5.6 Small-Signal Operation and Models

Reading **Assignment**: 443-458

Now let's examine how we use BJTs to construct **amplifiers**!

The first important design rule is that the BJT must be biased to the **active mode**.

HO: BJT GAIN AND THE ACTIVE REGION

For a BJT amplifier, we find that every current and every voltage has two components: the **DC (i.e., bias) component**—a value carefully selected and designed by a EE, and the **small-signal component**, which is the AC signal we are attempting to amplify (e.g., audio, video, etc.).

HO:DC AND SMALL-SIGNAL COMPONENTS

There are two extremely important circuit elements in small-signal amplifier design: the **Capacitor of Unusual Size (COUS)** and the **Inductor of Unusual Size (IOUS)**.

These devices are just realizable approximations of the **Unfathomably Large Capacitor (ULC)** and the **Unfathomably Large Inductor (ULI)**. These devices have radically different properties when considering DC and small-signal components!

HO:DC AND AC IMPEDANCE OF REACTIVE ELEMENTS

It turns out that separating **BJT** currents and voltages into DC and small-signal components is problematic!

HO:THE SMALL-SIGNAL CIRCUIT EQUATIONS

But, we can approximately determine the small-signal components if we use the **small-signal approximation**.

HO: A SMALL-SIGNAL ANALYSIS OF HUMAN GROWTH

HO: A SMALL-SIGNAL ANALYSIS OF A BJT

Let's do an **example** to illustrate the small-signal approximation.

EXAMPLE: SMALL-SIGNAL BJT APPROXIMATIONS

There are **several small-signal parameters** that can be extracted from a small-signal analysis of a BJT.

HO: BJT SMALL-SIGNAL PARAMETERS

HO: THE SMALL-SIGNAL EQUATION MATRIX

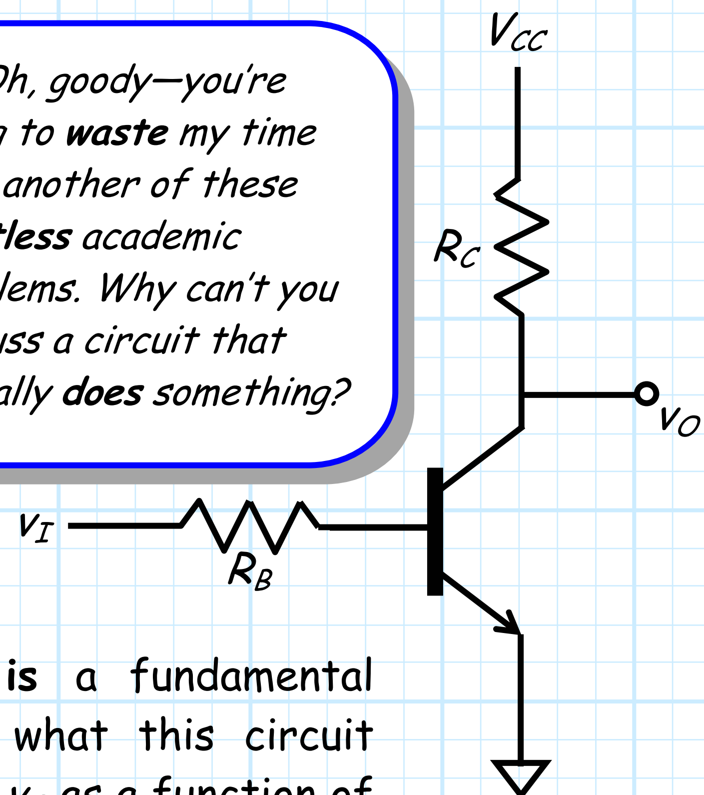
Let's do an **example!**

EXAMPLE: CALCULATING THE SMALL-SIGNAL GAIN

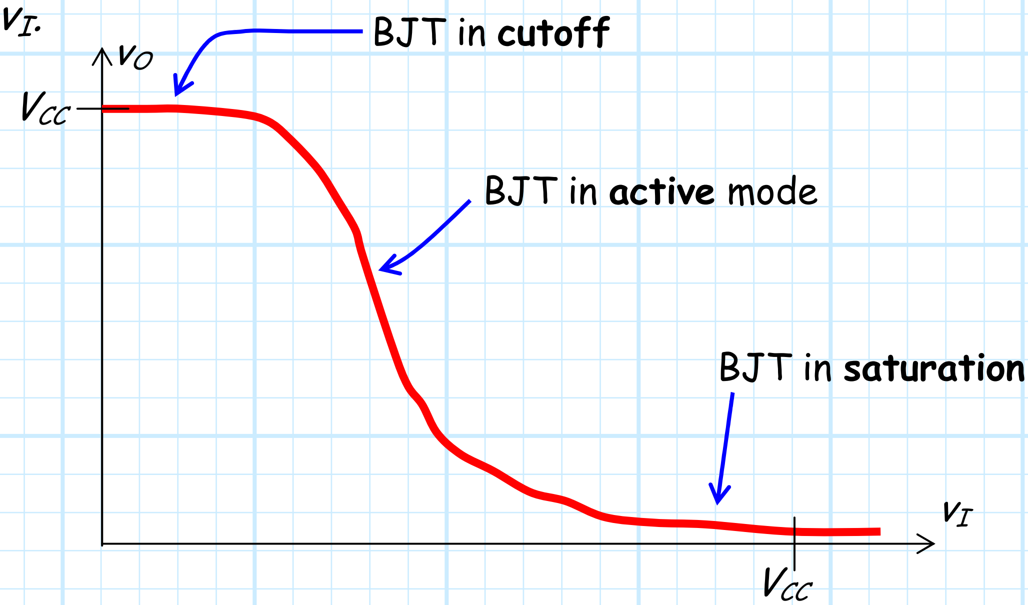
BJT Amplifier Gain and the Active Region

Consider this **simple** BJT circuit:

Q: *Oh, goody—you're going to **waste** my time with another of these **pointless** academic problems. Why can't you discuss a circuit that actually **does** something?*

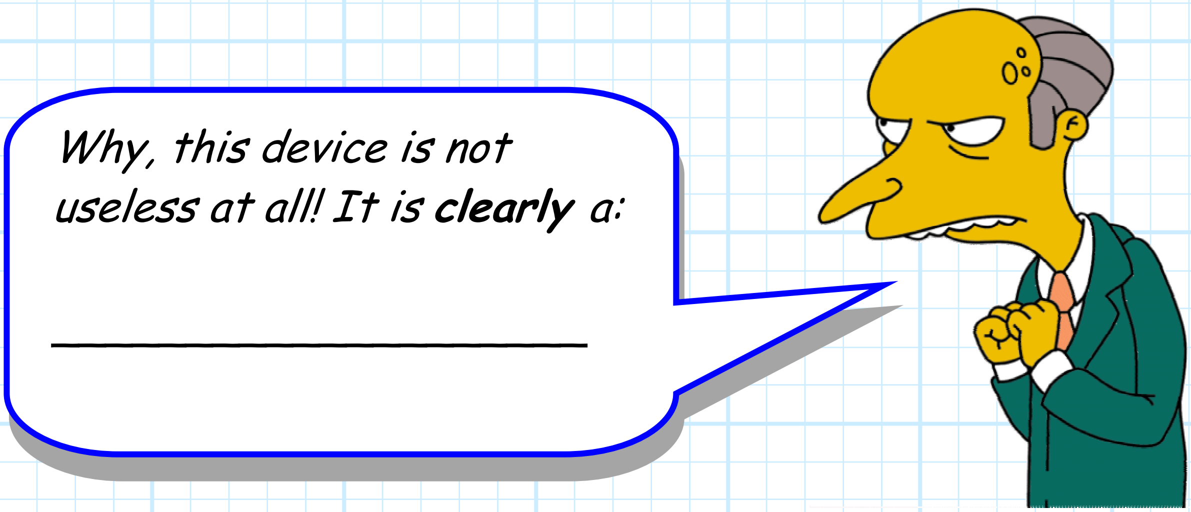


A: Actually, this circuit **is** a fundamental electronic device! To see what this circuit does, plot the output voltage v_O as a function of the input v_I.

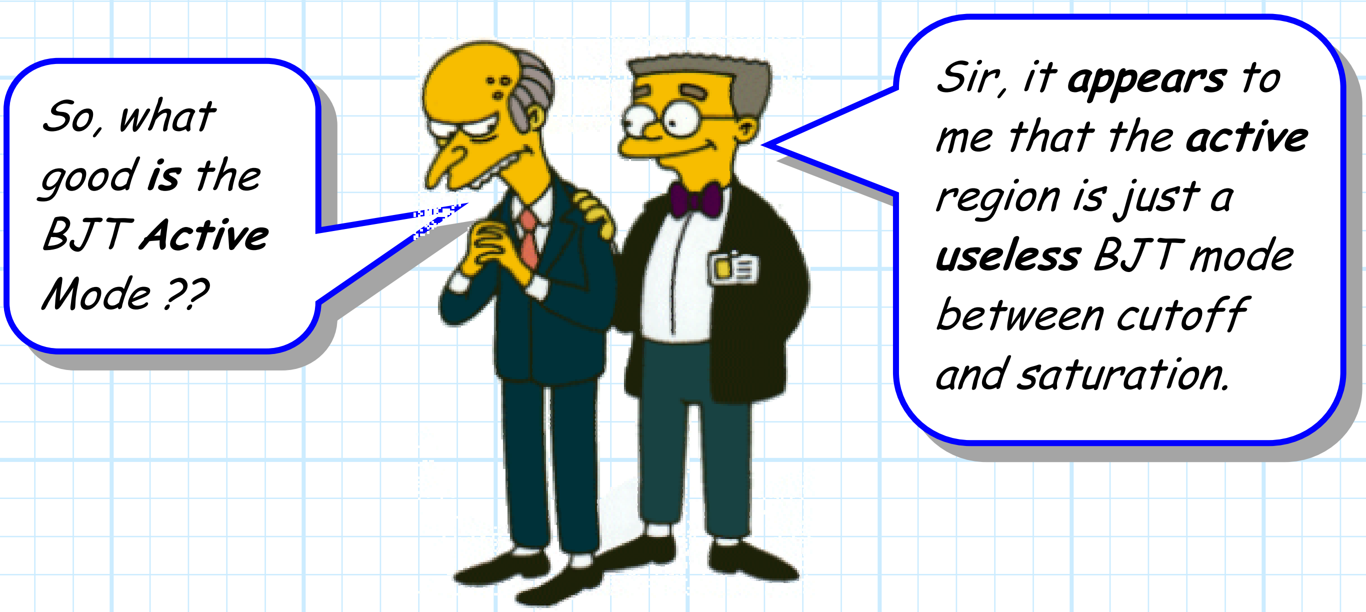


Note that:

v_I	v_O	**Mode**
0	V_{CC}	Cutoff
V_{CC}	0	Saturation

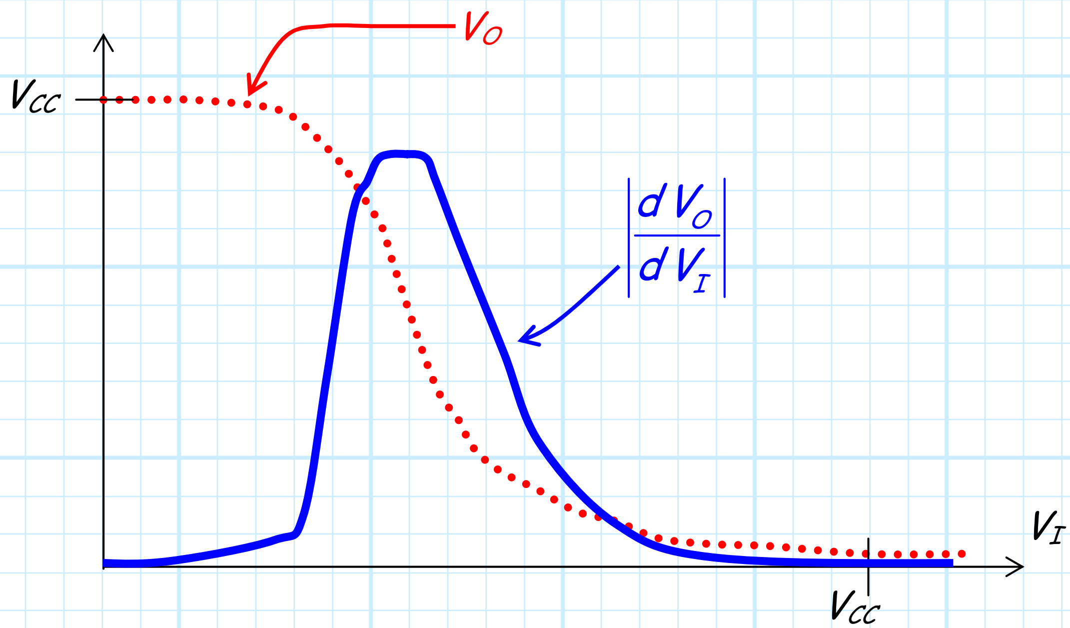


Digital devices made with BJTs typically work in either the **cutoff** of **saturation** regions.



Actually, we will find that the active mode is **extremely** useful!

To see why, take the **derivative** of the above circuit's transfer function (i.e., dV_O/dV_I):



We note that in **cutoff** and **saturation**:

$$\left|\frac{dV_O}{dV_I}\right| \approx 0$$

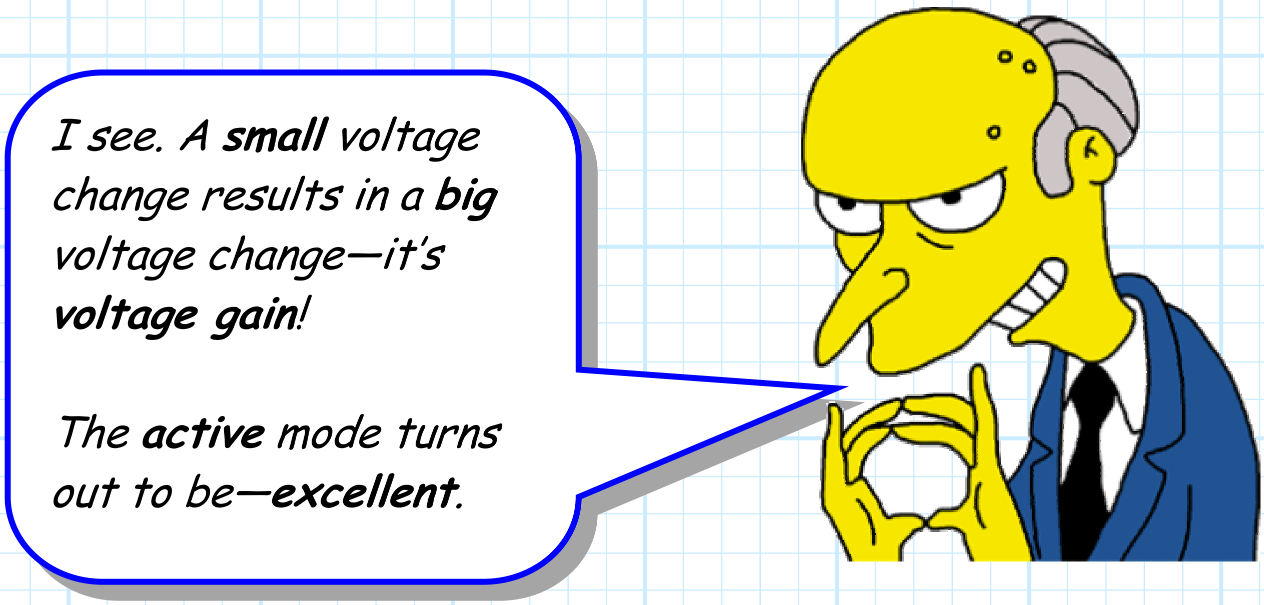
while in the **active** mode:

$$\left|\frac{dV_O}{dV_I}\right| >> 1$$

Q: *I've got better things to do than listen to some egghead professor mumble about derivatives. Are these results even **remotely** important?*

A: Since in **cutoff** and **saturation** $d V_O / d V_I = 0$, a small change in input voltage V_I will result in almost **no change** in output voltage V_O.

