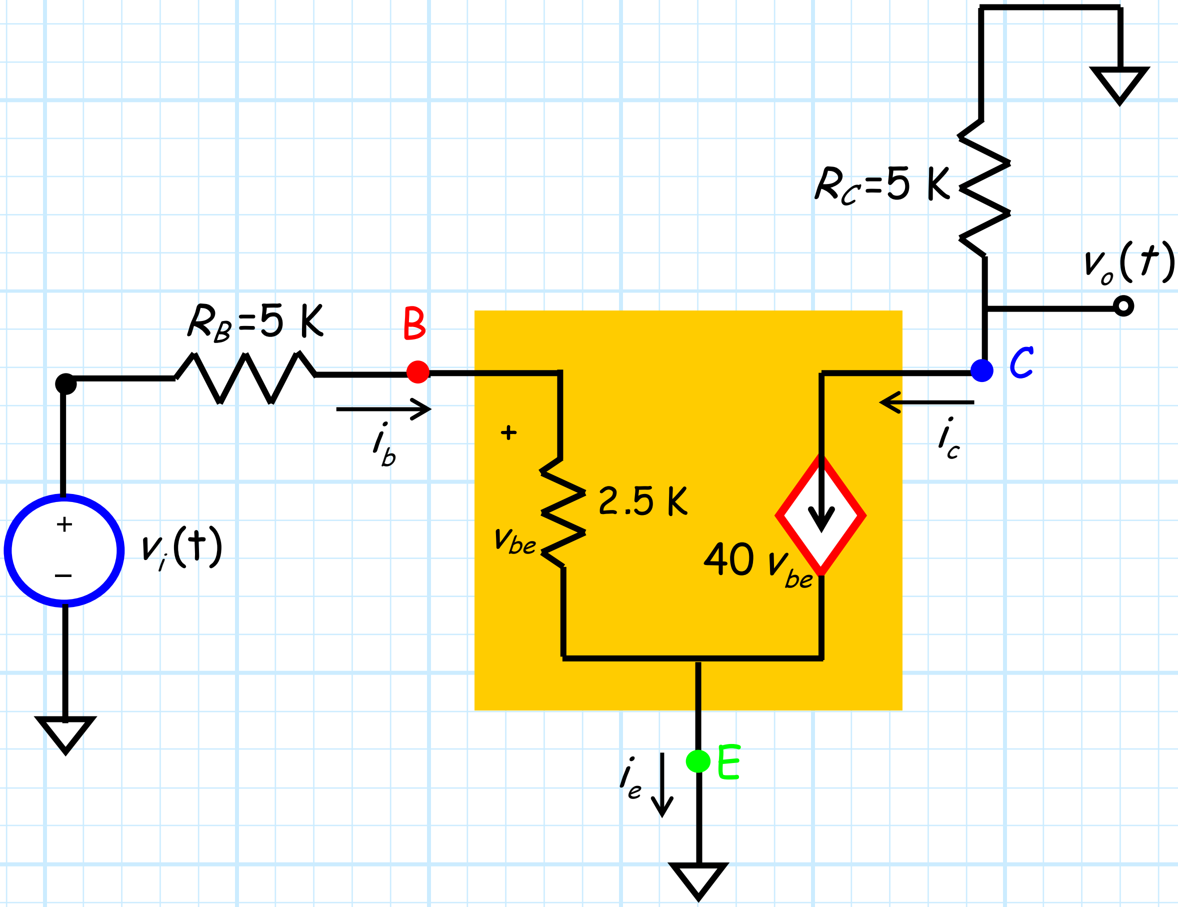
Step 3: Carefully replace all BJTs with their **small-signal circuit model.**



Step 4: Set all **D.C. sources** to zero.

