Example: Small-Signal BJT Approximations

Say that we wish to find the collector current i_C of a BJT biased in the active mode, with $I_S = 10^{-12} A$ and a **base-emitter voltage** of:

$$v_{BE} = 0.6 + 0.001\cos\omega t \quad V$$

Q: *Easy! Since:*

$$i_C = I_S \, e^{v_{BE}/V_T}$$

we find:

$$i_C(t) = \left(I_S \, e^{0.6/V_T} \right) e^{\left(0.001\cos\omega t / V_T\right)}$$

right?

A: Although this answer is definitely **correct**, it is **not very useful** to us as engineers. Clearly, the base-emitter voltage consists of a **D.C.** bias term (0.6 V) and a **small-signal** term ($0.001\cos\omega t$).

Accordingly, we are interested in the **D.C. collector current** I_C and the **small-signal collector current** i_c. The D.C. collector current is obviously:

$$\begin{aligned} I_C &= I_S \, e^{0.6/V_T} \\ &= 10^{-12} e^{0.6/0.025} \\ &= 26 \text{ mA} \end{aligned}$$

But **how** do we determine the **small-signal** collector current $i_c(t)$ from:

$$i_c(t) = \left(I_S \, e^{0.6/V_T} \right) e^{\left(0.001\cos\omega t / V_T\right)} \quad ???$$

The answer, of course, is to use the **small-signal approximation**.

We know that:

$$i_c(t) = g_m \, v_{be}(t)$$

where:

$$g_m = \frac{I_C}{V_T} = \frac{26mA}{25mV} = 1.06 \ \Omega^{-1}$$

Therefore, the **small-signal collector current** is approximately:

$$\begin{aligned} i_c(t) &= g_m \, v_{be}(t) \\ &= 1.06\left(0.001\cos\omega t\right) \\ &= 1.06\cos\omega t \qquad \text{mA} \end{aligned}$$

and therefore the **total** collector current is:

Q: *Say the D.C. bias voltage* ***increases*** *from* $V_{BE} = 0.6\text{ V}$ *to* $V_{BE} = 0.7\text{ V}$. *What happens to the BJT collector current?*

A: The **D.C. bias** current becomes:

$$I_C = I_S\, e^{0.7/V_T} = 10^{-12} e^{0.7/0.025} = 1446\text{ mA} \qquad !!!$$

since the **transconductance** is now:

$$g_m = \frac{I_C}{V_T} = \frac{1446mA}{25mV} = 57.84\ \Omega^{-1}$$

the **small-signal collector current** is:

$$\begin{aligned} i_c(t) &= g_m\, v_{be}(t) \\ &= 57.84\left(0.001\cos wt\right) \\ &= 57.8\cos wt \qquad \text{mA} \end{aligned}$$

Quite an **increase**!

Changing the transistor operating point (i.e., the DC bias point) will typically make a **big** difference in the small-signal result!