Contrast this with the **active** region, where $\left| d V_O / d V_I \right| >> 1$. This means that a **small** change in **input** voltage V_I results in a **large** change in the **output** voltage V_O!



Whereas the important BJT regions for **digital** devices are saturation and cutoff, bipolar junction transistors in **linear** (i.e., analog) devices are typically biased to the **active** region.

This is especially true for BJT **amplifier**. Almost all of the transistors in EECS 412 will be in the **active** region—this is where we get **amplifier gain**!

DC and Small-Signal Components

Note that we have used **DC sources** in all of our example circuits thus far.

We have done this just to **simplify** the analysis—generally speaking, realistic (i.e., useful) junction diode circuits will have sources that are **time-varying**!

The result will be voltages and currents in the circuit that will **likewise vary with time** (e.g., $i(t)$ and $v(t)$).

For example, we can express the forward bias junction diode equation as:

$$i_D(t) = I_s e^{v_D(t)/nV_T}$$

Although source voltages $v_S(t)$ or currents $i_S(t)$ can be **any** general function of time, we will find that often, in realistic and useful electronic circuits, that the source can be decomposed into **two** separate components—the **DC component** V_S, and the **small-signal component** $v_s(t)$. I.E.:

$$v_S(t) = V_S + v_s(t)$$

Let's look at each of these components **individually**:

* The **DC component** V_S is exactly what you would expect—the DC component of source $v_S(t)$!

Note this DC value is **not** a function of time (otherwise it would not be DC!) and therefore is expressed as a **constant** (e.g., $V_S = 12.3\,V$).

Mathematically, this DC value is the **time-averaged** value of $v_S(t)$:

$$V_S = \frac{1}{T}\int_0^T v_S(t)\,dt$$

where T is the **time duration** of function $v_S(t)$.

* As the notation indicates, the **small-signal component** $v_s(t)$ **is** a function of time!

Moreover, we can see that this signal is an ***AC* signal**, that is, its time-averaged value is **zero**! I.E.:

$$\frac{1}{T}\int_0^T v_s(t)\,dt = 0$$

This signal $v_s(t)$ is also referred to as the **small-signal component**.

* The **total** signal $v_S(t)$ is the **sum** of the DC and small signal components. Therefore, it is **neither** a DC nor an AC signal!

Pay attention to the **notation** we have used here. We will use this notation for the remainder of the course!

* **DC values** are denoted as **upper-case** variables (e.g., V_S, I_R, or V_D).

* **Time-varying** signals are denoted as **lower-case** variables (e.g., $v_S(t)$, $v_r(t)$, $i_D(t)$).

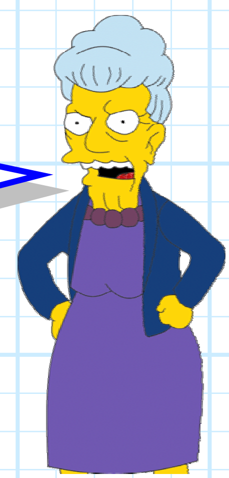
Also,

* **AC signals** (i.e., zero time average) are denoted with **lower-case subscripts** (e.g., $v_s(t)$, $v_d(t)$, $i_r(t)$).

* Signals that are **not AC** (i.e., they have a non-zero DC component!) are denoted with **upper-case subscripts** (e.g., $v_S(t)$, I_D, $i_R(t)$, V_D).

Note we should **never** use variables of the form V_i, I_e, V_b. Do **you** see why??

Q: *You say that we will often find sources with **both** components—a DC and small-signal component. **Why** is that? What is the significance or physical reason for each component?*



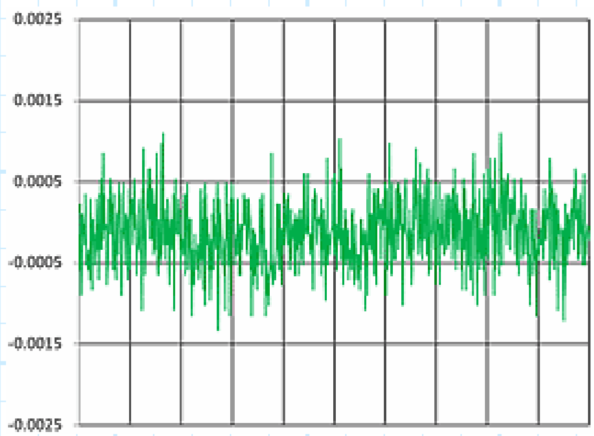
A1: First, the **DC component** is typically just a DC **bias**. It is a **known** value, selected and determined by the design engineer.

It carries or relates **no information**—the only reason it exists is to make the electronic devices work the way we want!

A2: Conversely, the **small signal component** is typically **unknown**!

It is the signal that we are often attempting to **process** in some manner (e.g., amplify, filter, integrate). The signal itself represents **information** such as audio, video, or data.

Sometimes, however, this small, AC, unknown signal represents not information—but **noise**!



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Noise is a **random**, unknown signal that in fact masks and **corrupts** information.

Our job as designers is to **suppress** it, or otherwise minimize it deleterious effects.

* This noise may be changing **very rapidly** with time (e.g., MHz), or may be changing **very slowly** (e.g., mHz).

* Rapidly changing noise is generally "**thermal noise**", whereas slowly varying noise is typically due to slowly varying environmental conditions, such as **temperature**.

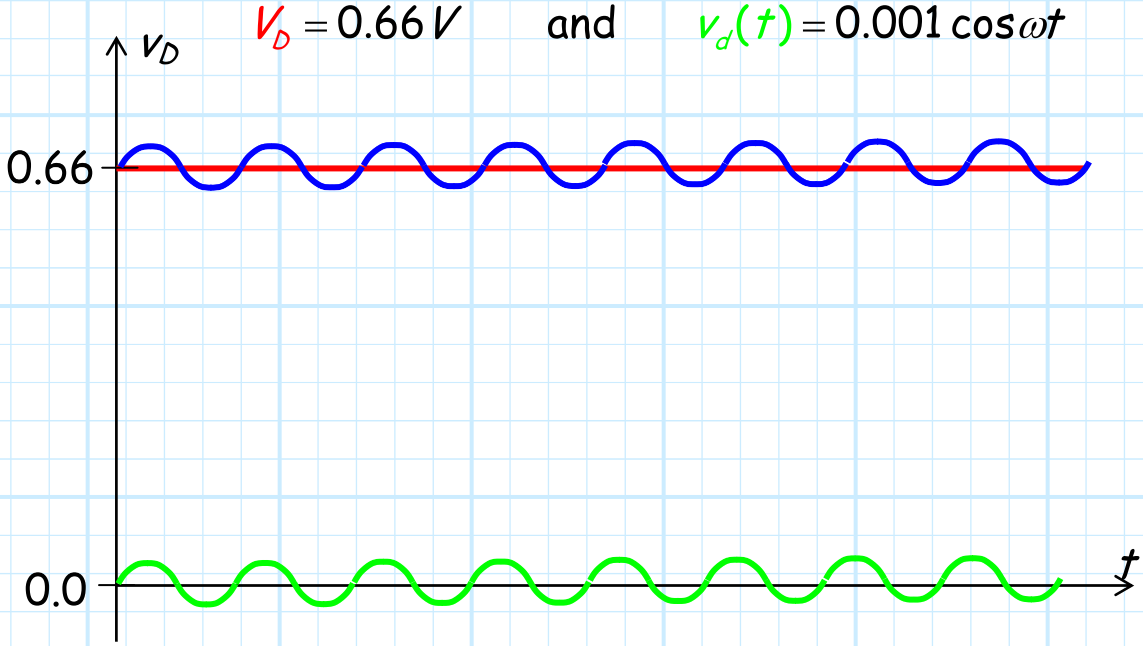
Note that in addition to (or perhaps because of) the source voltage $v_S(t)$ having both a DC bias and small-signal component, **all the currents and voltages** (e.g., $i_R(t)$, $v_D(t)$) within our circuits will likewise have **both** a DC bias and small-signal component!

For example, the junction **diode voltage** might have the form:

$$v_D(t) = 0.66 + 0.001\cos\omega t$$

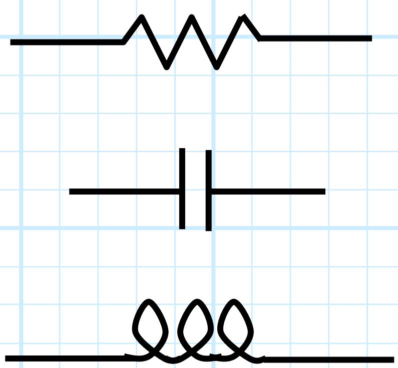
It is hopefully evident that:

$$V_D = 0.66\, V \qquad \text{and} \qquad v_d(t) = 0.001\cos\omega t$$



DC and AC Impedance of Reactive Elements

Now, recall from EECS 211 the **complex impedances** of our basic circuit elements:



$$Z_R = R$$

$$Z_C = \frac{1}{j\omega C}$$

$$Z_L = j\omega L$$

For a **DC** signal ($\omega = 0$), we find that:

$$Z_R = R$$

$$Z_C = \lim_{\omega \to 0} \frac{1}{j\omega C} = \infty$$

$$Z_L = j(0)L = 0$$

Thus, at **DC** we know that:

* a **capacitor** acts as an **open** circuit (I_C =0).

* an **inductor** acts as a **short** circuit (V_L = 0).

Now, let's consider **two** important cases:

1. A **capacitor** whose capacitance C is **unfathomably large**.

2. An **inductor** whose inductance L is **unfathomably large**.

1. The Unfathomably Large Capacitor

In this case, we consider a capacitor whose capacitance is **finite**, but **very, very, very** large.

For **DC** signals ($\omega = 0$), this device acts **still** acts like an **open** circuit.

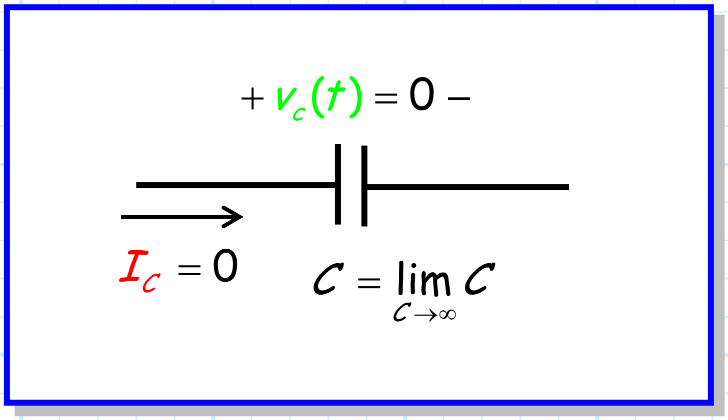
However, now consider the **AC signal case** (e.g., a small signal), where $\omega \neq 0$. The **impedance** of an unfathomably large capacitor is:

$$Z_C = \lim_{C \to \infty} \frac{1}{j\omega C} = 0$$

Zero impedance!

→ An unfathomably large capacitor acts like an **AC short**.

Quite a trick! The unfathomably large capacitance acts like an **open** to **DC** signals, but likewise acts like a **short** to **AC** (small) signals!

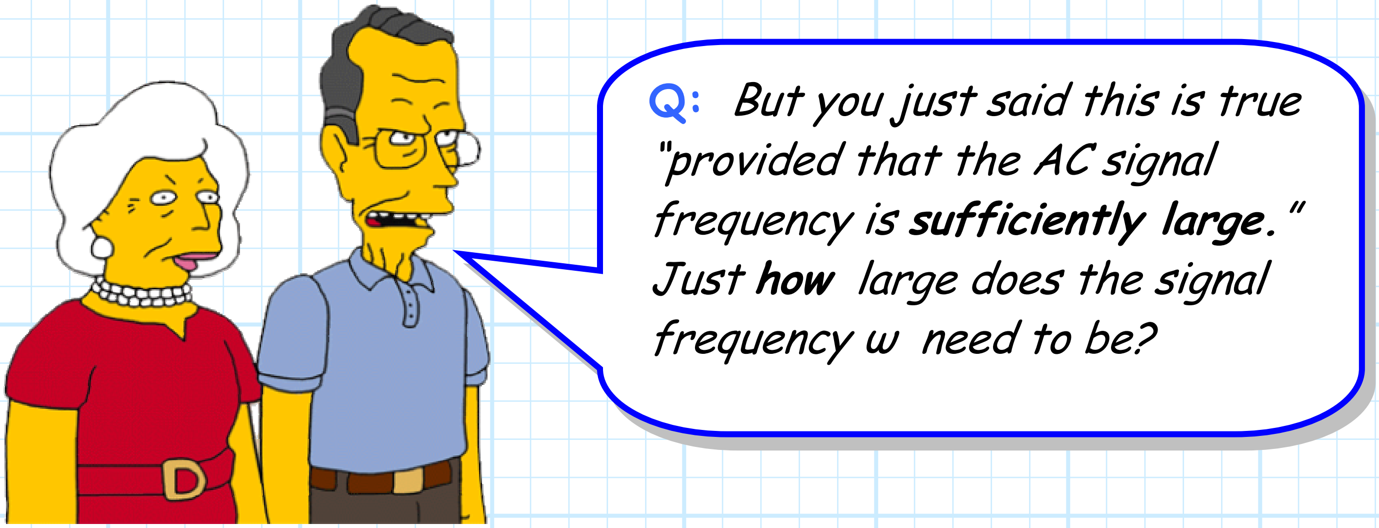


Q: *I fail to see the **relevance** of this analysis at this juncture. After all, **unfathomably** large capacitors do **not** exist, and are **impossible** to make (being unfathomable and all).*

A: True enough! However, we can make **very big** (but **fathomably** large) capacitors. Big capacitors will not act as a **perfect** AC short circuit, but **will** exhibit an impedance of **very small** magnitude (e.g., a few Ohms), provided that the AC signal frequency is sufficiently large.