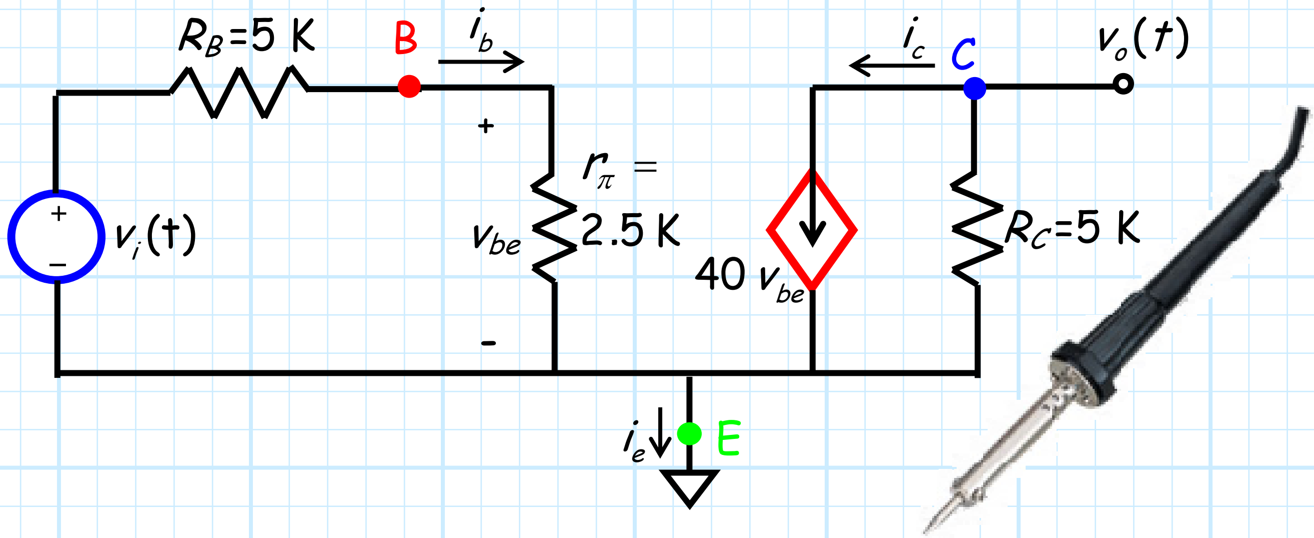


We likewise notice that the **large capacitor** (COUS) is an approximate ***AC* short**, and thus we can further simplify the schematic by replacing it with a short circuit.



We notice that one terminal of the small-signal voltage source, the emitter terminal, and one terminal of the collector resistor R_C are all connected to **ground**—thus they are **all** collected to **each other**!

We can use this fact to **simplify** the small-signal schematic.



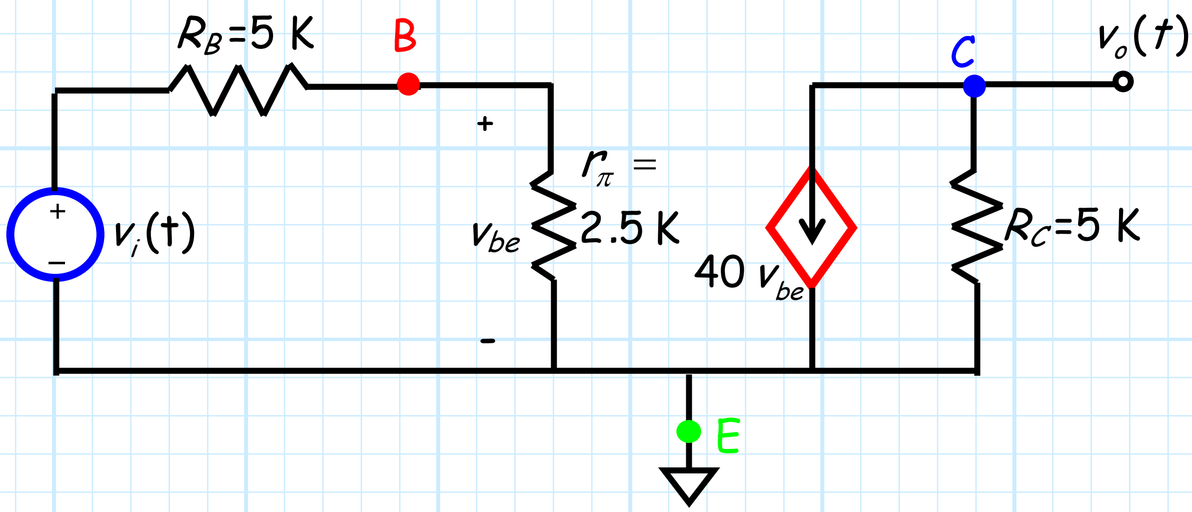
The schematic above is the **small-signal circuit** of this amplifier. We are ready to continue to **step 5**!

Step 5: Analyze **small-signal circuit**.

This is just a simple **EECS 211** problem! The **left** side of the circuit provides the **voltage divider** equation:

$$\begin{aligned} v_{be} &= \frac{r_\pi}{R_B + r_\pi} v_i \\ &= \frac{2.5}{5.0 + 2.5} v_i \\ &= \frac{v_i}{3} \end{aligned}$$

a result that relates the **input** signal to the base-emitter voltage.



The **right** side of the schematic allows us to determine the **output** voltage in terms of the base-emitter voltage:

$$\begin{aligned} v_o &= -i_c R_C \\ &= -(g_m v_{be}) R_C \\ &= -40(5) v_{be} \\ &= -200\, v_{be} \end{aligned}$$

Combining these two equations, we find:

$$\begin{aligned} v_o &= -200\, v_{be} \\ &= -200 \frac{v_i}{3} \\ &= -66.7\, v_i \end{aligned}$$

The **open-circuit**, small-signal **voltage gain** of this amplifier gain is therefore:

$$A_{vo} = \frac{v_o}{v_i} = -66.7$$