Small-Signal Output Resistance

Recall that due to the **Early effect**, the collector current i_C is **slightly** dependent on v_{CE}:

$$i_C = \beta\, i_B \left(1 + \frac{v_{CE}}{V_A}\right)$$

where we recall that V_A is a BJT device parameter, called the **Early Voltage**.

Q: *How does this affect the **small-signal response** of the BJT?*

A: Well, if $i_C(t) = I_C + i_c(t)$ and $v_{CE}(t) = V_{CE} + v_{ce}(t)$, then with the small-signal approximation:

$$\begin{aligned} I_C + i_c &= \beta\, i_B \left(1 + \frac{v_{CE}}{V_A}\right)\Bigg|_{v_{CE}=V_{CE}} + \left(\frac{\partial i_C}{\partial v_{CE}}\Bigg|_{v_{CE}=V_{CE}}\right) v_{ce} \\ &= \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right) + \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right)\left(\frac{1}{V_A}\right) v_{ce} \end{aligned}$$

Small-signal base resistance

Equating the **DC components**:

$$I_C = \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right)$$

And equating the **small-signal components**:

$$i_c = \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right)\left(\frac{1}{V_A}\right) v_{ce}$$

Note that by inserting the DC result, this expression can be simplified to:

$$i_c = I_C \left(\frac{1}{V_A}\right) v_{ce} = \left(\frac{I_C}{V_A}\right) v_{ce}$$

Therefore, another **small-signal** equation is found, one that expresses the small-signal response of the **Early effect**:

$$i_c = \left(\frac{I_C}{V_A}\right) v_{ce}$$

Small-signal base resistance

Recall that we defined (in EECS 312) the BJT **output resistance** r_o:

$$\frac{I_C}{V_A} \doteq \frac{1}{r_o}$$

Be careful! Although the Early Voltage V_A **is** a **device parameter**, the **output resistance** r_o—since it depends on DC collector current I_C—is **not a device parameter!**

Therefore, the **small-signal collector current** resulting **from the Early effect** can likewise be expressed as:

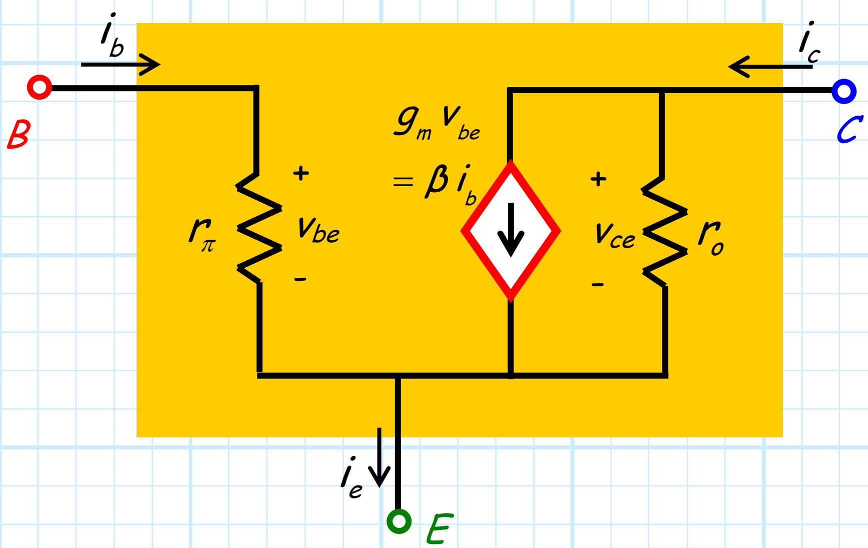
$$i_c = \frac{v_{ce}}{r_o}$$

Small-signal base resistance

Combining this result with an earlier result (i.e., $i_c = g_m v_{be}$), we find that the **total** small-signal collector current is:

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_o} = \beta i_b + \frac{v_{ce}}{r_o}$$

We can account for this effect in our small-signal **circuit models**. For example, the Hybrid-Π becomes:



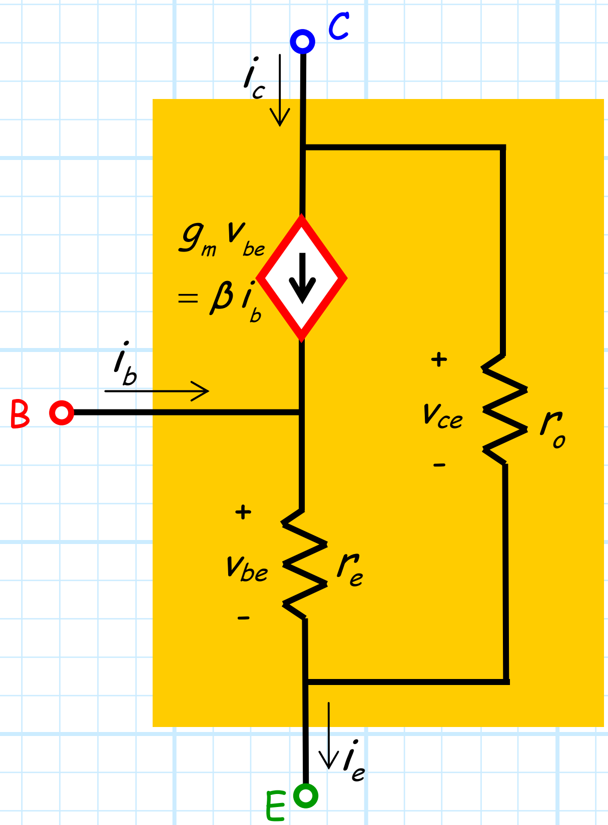
$$i_b = v_{be} / r_\pi$$

$$i_c = g_m v_{be} + \frac{v_{be}}{r_o}$$

$$i_e = i_b + i_c$$

Small-signal base resistance

And for the **T**-model:



Often, r_o is so **large** that it can be ignored (**caution:** ignoring the output resistance means approximating it as an **open** circuit, i.e., $r_o = \infty$).