In this way, a **very large** capacitor acts as an **approximate AC short**, and as a **perfect DC open**.

We call these large capacitors **DC blocking capacitors**, as they allow **no DC current** to flow through them, while allowing AC current to flow **nearly unimpeded**!



A: Say we desire the AC impedance of our capacitor to have a magnitude of **less than ten Ohms**:

$$|Z_C| < 10$$

Rearranging, we find that this will occur **if** the frequency ω is:

$$\begin{aligned} 10 &> |Z_C| \\ 10 &> \frac{1}{\omega C} \\ \omega &> \frac{1}{10C} \end{aligned}$$

For **example**, a 50 μF capacitor will exhibit an impedance whose magnitude is less than 10 Ohms for all AC signal frequencies above **320 Hz**.

Likewise, **almost** all AC signals in modern electronics will operate in a spectrum much higher than 320 Hz.

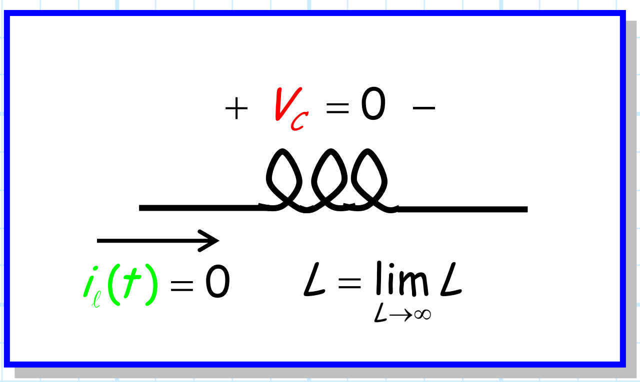
Thus, a 50 μF blocking capacitor will **approximately** act as an AC short and (precisely) act as a DC open.

2. The Unfathomably Large Inductor

Similarly, we can consider an **unfathomably large inductor**. In addition to a **DC** impedance of **zero** (a DC short), we find for the **AC** case (where $\omega \neq 0$):

$$Z_L = \lim_{L \to \infty} j\omega L = \infty$$

In other words, an unfathomably large inductor acts like an ***AC* open circuit!**



The unfathomably large inductor acts like an **short** to **DC** signals, but likewise acts like an **open** to ***AC*** (small) signals!

As before, an unfathomably large inductor is **impossible** to build.

However, a **very large** inductor will typically exhibit a **very large AC impedance** for all but the lowest of signal frequencies ω.

We call these large inductors "AC chokes" (also known RF chokes), as they act as a **perfect short** to **DC** signals, yet so effectively impede AC signals (with sufficiently high frequency) that they act **approximately** as an **AC open circuit**.

For example, if we desire an **AC choke** with an impedance magnitude greater than 100 kΩ, we find that:

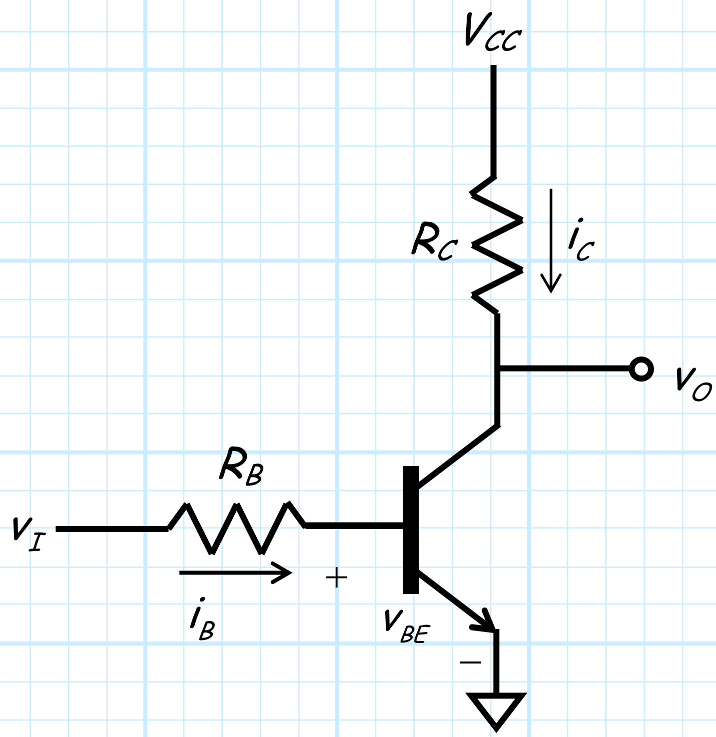
$$
\begin{aligned}
|Z_L| &> 10^5 \\
\omega L &> 10^5 \\
\omega &> \frac{10^5}{L}
\end{aligned}
$$

Thus, an AC choke of 50 mH would exhibit an impedance magnitude of greater than 100 kΩ for all signal frequencies greater than **320 kHz**.

Note that this is still a fairly low signal frequency for **many** modern electronic applications, and thus this inductor would be an adequate AC choke. Note however, that building and AC choke for **audio** signals (20 Hz to 20 kHz) is typically **very** difficult!

The Small-Signal Circuit Equations

Now let's again consider this circuit, where we assume the BJT is in the **active** mode:



The **four** equations describing this circuit are:

1) $v_I - R_B\, i_B - v_{BE} = 0$ (KVL)

2) $i_C = \beta\, i_B$ (BJT)

3) $v_O = V_{CC} - R_C\, i_C$ (KVL)

4) $i_C = I_s\, e^{v_{BE}/V_T}$ (BJT)

Now, we assume that each current and voltage has **both** a small-signal and DC component. Writing each equation **explicitly** in terms of these components, we find that the four circuit equations become:

(1)
$$(V_I + v_i) - R_B(I_B + i_b) - (V_{BE} + v_{be}) = 0$$
$$(V_I - R_B I_B - V_{BE}) + (v_i - R_B i_b - v_{be}) = 0$$

(2)
$$I_C + i_c = \beta(I_B + i_b)$$
$$I_C + i_c = \beta I_B + \beta i_b$$

(3)
$$V_O + v_o = V_{CC} - R_C(I_C + i_c)$$
$$V_O + v_o = (V_{CC} - R_C I_C) - R_C i_c$$

(4)
$$I_C + i_c = I_s\, e^{(V_{BE}+v_{be})/V_T}$$
$$I_C + i_c = I_s\, e^{V_{BE}/V_T}\, e^{v_{be}/V_T}$$

Note that each equation is really **two** equations!

1. The sum of the **DC** components on **one** side of the equal sign must equal the sum of the **DC** components on the **other**.

2. The sum of the **small-signal** components on **one** side of the equal sign must equal the sum of the **small-signal** components on the **other**.

This result can greatly **simplify** our quest to determine the **small-signal** amplifier parameters!

You see, all we need to do is determine four ***small-signal*** *equations, and we can then solve for the four* ***small-signal*** *values i_b, i_c, v_{be}, v_o!*

From (1) we find that the **DC** equation is:

$$V_I - R_B I_B - V_{BE} = 0$$

while the **small-signal** equation from 1) is:

$$v_i - R_B i_b - v_{be} = 0$$

Similarly, from equation (2) we get these equations:

$$I_C = \beta I_B \qquad \text{(DC)}$$

$$i_c = \beta i_b \qquad \text{(small signal)}$$

And from equation (3):

$$V_O = V_{CC} - R_C I_C \qquad \text{(DC)}$$

$$v_o = R_C i_c \qquad \text{(small-signal)}$$

Finally, from equation (4) we, um, get, er—just what the heck **do** we get?

$$(4) \qquad I_C + i_c = I_s \, e^{(V_{BE}+v_{be})/V_T}$$

$$I_C + i_c = I_s \, e^{V_{BE}/V_T} \, e^{v_{be}/V_T} \qquad ????$$

Q: *Jeepers! Just what are the* ***DC*** *and* ***small-signal*** *components of:*

$$I_s \, e^{V_{BE}/V_T} \, e^{v_{be}/V_T} \qquad ???$$

A: Precisely speaking, we **cannot** express the above expression as the sum of a **DC** and **small-signal** component. Yet, we **must** determine a **fourth** small-signal equation in order to determine the **four** small signal values i_b, i_c, v_{be}, v_o!

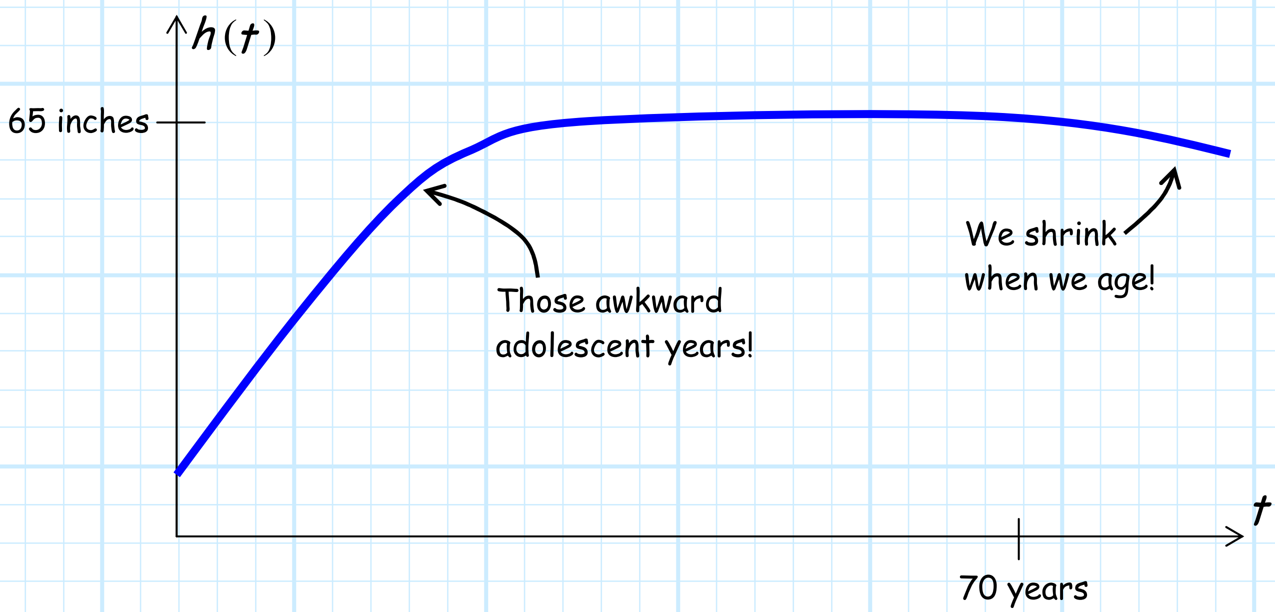
However, we can **approximate** the above expression as the sum of DC and small-signal components. To accomplish this, we must apply the **small-signal approximation** (essentially a Taylor series approx.).

We will find that the **small-signal approximation** provides an accurate small-signal equation for expressions such (4). We will likewise find that this approximate equation is accurate **if** the small-signal voltage v_{be} is, well, **small**!

A "Small-Signal Analysis" of Human Growth

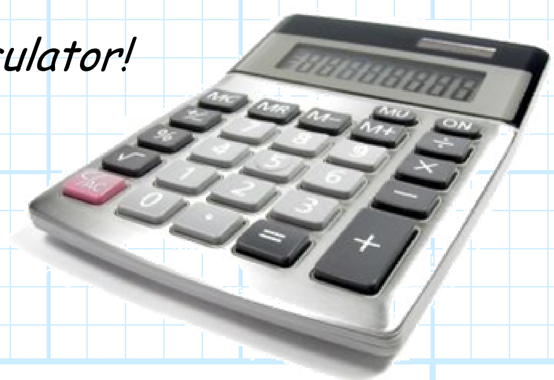
Say the **average height** h of a human (in **inches**) is related to his/her age t in **months** by this equation:

$$h(t) = 65 - 3.66\text{x}10^{-10}(45 - t/12)^{6.75} \text{ inches}$$



Say that we want to **calculate** the average height of a human at an age of t =58, 59, 59.5, 60, 60.5, 61, and 62 months.

Whew! Let me get out my calculator!



$$h(t = 58.0) = 40.48 \text{ inches}$$
$$h(t = 59.0) = 40.82 \text{ inches}$$
$$h(t = 59.5) = 40.99 \text{ inches}$$
$$h(t = 60.0) = 41.16 \text{ inches}$$
$$h(t = 60.5) = 41.32 \text{ inches}$$
$$h(t = 61.0) = 41.49 \text{ inches}$$
$$h(t = 62.0) = 41.82 \text{ inches}$$

Q: *Wow, this was hard. Isn't there an* ***easier*** *way to calculate these values?*

A: Yes, there is! We can make a "**small-signal**" approximation.

For a small-signal approximation, we simply need to calculate **two** values. First:

$$h(t)\Big|_{t=60} = h(t = 60) = 41.16 \text{ inches}$$

In other words, the average height of a human at **60 months** (i.e., 5 years) is **41.16 inches**.

Likewise, we calculate the **time derivative** of $h(t)$, and **then evaluate** the result at 60 months:

$$\begin{aligned}\left.\frac{d\,h(t)}{dt}\right|_{t=60} &= \left(2.059 \times 10^{-10} (45 - t/12)^{5.75}\right)\Big|_{t=60} \\ &= 2.059 \times 10^{-10} (45 - 60/12)^{5.75} \\ &= 0.34 \text{ inches/month}\end{aligned}$$

In other words, the average 5 year old **grows** at a rate of **0.34 inches/month**!

Now let's again consider the earlier problem.

If we know that an average 5-year old is 41.16 inches tall, and grows at a rate of 0.34 inches/month, then at 5 years **and one month** (i.e., 61 months), the little bugger will approximately be:

$$41.16 + (0.34)(1) = 41.50 \text{ inches}$$

Compare this to the exact value of 41.49 inches—a **very accurate approximation**.

We can likewise **approximate** the average height of a **59-month** old (i.e., 5 years **minus one** month):

$$41.16 + (0.34)(-1) = 40.83 \text{ inches}$$

or of a **62-month** old (i.e., 5 years **plus two** months):

$$41.16 + (0.34)(2) = 41.83 \text{ inches}$$

Note again the **accuracy** of these approximations!

