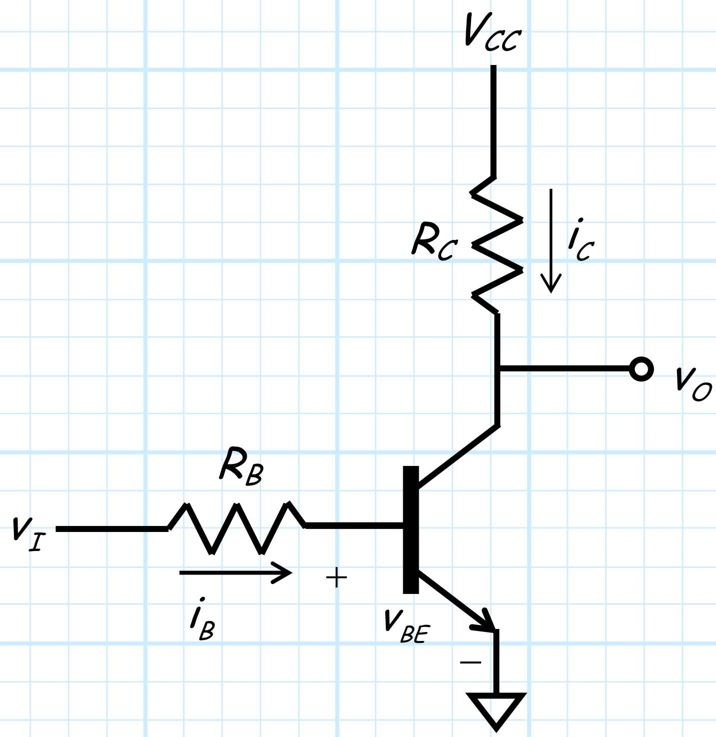
The Small-Signal Circuit Equations

Now let's again consider this circuit, where we assume the BJT is in the **active** mode:



The **four** equations describing this circuit are:

1) $v_I - R_B\, i_B - v_{BE} = 0$ (KVL)

2) $i_C = \beta\, i_B$ (BJT)

3) $v_O = V_{CC} - R_C\, i_C$ (KVL)

4) $i_C = I_s\, e^{v_{BE}/V_T}$ (BJT)

Now, we assume that each current and voltage has **both** a small-signal and DC component. Writing each equation **explicitly** in terms of these components, we find that the four circuit equations become:

(1)
$$(V_I + v_i) - R_B(I_B + i_b) - (V_{BE} + v_{be}) = 0$$
$$(V_I - R_B I_B - V_{BE}) + (v_i - R_B i_b - v_{be}) = 0$$

(2)
$$I_C + i_c = \beta(I_B + i_b)$$
$$I_C + i_c = \beta I_B + \beta i_b$$

(3)
$$V_O + v_o = V_{CC} - R_C(I_C + i_c)$$
$$V_O + v_o = (V_{CC} - R_C I_C) - R_C i_c$$

(4)
$$I_C + i_c = I_s e^{(V_{BE} + v_{be})/V_T}$$
$$I_C + i_c = I_s e^{V_{BE}/V_T} e^{v_{be}/V_T}$$

Note that each equation is really **two** equations!

1. The sum of the **DC** components on **one** side of the equal sign must equal the sum of the **DC** components on the **other**.

2. The sum of the **small-signal** components on **one** side of the equal sign must equal the sum of the **small-signal** components on the **other**.

This result can greatly **simplify** our quest to determine the **small-signal** amplifier parameters!

*You see, all we need to do is determine four **small-signal** equations, and we can then solve for the four **small-signal** values i_b, i_c, v_{be}, v_o!*

From (1) we find that the DC equation is:

$$V_I - R_B I_B - V_{BE} = 0$$

while the small-signal equation from 1) is:

$$v_i - R_B i_b - v_{be} = 0$$

Similarly, from equation (2) we get these equations:

$$I_C = \beta I_B \qquad \text{(DC)}$$

$$i_c = \beta i_b \qquad \text{(small signal)}$$

And from equation (3):

$$V_O = V_{CC} - R_C I_C \qquad \text{(DC)}$$

$$v_o = -R_C i_c \qquad \text{(small-signal)}$$

Finally, from equation (4) we, um, get, er—just what the heck **do** we get?

$$(4) \qquad I_C + i_c = I_s \, e^{(V_{BE}+v_{be})/V_T}$$

$$I_C + i_c = I_s \, e^{V_{BE}/V_T} \, e^{v_{be}/V_T} \qquad ????$$

Q: *Jeepers! Just what are the **DC** and **small-signal** components of:*

$$I_s \, e^{V_{BE}/V_T} \, e^{v_{be}/V_T} \qquad ???$$

A: Precisely speaking, we **cannot** express the above expression as the sum of a **DC** and **small-signal** component. Yet, we **must** determine a **fourth** small-signal equation in order to determine the **four** small signal values i_b, i_c, v_{be}, v_o!

However, we can **approximate** the above expression as the sum of DC and small-signal components. To accomplish this, we must apply the **small-signal approximation** (essentially a Taylor series approx.).

We will find that the **small-signal approximation** provides an accurate small-signal equation for expressions such (4). We will likewise find that this approximate equation is accurate **if** the small-signal voltage v_{be} is, well, **small**!