For this approximation, let us define time t=60 as the **evaluation point**, or bias point T:

$$T \doteq \text{evaluation point}$$

We can then define:

$$\Delta t = t - T$$

In this example then, T = 60 months, and the values of Δt range from –2 to +2 months.

For example, t = 59 months can be expressed as $t = T + \Delta t$, where $T = 60$ months and $\Delta t = -1$ month.

We can therefore write our approximation as:

$$\boxed{h(t) \approx h(t)\Big|_{t=T} + \left.\frac{d\,h(t)}{dt}\right|_{t=T} \Delta t}$$

For the example where T=60 months we find:

$$\begin{aligned} h(t) &\approx h(t)\Big|_{t=60} + \left.\frac{d\,h(t)}{dt}\right|_{t=60} \Delta t \\ &= 41.16 + 0.34\ \Delta t \end{aligned}$$

This approximation is **not accurate**, however, if $|\Delta t|$ is **large**.

For example, we can determine from the **exact** equation that the average height of a **forty-year old** human is:

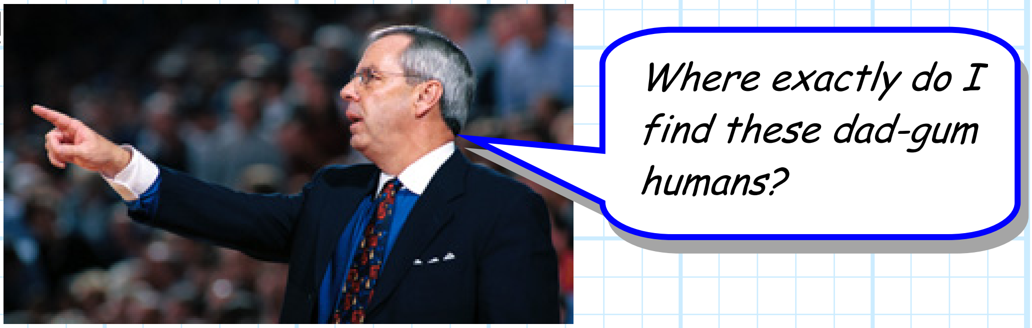
$$h(t = 480) = 65 \text{ inches}$$

or about **5 feet 5 inches**.

However, if we were to use our **approximation** to determine the average height of a 40-year old ($\Delta t = t - T = 480 - 60 = 420$), we would find:

$$h(t) \approx 41.16 + 0.34\,(420)$$
$$= 181.86 \text{ inches}$$

The approximation says that the average 40-year old human is **over 15 feet tall!**



The reason that the above approximation provides an **inaccurate** answer is because it is based on the assumption that humans grow at a rate of 0.34 inches/month.

This **is** true for 5-year olds, but **not** for 40-year olds (unless, of course, you are referring to their **waistlines**)!

We thus refer to the approximation function as a "**small-signal**" approximation, as it is valid only for times that are **slightly different** from the nominal (evaluation) time T (i.e., Δt is small).

If we wish to have an **approximate** function for the growth of humans who are near the age of forty, we would need to **construct a new approximation**:

$$h(t) \approx h(t)\big|_{t=480} + \left.\frac{d\,h(t)}{dt}\right|_{t=480} \Delta t$$

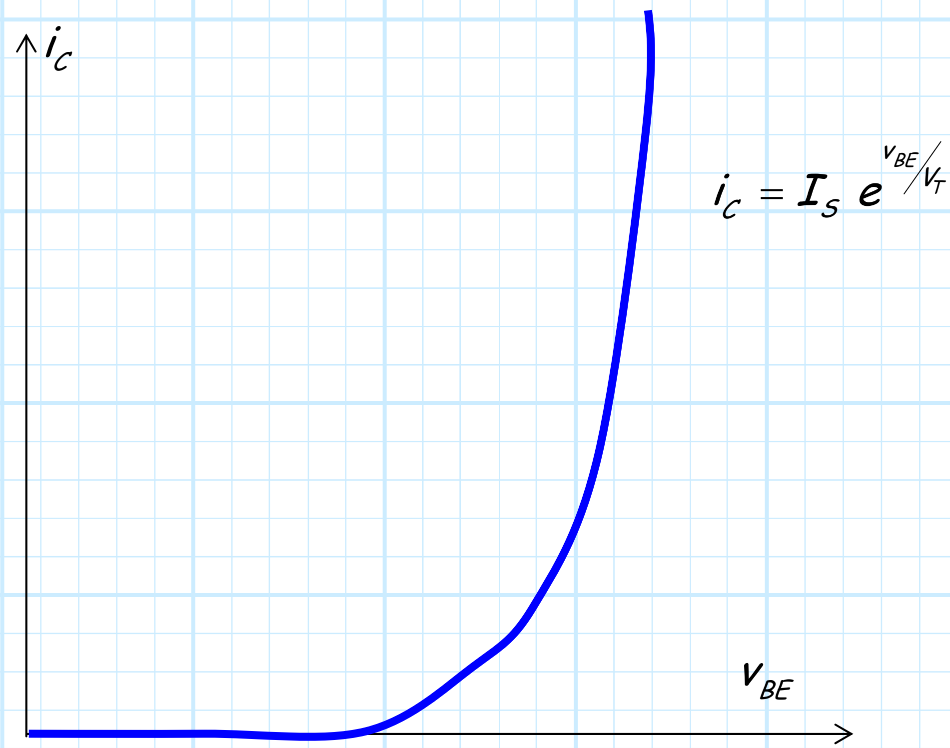
$$= 65.0 + 2.2 \times 10^{-6}\ \Delta t$$

Note that forty-year old humans have **stopped growing**!

The mathematically astute will recognize the small-signal model as a first-order **Taylor Series** approximation!

A Small-Signal Analysis of a BJT

The **collector current** i_C of a BJT is related to its **base-emitter voltage** v_{BE} as:



One messy result

Say the current and voltage have both **D.C.** (I_C, V_{BE}) and **small-signal** (i_c, v_{be}) components:

$$i_C(t) = I_C + i_c(t)$$

and

$$v_{BE}(t) = V_{BE} + v_{be}(t)$$

Therefore, the **total** collector current is:

$$i_C(t) = I_S \, e^{v_{BE}(t)/V_T}$$

$$I_C + i_c(t) = I_S \, e^{\frac{V_{BE} + v_{be}(t)}{V_T}}$$

Apply the Small-Signal Approximation

Q: *Yikes! The exponential term is very messy. Is there some way to **approximate** it?*

A: Yes! The collector current i_c is a **function** of base emitter voltage v_{BE}.

Let's perform a **small-signal analysis** to determine an **approximate** relationship between i_c and v_{BE}.

Note that the value of $v_{BE}(t) = V_{BE} + v_{be}(t)$ is **always** very close to the D.C. voltage for all time t (since $v_{be}(t)$ is very **small**).

We therefore will use this D.C. voltage as the **evaluation point** (i.e., bias point) for our small-signal analysis.

How fast it grows!

We first determine the value of the collector current i_C when the base emitter voltage v_{BE} is equal to the **DC value** V_{BE} :

$$i_C\Big|_{v_{BE}=V_{BE}} = I_s\, e^{\frac{v_{BE}}{V_T}}\Big|_{v_{BE}=V_{BE}} = I_s\, e^{\frac{V_{BE}}{V_T}} = I_C$$

Of course, the result is the **D.C. collector current** I_C .

We now determine the **change** in collector current due to a **change** in base-emitter voltage (i.e., a first **derivative**), **evaluated** at the D.C. voltage V_{BE} :

$$\begin{aligned}
\frac{d\, i_C}{d\, v_{BE}}\Bigg|_{v_{BE}=V_{BE}} &= \frac{d\left(I_S \exp\left[v_{BE}/V_T\right]\right)}{d\, v_{BE}}\Bigg|_{v_{BE}=V_{BE}} \\
&= \frac{I_S}{V_T}\, e^{v_{BE}/V_T}\Bigg|_{v_{BE}=V_{BE}} \\
&= \frac{I_S}{V_T}\, e^{V_{BE}/V_T} \qquad \left[\tfrac{A}{V}\right]
\end{aligned}$$

A simple approximation

Thus, when the base-emitter voltage is equal to the D.C. "bias" voltage V_{BE}, the collector current i_C will equal the D.C. "bias" current I_C.

Likewise, this collector current will increase (decrease) by an amount of $(I_S/V_T)\, e^{V_{BE}/V_T}$ mA for every 1mV increase (decrease) in v_{BE}.

Thus, we can easily **approximate** the collector current when the base-emitter voltage is equal to values such as:

$$v_{BE} = V_{BE} + 1\text{ mV}$$
$$v_{BE} = V_{BE} + 3\text{ mV}$$
$$v_{BE} = V_{BE} - 2\text{ mV}$$
$$v_{BE} = V_{BE} - 0.5\text{ mV}$$

Respectively, the answers are:

$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(1) \quad \text{mA}$$
$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(3) \quad \text{mA}$$
$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(-2) \quad \text{mA}$$
$$i_C = I_C + (I_S/V_T)\, e^{V_{BE}/V_T}\,(-0.5) \quad \text{mA}$$

where we have assumed that scale current I_S is expressed in mA, and thermal voltage V_T is expressed in mV.

The small signal approximation

Recall that the **small-signal voltage** $v_{be}(t)$ represents a small **change** in $v_{BE}(t)$ from its nominal (i.e., bias) voltage V_{BE}.

For example, we might find that the value of $v_{be}(t)$ at four different times t are:

$$\begin{aligned} v_{be}(t_1) &= 1 \text{ mV} \\ v_{be}(t_2) &= 3 \text{ mV} \\ v_{be}(t_3) &= -2 \text{ mV} \\ v_{be}(t_4) &= -0.5 \text{ mV} \end{aligned}$$

Thus, we can approximate the collector current using the **small-signal approximation** as:

$$i_C(t) = I_C + \left(I_S / V_T\right) e^{V_{BE}/V_T} \; v_{be}(t)$$

where of course $I_C = I_S e^{V_{BE}/V_T}$.

This is a very useful result, as we can now **explicitly** determine an expression for the **small-signal current** $i_c(t)$!

The small-signal collector current

Recall $i_C(t) = I_C + i_c(t)$, therefore:

$$i_C(t) = I_C + i_c(t) = I_C + \left(I_S / V_T\right) e^{V_{BE}/V_T} \; v_{be}(t)$$

Subtracting the D.C. current from each side, we are left with an expression for the **small-signal current** $i_c(t)$, in terms of the **small-signal voltage** $v_{be}(t)$:

$$i_c(t) = \left(I_S / V_T\right) e^{V_{BE}/V_T} \; v_{be}(t)$$

We can **simplify** this expression by noting that $I_C = I_S e^{V_{BE}/V_T}$, resulting in:

$$\begin{aligned}\left(I_S / V_T\right) e^{V_{BE}/V_T} &= \frac{I_S \, e^{V_{BE}/V_T}}{V_T} \\ &= \frac{I_C}{V_T}\end{aligned}$$

and thus:

$$i_c(t) = \frac{I_C}{V_T} \, v_{be}(t)$$

Transconductance: A small signal parameter

We define the value I_C/V_T as the **transconductance** g_m:

$$\boxed{g_m = \frac{I_C}{V_T} \qquad \left[A/V \right]}$$

and thus the **small-signal equation** simply becomes:

$$\boxed{i_c(t) = g_m \, v_{be}(t)}$$

How transistors got their name

Let's now consider for a moment the transconductance g_m.

The term is short for transfer conductance: conductance because its units are amps/volt, and transfer because it relates the **collector** current to the voltage from **base to emitter**—the collector voltage is **not relevant** (if in **active** mode)!

Note we can rewrite the small-signal equation as:

$$\frac{v_{be}(t)}{i_c(t)} = \frac{1}{g_m}$$

The value ($1/g_m$) can thus be considered as transfer resistance, the value describing a **transfer resistor**.

Transfer **Resistor**—we can shorten this term to **Transistor** (this is how these devices were named)!

Summarizing

We can **summarize** our results as:

$$I_C = I_S \, e^{V_{BE}/V_T} \qquad \text{D.C. Equation}$$

$$i_c(t) = g_m \, v_{be}(t) \qquad \text{Small-Signal Equation}$$

$$i_C(t) = I_C + g_m \, v_{be}(t) \qquad \text{Small-Signal Approximation}$$

Note that we know have **two** expressions for the **total** (D.C. plus small-signal) collector current. The **exact** expression:

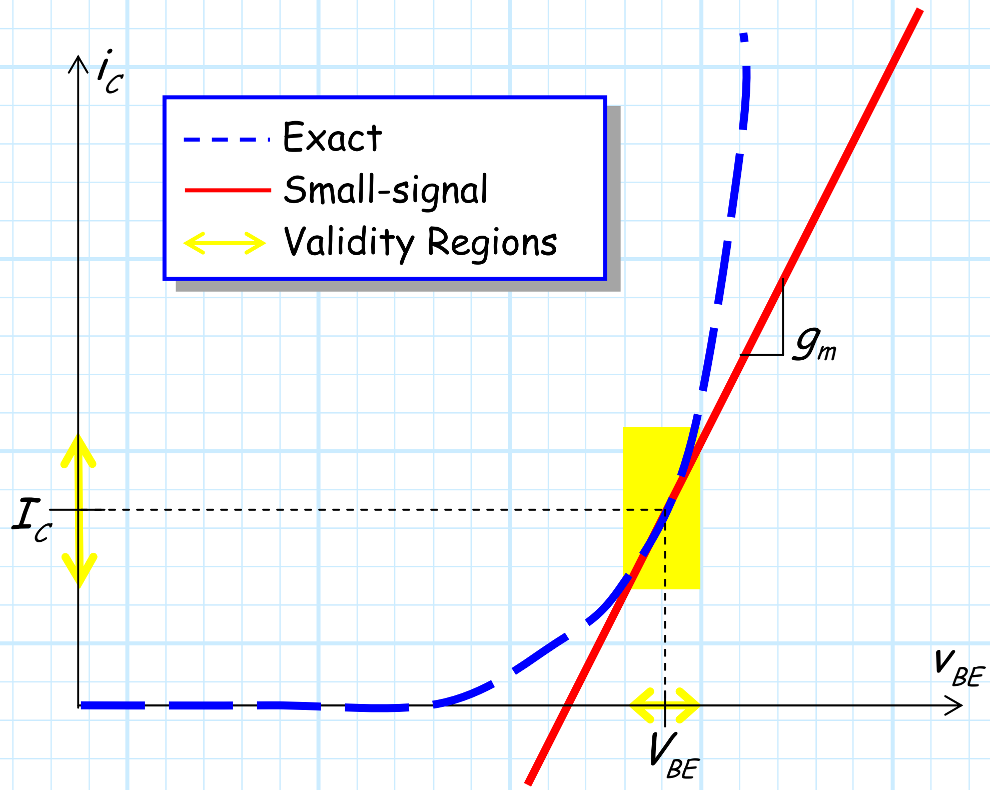
$$i_C(t) = I_S \, e^{\frac{V_{BE} + v_{be}(t)}{V_T}}$$

and the **small-signal approximation**:

$$i_C(t) = I_C + g_m \, v_{be}(t)$$

Accurate over a small region

Let's **plot** these two expressions and see how they compare:



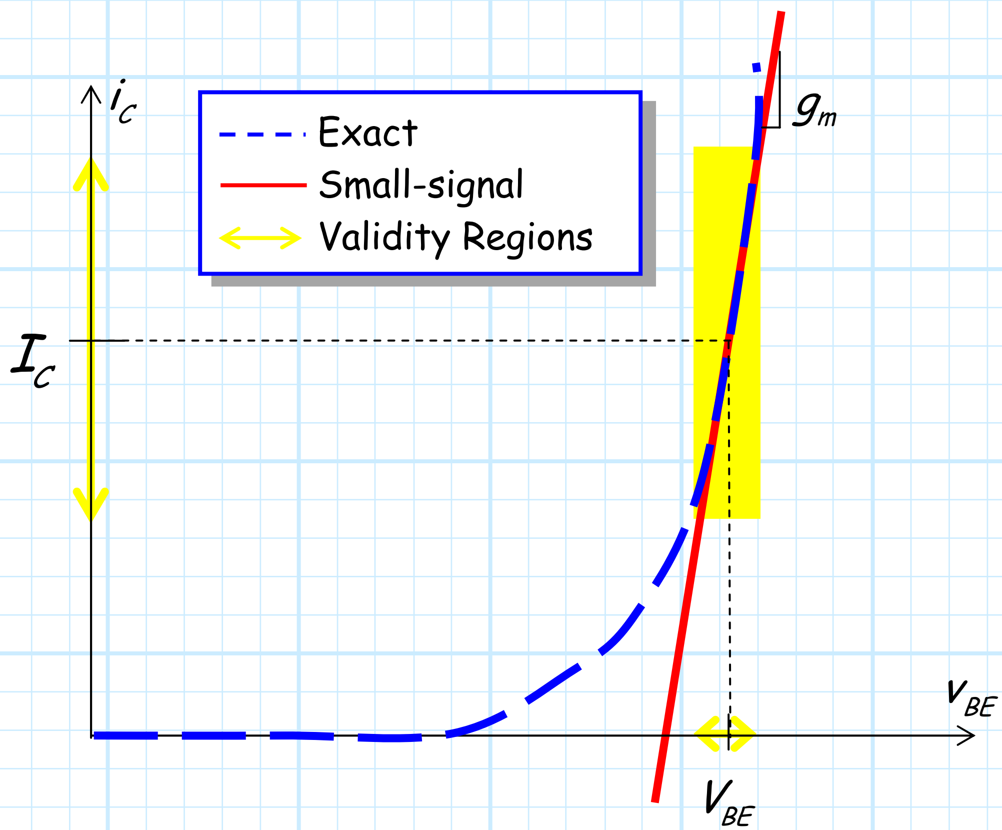
It is evident that the small-signal approximation is accurate (it provides **nearly** the exact values) **only** for values of i_C **near** the D.C bias value I_C, and **only** for values of v_C **near** the D.C bias value V_C.

The point (V_{BE}, I_C) is alternately known as the **D.C. bias point**, the **transistor operating point**, or the **Q-point**.

Change the DC bias, change the transconductance

Note if we **change the D.C. bias** of a transistor circuit, the transistor operating point will change.

The small-signal model will **likewise** change, so that it provides accurate results in the region of this new operating point:



Example: Small-Signal BJT Approximations

Say that we wish to find the collector current i_C of a BJT biased in the active mode, with $I_S = 10^{-12} A$ and a **base-emitter voltage** of:

$$v_{BE} = 0.6 + 0.001\cos\omega t \quad V$$

Q: *Easy! Since:*

$$i_C = I_S\, e^{v_{BE}/V_T}$$

we find:

$$i_C(t) = \left(I_S\, e^{0.6/V_T} \right) e^{(0.001\cos\omega t / V_T)}$$

right?

A: Although this answer is definitely **correct**, it is **not very useful** to us as engineers. Clearly, the base-emitter voltage consists of a **D.C.** bias term (0.6 V) and a **small-signal** term ($0.001\cos\omega t$).

Accordingly, we are interested in the **D.C. collector current** I_C and the **small-signal collector current** i_c. The D.C. collector current is obviously:

$$
\begin{aligned}
I_C &= I_S \, e^{0.6/V_T} \\
&= 10^{-12} e^{0.6/0.025} \\
&= 26 \text{ mA}
\end{aligned}
$$

But **how** do we determine the **small-signal** collector current $i_c(t)$ from:

$$i_c(t) = \left(I_S \, e^{0.6/V_T} \right) e^{\left(0.001 \cos \omega t / V_T\right)} \quad ???$$

The answer, of course, is to use the **small-signal approximation**.

We know that:

$$i_c(t) = g_m \, v_{be}(t)$$

where:

$$g_m = \frac{I_C}{V_T} = \frac{26mA}{25mV} = 1.06 \ \Omega^{-1}$$

Therefore, the **small-signal collector current** is approximately:

$$
\begin{aligned}
i_c(t) &= g_m \, v_{be}(t) \\
&= 1.06 \left(0.001 \cos \omega t\right) \\
&= 1.06 \cos \omega t \qquad \text{mA}
\end{aligned}
$$

and therefore the **total** collector current is:

Q: *Say the D.C. bias voltage* ***increases*** *from* $V_{BE} = 0.6\ \text{V}$ *to* $V_{BE} = 0.7\ \text{V}$. *What happens to the BJT collector current?*

A: The **D.C. bias** current becomes:

$$I_C = I_S\, e^{0.7/V_T} = 10^{-12} e^{0.7/0.025} = 1446\ \text{mA} \qquad !!!$$

since the **transconductance** is now:

$$g_m = \frac{I_C}{V_T} = \frac{1446 mA}{25 mV} = 57.84\ \Omega^{-1}$$

the **small-signal collector current** is:

$$\begin{aligned} i_c(t) &= g_m\, v_{be}(t) \\ &= 57.84\left(0.001 \cos \omega t\right) \\ &= 57.8 \cos \omega t \qquad \text{mA} \end{aligned}$$

Quite an **increase**!

Changing the transistor operating point (i.e., the DC bias point) will typically make a **big** difference in the small-signal result!

BJT Small-Signal Parameters

We know that the following **small-signal** relationships are true for **BJTs**:

$$i_c = \beta\, i_b \qquad\qquad i_c = g_m v_{be}$$

Q: *What **other** relationship can be derived from these two??*

A: Well, **one** obvious relationship is determined by equating the two equations above:

$$i_c = \beta\, i_b = g_m v_{be} \qquad\qquad \therefore\ v_{be} = \left(\frac{\beta}{g_m}\right) i_b$$

We can thus define the small-signal parameter r_π as:

$$\boxed{\frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{V_T}{I_B} \doteq r_\pi}$$

Small-signal base resistance

Therefore, we can write the **new** BJT small-signal equation:

$$\boxed{v_{be} = r_\pi \, i_b}$$

The value r_π is commonly thought of as the small-signal **base resistance.**

We can likewise define a small-signal **emitter resistance**:

$$r_e \doteq \frac{v_{be}}{i_e}$$

We begin with the small-signal equation $i_c = \alpha \, i_e$. Combining this with $i_c = g_m v_{be}$, we find:

$$i_c = \alpha \, i_e = g_m v_{be} \qquad \therefore \quad v_{be} = \left(\frac{\alpha}{g_m} \right) i_e$$

Small-signal emitter resistance

We can thus **define** the small-signal parameter r_e as:

$$\boxed{\frac{\alpha}{g_m} = \frac{\alpha V_T}{I_C} = \frac{V_T}{I_E} \doteq r_e}$$

Therefore, we can write **another new** BJT small-signal equation:

$$\boxed{v_{be} = r_e \, i_e}$$

Note that **in addition** to β, we now have **three** fundamental BJT small-signal parameters:

$$\boxed{g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E}}$$

These results are not independent!

Since $I_C = \beta\, I_B$ ($I_C = \alpha\, I_E$), we find that these small signal values are **not** independent.

If we know **two** of the four values β, g_m, r_π, r_e, we can determine **all four**!

$$g_m = \frac{\alpha}{r_e} = \frac{\beta}{r_\pi} = \frac{r_\pi - r_e}{r_\pi\, r_e}$$

$$r_\pi = \frac{\beta}{g_m} = \left(\beta + 1\right) r_e = \frac{r_e}{1 - g_m\, r_e}$$

$$r_e = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta + 1} = \frac{r_\pi}{1 + g_m\, r_\pi}$$

Make sure you can derive these!

The results on the previous page are easily determined from the equations:

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E}$$

$$I_E = I_C + I_B$$

$$I_C = \beta I_B$$

$$I_C = \alpha I_E$$

Make sure **you** can derive them!

The Small-Signal Equation Matrix

We can **summarize** our small-signal equations with the small-signal equation matrix. Note this matrix relates the **small-signal** BJT parameters v_{be}, i_b, i_c, and i_e.

Column Parameters

Row Parameters

	v_{be}	i_b	i_c	i_e
v_{be}	1	$r_\pi = \beta / g_m$	$1/g_m$	$r_e = \alpha / g_m$
i_b	$\dfrac{1}{r_\pi} = \dfrac{g_m}{\beta}$	1	$1/\beta$	$1/(\beta+1)$
i_c	g_m	β	1	$\alpha = \dfrac{\beta}{\beta+1}$
i_e	$\dfrac{1}{r_e} = \dfrac{g_m}{\alpha}$	$\beta+1$	$1/\alpha = \dfrac{\beta+1}{\beta}$	1

Here's how you use this

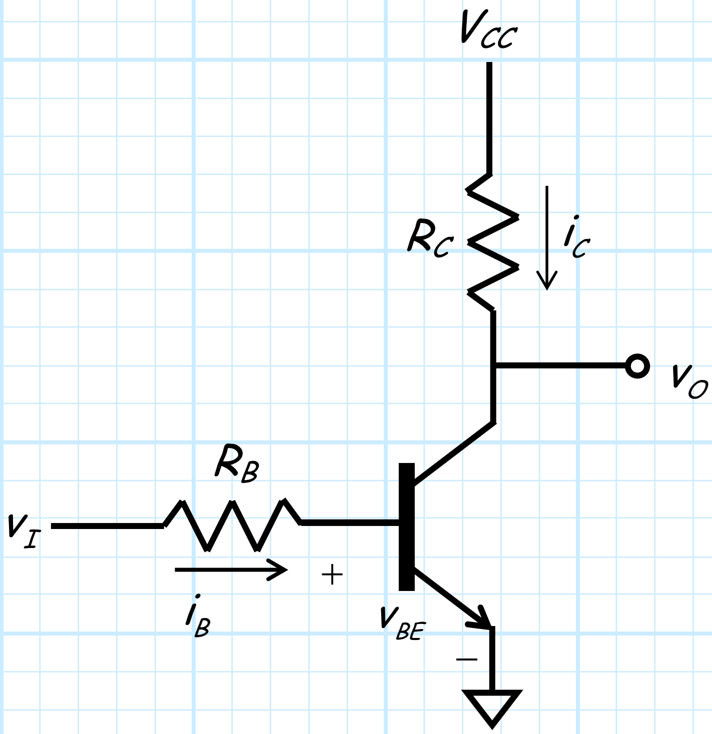
To use this matrix, note that the **row parameter** is equal to the product of the **column parameter** and the **matrix element**. For example:

$$i_b = \frac{1}{r_\pi} v_{be}$$

	v_{be}	i_b	i_c	i_e
v_{be}	1	$r_\pi = \beta / g_m$	$1 / g_m$	$r_e = \alpha / g_m$
i_b	$\frac{1}{r_\pi} = \frac{g_m}{\beta}$	1	$1 / \beta$	$1 / (\beta + 1)$
i_c	g_m	β	1	$\alpha = \frac{\beta}{\beta + 1}$
i_e	$\frac{1}{r_e} = \frac{g_m}{\alpha}$	$\beta + 1$	$1 / \alpha = \frac{\beta + 1}{\beta}$	1

Example: Calculating the Small-Signal Gain

For this circuit, we have **now** determined (**if** BJT is in active mode), the **small-signal equations** are:



1) $v_i = i_b R_B + v_{be}$

2) $i_c = \beta i_b$

3) $v_o = -R_c i_c$

4) $i_c \cong g_m v_{be}$

Q: *So, can we now determine the **small-signal** open-circuit voltage **gain** of this amplifier? I.E.:*

$$A_{vo} = \frac{v_o(t)}{v_i(t)}$$

A: Look at the **four** small-signal equations—there are **four** unknowns (i.e., i_b, v_{be}, i_c, v_o)!

Combining equations 2) and 4), we get:

$$v_{be} = \frac{\beta}{g_m} i_b = r_\pi i_b$$

Inserting this result in equation 1), we find:

$$v_i = \left(R_B + r_\pi\right) i_b$$

Therefore:

$$i_b = \frac{v_i}{R_B + r_\pi}$$

and since $i_c = \beta\, i_b$:

$$i_c = \frac{\beta}{R_B + r_\pi} v_i$$

which we insert into equation 3):

$$v_o = -i_c R_C = \frac{-\beta R_C}{R_B + r_\pi} v_i$$

Therefore, the **small-signal gain** of **this** amplifier is:

$$A_{vo} = \frac{v_o(t)}{v_i(t)} = \frac{-\beta R_C}{R_B + r_\pi}$$

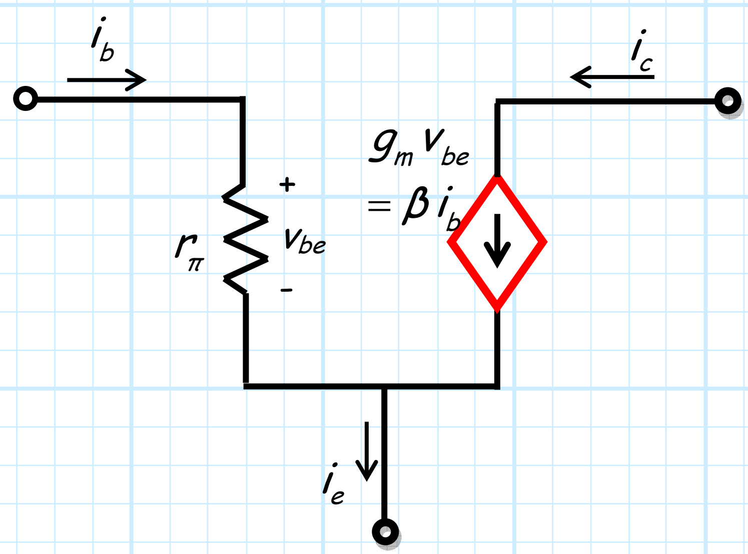
Note this is the small signal gain of **this** amplifier—and this amplifier **only**!

The Hybrid-Π and T Models

Consider again the small-small signal equations for an npn BJT biased in the active mode:

$$i_b = \frac{v_{be}}{r_\pi} \qquad i_c = g_m v_{be} = \beta i_b \qquad i_e = i_b + i_c \quad (KCL)$$

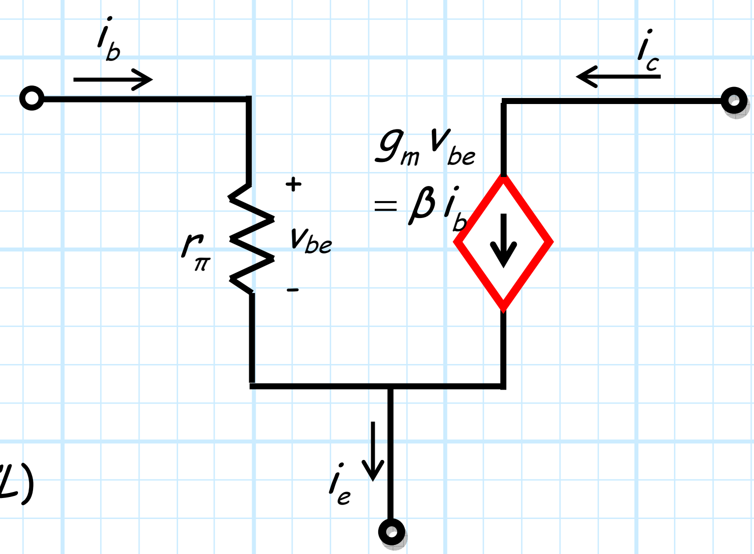
Now, analyze **this** circuit:



Do these equations look familiar?

From **Ohm's Law**:

$$i_b = \frac{v_{be}}{r_\pi}$$



From **KCL**:

$$i_c = g_m v_{be} = \beta i_b$$

And **also** from **KCL**:

$$i_e = i_b + i_c \qquad (KCL)$$

Q: *Hey! Aren't these the* ***same*** *three equations as the npn BJT* ***small-signal*** *equations?*

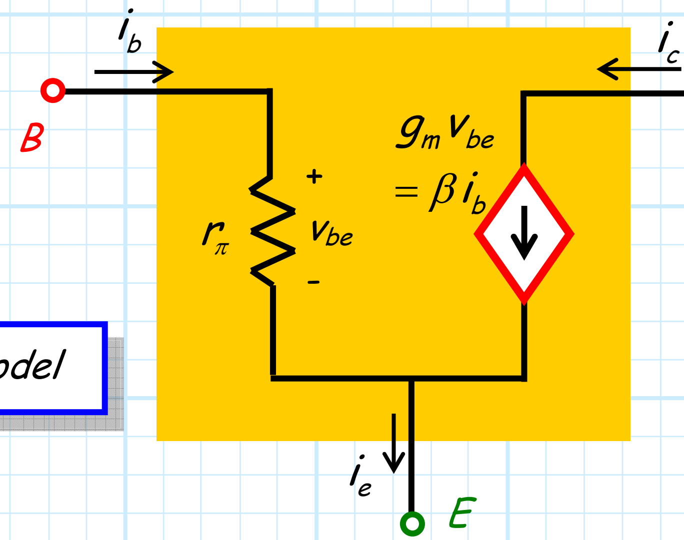
A: They **are** indeed!

With respect to the small-signal currents and voltages in a circuit (but **only** small-signal voltages and currents) , an *npn* BJT in active mode might as well be **this** circuit.

Two equivalent circuits

Thus, this circuit can be used as an **equivalent circuit** for BJT small-signal analysis (but **only** for small signal analysis!).

This equivalent circuit is called the Hybrid-Π model for a BJT biased in the **active** mode:

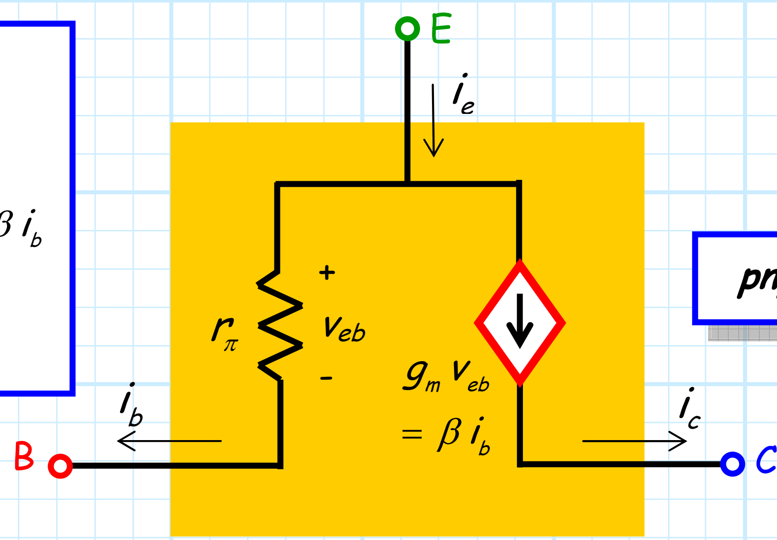


***npn** Hybrid-Π Model*

$$i_b = {v_{be}}/{r_\pi}$$

$$i_c = g_m v_{be} = \beta i_b$$

$$i_e = i_b + i_c$$



***pnp** Hybrid-Π Model*

$$i_b = {v_{eb}}/{r_\pi}$$

$$i_c = g_m v_{eb} = \beta i_b$$

$$i_e = i_b + i_c$$

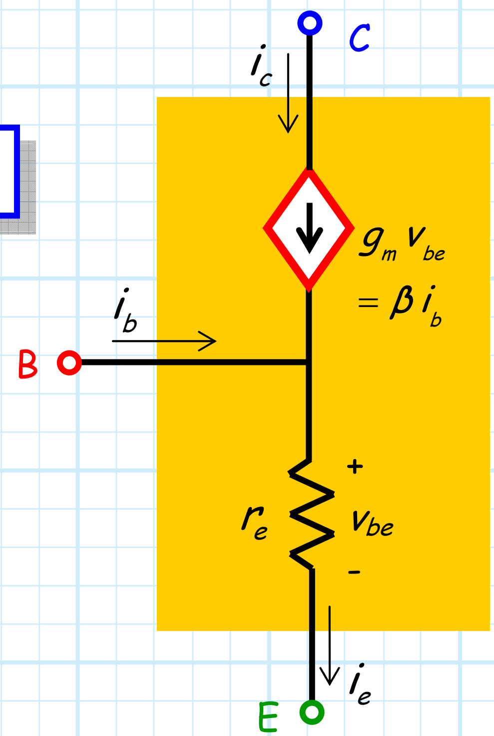
An alternative equivalent circuit

Note however, that we can **alternatively** express the small-signal circuit equations as:

$$i_b = i_e - i_c \qquad i_c = g_m v_{be} = \beta\, i_b \qquad i_e = \frac{v_{be}}{r_e}$$

These equations likewise describes the **T-Model**—an **alternative** but **equivalent** model to the Hybrid-Π.

***npn* T-Model**

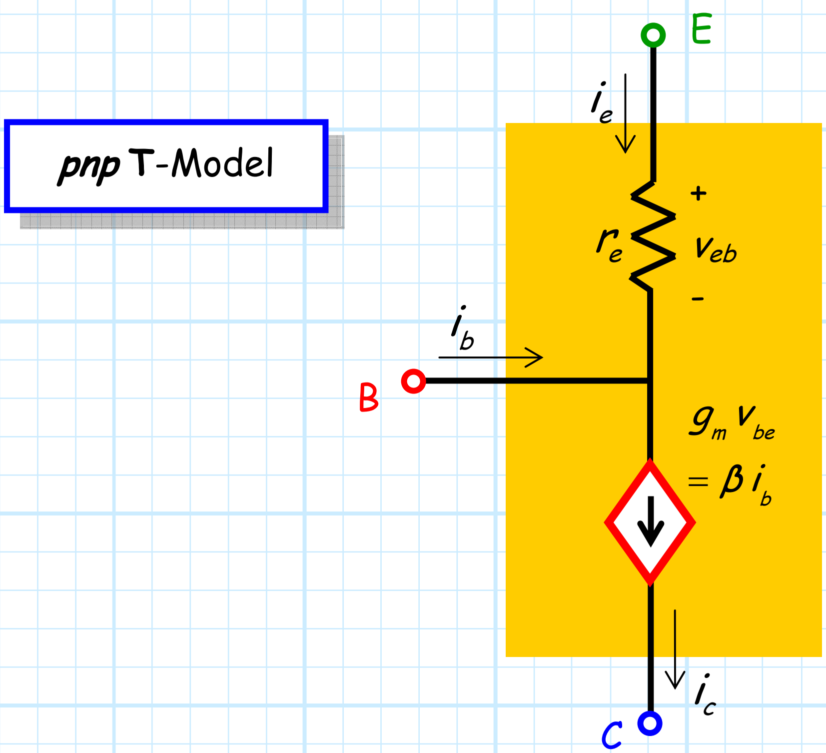


$$i_b = i_e - i_c$$

$$i_c = g_m v_{be} = \beta\, i_b$$

$$i_e = \frac{v_{be}}{r_e}$$

I just couldn't fit the *pnp* T-model on the previous page



$$i_b = i_e - i_c$$

$$i_c = g_m v_{eb} = \beta i_b$$

$$i_e = \frac{v_{eb}}{r_e}$$

So many choices; which should I use?

The Hybrid-Π and the **T** circuit models are equivalent—they **both** will result in the **same** correct answer!

Therefore, you do **not** need to worry about which one to use for a particular small-signal circuit analysis, **either one** will work.

However, you will find that a particular analysis is **easier** with one model or the other; a result that is dependent **completely** on the type of amplifier being analyzed.

For time being, use the **Hybrid-Π model**; later on, we will discuss the types of amplifiers where the **T**-model is simplest to use.

Small-Signal Output Resistance

Recall that due to the **Early effect**, the collector current i_C is **slightly** dependent on v_{CE}:

$$i_C = \beta\, i_B \left(1 + \frac{v_{CE}}{V_A}\right)$$

where we recall that V_A is a BJT device parameter, called the **Early Voltage**.

Q: *How does this affect the **small-signal response** of the BJT?*

A: Well, if $i_C(t) = I_C + i_c(t)$ and $v_{CE}(t) = V_{CE} + v_{ce}(t)$, then with the small-signal approximation:

$$\begin{aligned} I_C + i_c &= \beta\, i_B \left(1 + \frac{v_{CE}}{V_A}\right)\Bigg|_{v_{CE}=V_{CE}} + \left(\frac{\partial i_C}{\partial v_{CE}}\Bigg|_{v_{CE}=V_{CE}}\right) v_{ce} \\ &= \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right) + \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right)\left(\frac{1}{V_A}\right) v_{ce} \end{aligned}$$

Small-signal base resistance

Equating the **DC components**:

$$I_C = \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right)$$

And equating the **small-signal components**:

$$i_c = \beta\, I_B \left(1 + \frac{V_{CE}}{V_A}\right)\left(\frac{1}{V_A}\right) v_{ce}$$

Note that by inserting the DC result, this expression can be simplified to:

$$i_c = I_C \left(\frac{1}{V_A}\right) v_{ce} = \left(\frac{I_C}{V_A}\right) v_{ce}$$

Therefore, another **small-signal** equation is found, one that expresses the small-signal response of the **Early effect**:

$$\boxed{i_c = \left(\frac{I_C}{V_A}\right) v_{ce}}$$

Small-signal base resistance

Recall that we defined (in EECS 312) the BJT **output resistance** r_o:

$$\boxed{\frac{I_C}{V_A} \doteq \frac{1}{r_o}}$$

Be careful! Although the Early Voltage V_A **is** a **device parameter**, the **output resistance** r_o—since it depends on DC collector current I_C—is **not a device parameter!**

Therefore, the **small-signal collector current** resulting **from the Early effect** can likewise be expressed as:

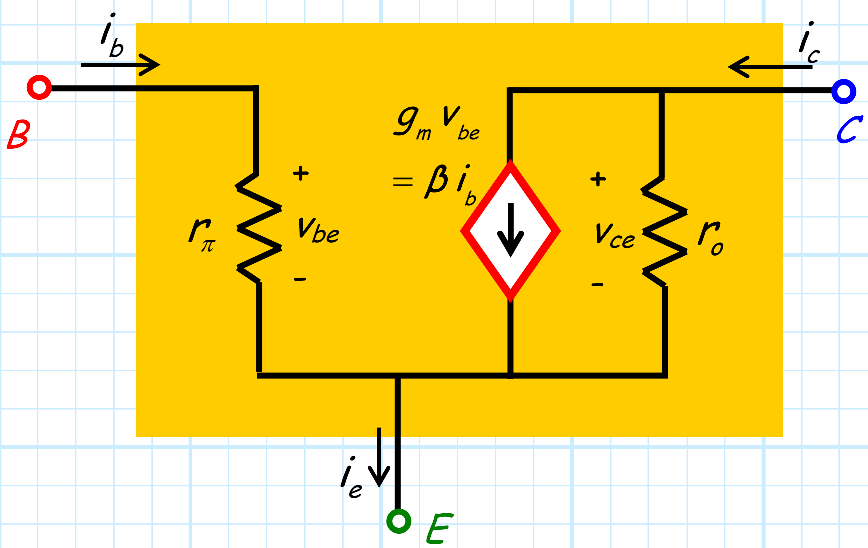
$$i_c = \frac{v_{ce}}{r_o}$$

Small-signal base resistance

Combining this result with an earlier result (i.e., $i_c = g_m v_{be}$), we find that the **total** small-signal collector current is:

$$\boxed{i_c = g_m v_{be} + \frac{v_{ce}}{r_o} = \beta\, i_b + \frac{v_{ce}}{r_o}}$$

We can account for this effect in our small-signal **circuit models**. For example, the Hybrid-Π becomes:



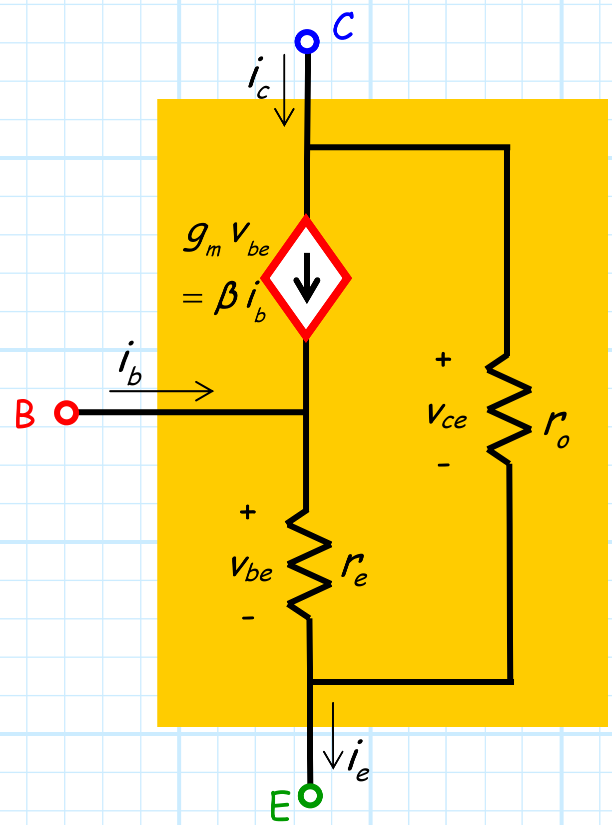
$$i_b = v_{be} / r_\pi$$

$$i_c = g_m v_{be} + \frac{v_{be}}{r_o}$$

$$i_e = i_b + i_c$$

Small-signal base resistance

And for the **T**-model:



Often, r_o is so **large** that it can be ignored (**caution:** ignoring the output resistance means approximating it as an **open** circuit, i.e., $r_o = \infty$).

BJT Small-Signal Analysis Steps

Complete **each** of these steps if you choose to correctly complete a BJT Amplifier **small-signal** analysis.

Step 1: Complete a **D.C. Analysis**

Turn **off** all **small-signal** sources, and then complete a circuit analysis with the remaining **D.C. sources** only.

* Complete this DC analysis exactly, precisely, the same way you performed the DC analysis in section 5.4.

That is, you assume (the active mode), enforce, analyze, and **check (do not forget to check!)**.

* Note that you enforce and check exactly, precisely the same the same equalities and inequalities as discussed in section 5.4 (e.g., $V_{BE} = 0.7$ V , $V_{CB} > 0$).

You must remember this

* Remember, if we "turn off" a **voltage** source (e.g.,$v_i(t)=0$), it becomes a **short** circuit.

* However, if we "turn off" a **current** source (e.g., $i_i(t)=0$), it becomes an **open** circuit!

* Small-signal amplifiers frequently employ Capacitors of Unusual Sizes (**COUS**), we'll discuss why later.

Remember, the impedance of a capacitor at **DC** is infinity—a DC **open** circuit.

The goals of DC analysis—and don't forget to CHECK

The **goal** of this DC analysis is to determine:

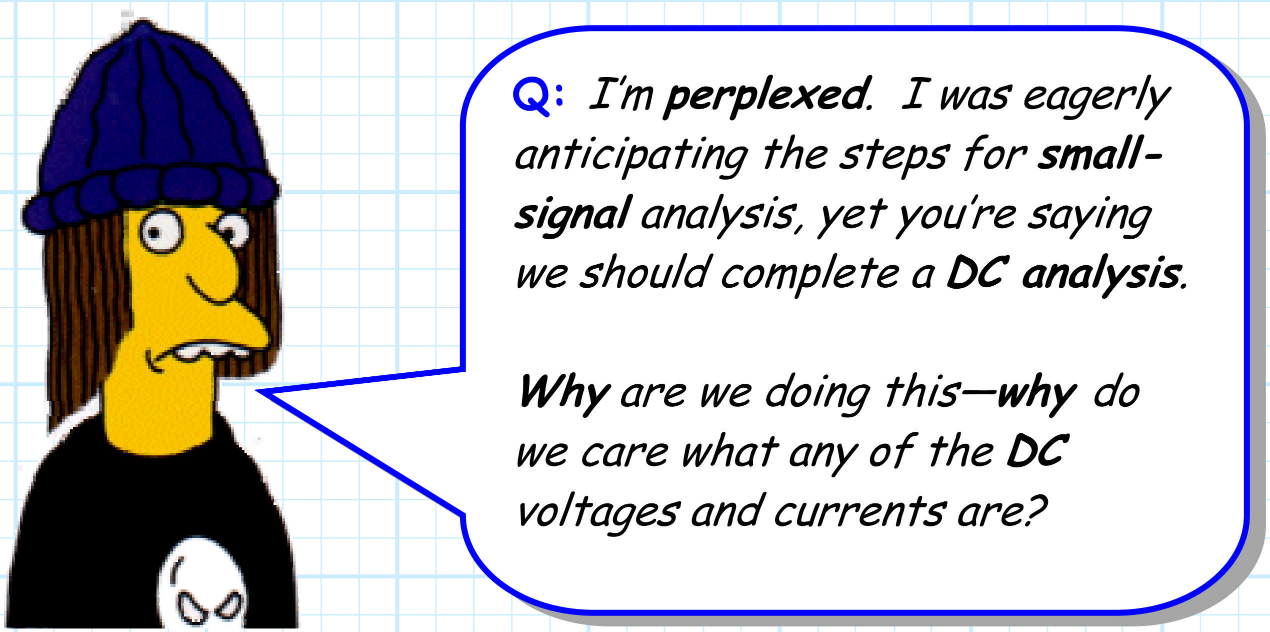
1) **One** of the DC BJT currents (I_B, I_C, I_E) for **each** BJT.

2) Either the voltage V_{CB} or V_{CE} for **each** BJT.

You do not **necessarily** need to determine any other DC currents or voltages within the amplifier circuit!

Once you have found these values, you can **CHECK** your active assumption, and then move on to **step 2**.

The DC bias terms are required to determine our small-signal parameters



A: Remember, all of the **small-signal** BJT parameters (e.g., g_m, r_π, r_e, r_o) are dependent on **D.C.** values (e.g., I_C, I_B, I_E).

In other words, we must **first** determine the operating (i.e., **bias**) point of the transistor in order to determine its **small-signal** performance!

Now for step 2

Step 2: Calculate the **small-signal circuit parameters** for each BJT.

Recall that we now understand **4** small-signal parameters:

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{V_T}{I_B} \qquad r_e = \frac{V_T}{I_E} \qquad r_o = \frac{V_A}{I_C}$$

Q: *Yikes! Do we need to calculate* ***all*** *four?*

A: Typically no. You need to calculate **only** the small signal parameters required by the small-signal circuit **model** that you plan to implement.

For example, if you plan to:

a) use the **Hybrid-Π** model, you must determine g_m and r_π.

b) use the **T-model**, you must determine g_m and r_e.

c) account for the **Early effect** (in either model) you must determine r_o.

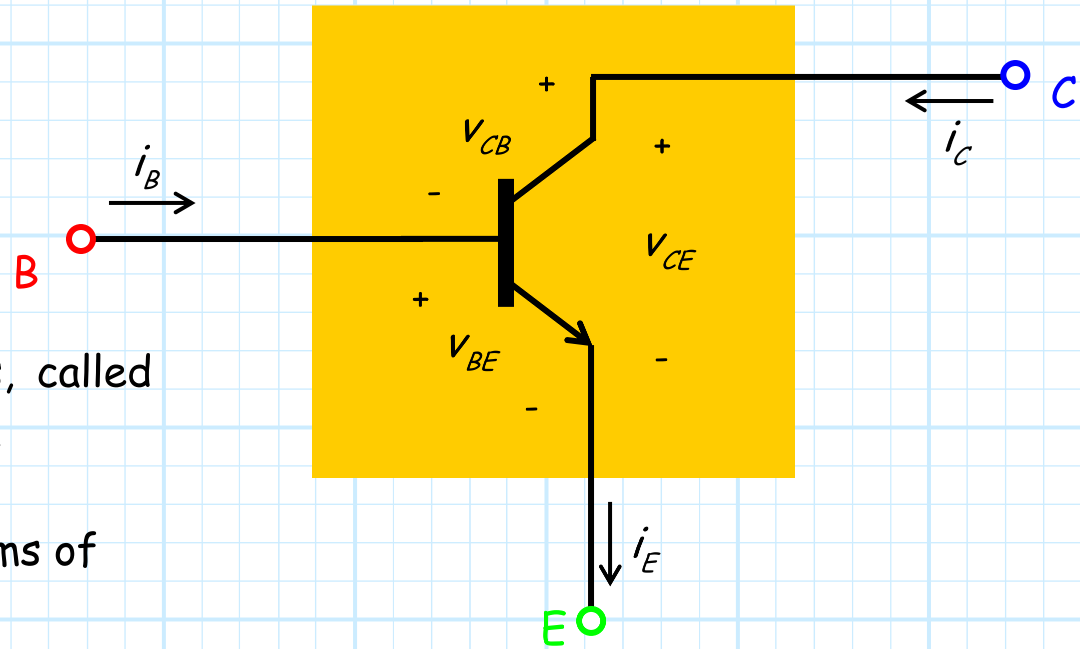
The four "Pees"

Step 3: Carefully replace all BJTs with their **small-signal circuit model**.

This step often gives students **fits**!

However, it is actually a **very simple** and straight-forward step.

It does require four important things from the student—**patience**, **precision**, **persistence** and **professionalism**!



First, note that a **BJT** is:

A device with **three** terminals, called the base, collector, and emitter.

Its behavior is described in terms of currents i_B, i_C, i_E and voltages v_{BE}, v_{CB}, v_{CE}.

They're both so different—not!

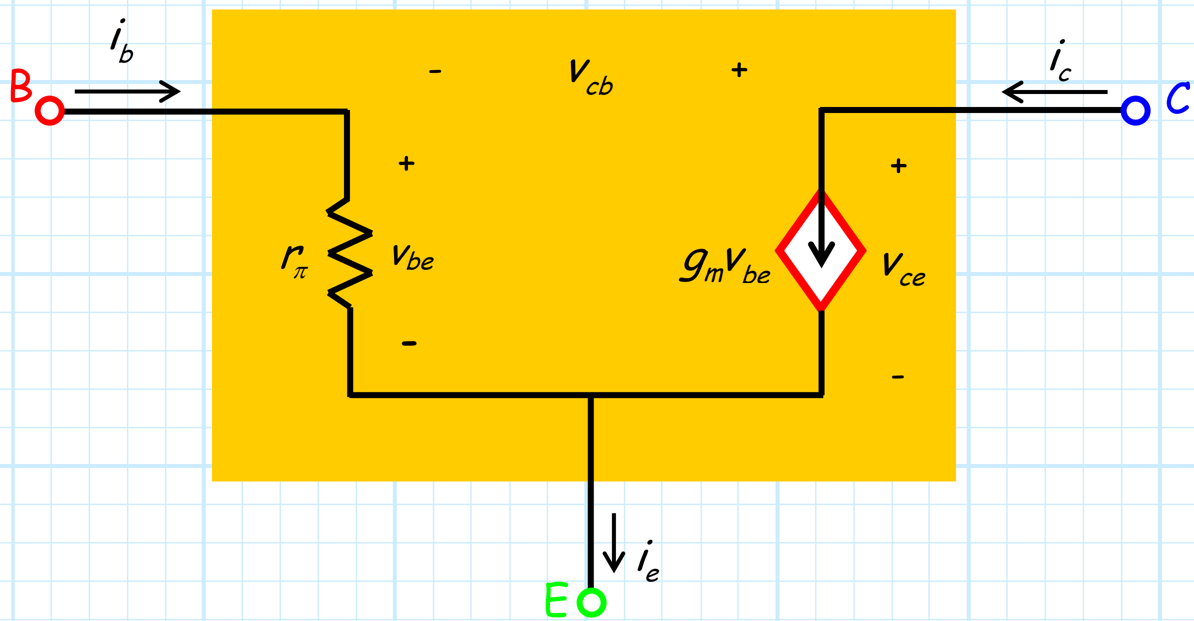
Now, **contrast** the BJT with its small-signal circuit model.

A BJT small-signal circuit model is:

A device with **three** terminals, called the base, collector, and emitter.

Its behavior is described in terms of currents i_b, i_c, i_e and voltages v_{be}, v_{cb}, v_{ce}.

Exactly the **same**—what a coincidence!



Am I making this clear?

Therefore, replacing a BJT with its small-signal circuit model is very simple—you simply change the stuff **within** the orange box!

Note the parts of the circuit **external** to the orange box do not change! In other words:

1) **every** device attached to the BJT **base** is attached in **precisely** the same way to the base terminal of the **circuit model**.

2) **every** device attached to the BJT **collector** is attached in **precisely** the same way to the collector terminal of the **circuit mode**

3) **every** device attached to the BJT **emitter** is attached in **precisely** the same way to the emitter terminal of the **circuit model**.

4) **every** external voltage or current (e.g., v_i, v_o, i_R) is defined in **precisely** the same way both before and after the BJT is replaced with its circuit model is (e.g., if the output voltage is the collector voltage in the BJT circuit, then the output voltage is **still** the collector voltage in the small-signal circuit!).

It's just like working in the lab

You can think of replacing a BJT with its small-signal circuit model as a **laboratory** operation:

1) Disconnect the red wire (base) of the BJT from the circuit and then "solder" the red wire (base) of the circuit model to the same point in the circuit.

2) Disconnect the blue wire (collector) of the BJT from the circuit and then "solder" the blue wire (collector) of the circuit model to the same point in the circuit.

3) Disconnect the green wire (emitter) of the BJT from the circuit and then "solder" the green wire (emitter) of the circuit model to the same point in the circuit.

This is superposition—turn off the DC sources!

Step 4: Set all **D.C. sources** to zero.

Remember:

A zero-**voltage** DC source is a **short circuit**.

A zero-**current** DC source is an **open circuit**.

The schematic in now in front of you is called the **small-signal circuit**. Note that it is **missing** two things—**DC sources** and bipolar junction **transistors**!

* Note that steps three and four are **reversible**.

You could turn off the DC sources **first**, and then replace all BJTs with their small-signal models—the resulting small-signal circuit will be the **same**!

* You will find that the small-signal circuit schematic can often be greatly **simplified**.

Many things will be connected to ground!

Once the DC voltage sources are turned **off**, you will find that the terminals of many devices are **connected to ground**.

* Remember, all terminals connected to ground are **also** connected to each other!

For **example**, if the emitter terminal is connected to ground, and one terminal of a resistor is connected to ground, then that resistor terminal is connected to the emitter!

* As a result, you often find that resistors in different parts of the circuit are actually connected in **parallel**, and thus can be **combined** to simplify the circuit schematic!

* Finally, note that the AC impedance of a **COUS** (i.e., $|Z_c| = 1/\omega C$) is small for all but the lowest frequencies ω.

If this impedance is smaller than the other circuit elements (e.g., < 10Ω), we can view the impedance as **approximately zero**, and thus replace the **large** capacitor with a (AC) **short**!

Organize and simplify or perish!

Organizing and **simplifying** the small-signal circuit will pay **big** rewards in the next step, when we **analyze** the small-signal circuit.

However, correctly organizing and simplifying the small-signal circuit requires **patience**, **precision**, **persistence** and **professionalism**.

Students frequently run into problems when they try to accomplish **all** the goals (i.e., replace the BJT with its small-signal model, turn off DC sources, simplify, organize) in **one** big step!

*Steps 3 and 4 are **not** rocket science!*

*Failure to correctly determine the simplified small-signal circuit is **almost always** attributable to an engineer's patience, precision and/or persistence (or, more specifically, the lack of same).*

This is a EECS 211 problem, and *only* a 211 problem

Step 5: Analyze **small-signal circuit**.

We now can **analyze** the small-signal **circuit** to find all small-signal **voltages** and **currents**.

* For small-signal **amplifiers**, we typically attempt to find the small-signal output voltage v_o in terms of the small-signal input voltage v_i.

From this result, we can find the **voltage gain** of the amplifier.

* Note that this analysis requires **only** the knowledge you acquired in **EECS 211**!

The small-signal circuit will consist **entirely** of resistors and (small-signal) voltage/current sources.

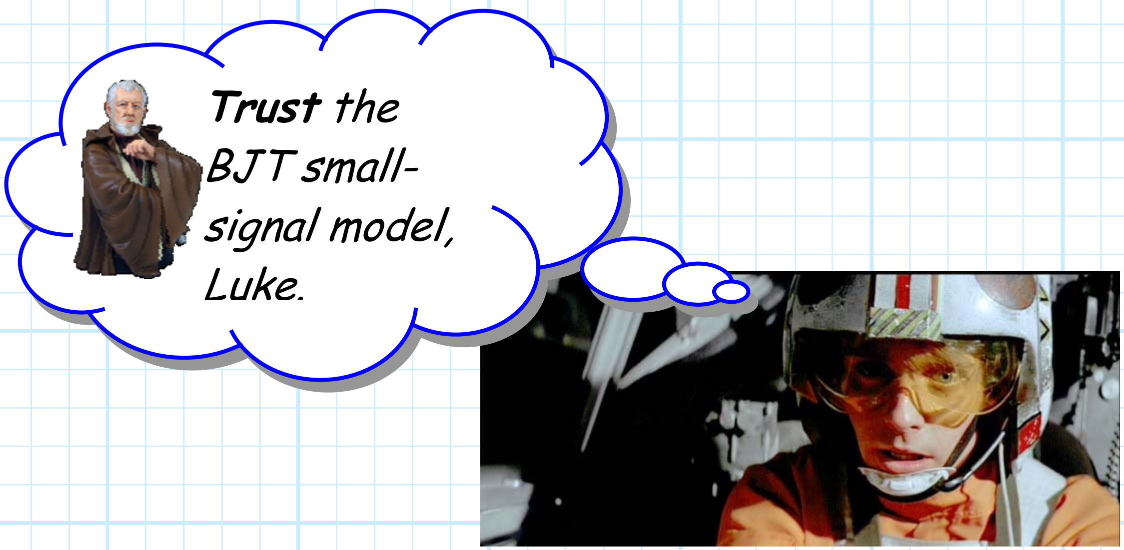
These are **precisely** the same resistors and sources that you learned about in EECS 211. You analyze them in **precisely** the same way.

Trust me, this works!

* Do **not** attempt to insert any BJT knowledge into your small-signal circuit analysis—there are **no** BJTs in a small-signal circuit!!!!!

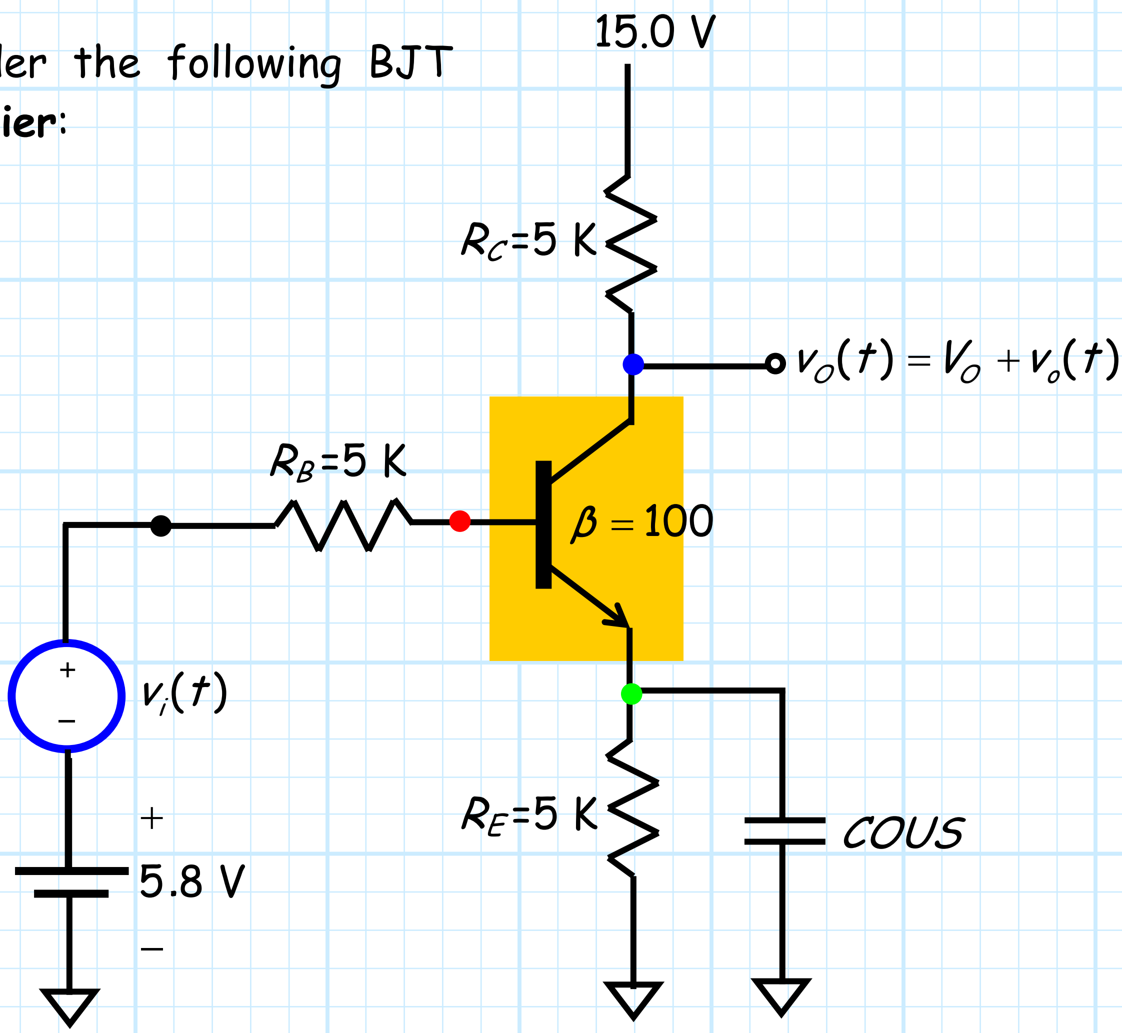
* Remember, the BJT circuit model contains **all** of our BJT small-signal knowledge, we **do** not—indeed **must** not—add any more information to the analysis.

You must **trust** completely the BJT small-circuit model. It **will** give you the correct answer!



Example: A Small-Signal Analysis of a BJT Amplifier

Consider the following BJT **amplifier**:



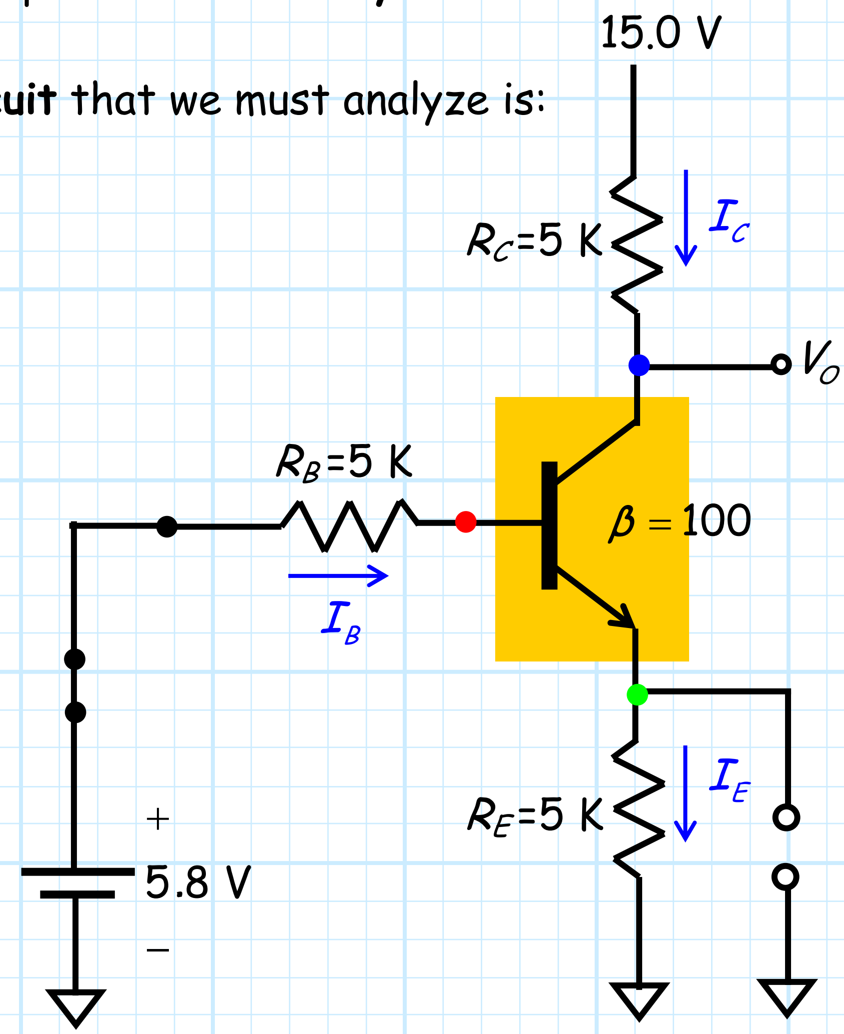
Let's determine its **small-signal, open-circuit voltage gain**:

$$A_{vo} = \frac{v_o(t)}{v_i(t)}$$

To do this, we must follow each of our **five** small-signal analysis **steps**!

Step 1: Complete a **D.C. Analysis**

The **DC circuit** that we must analyze is:



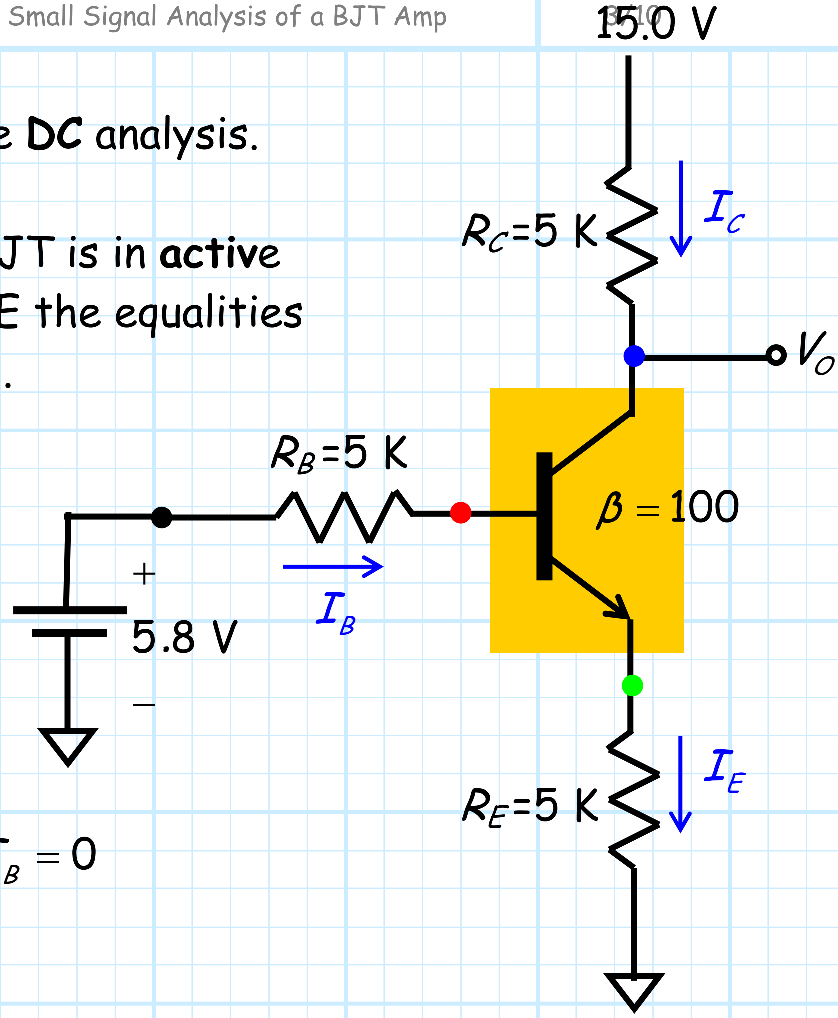
Note what we have done to the original circuit:

1) We turned **off** the **small-signal** voltage source ($v_i(t) = 0$), thus replacing it with a **short** circuit.

2) We replaced the **capacitor** with an **open** circuit—its DC impedance.

Now we proceed with the **DC** analysis.

We ASSUME that the BJT is in **active** mode, and thus ENFORCE the equalities $V_{BE} = 0.7$ V and $I_C = \beta I_B$.



We now begin to ANALYZE the circuit by writing the Base-Emitter Leg KVL:

$$5.8 - 5I_B - 0.7 - 5(\beta + 1)I_B = 0$$

Therefore:

$$I_B = \frac{5.1}{5 + 5(101)} = 0.01 \text{ mA}$$

and thus:

$$I_C = \beta I_B = 1.0 \text{ mA}$$

$$I_E = I_B + I_C = 1.01 \text{ mA}$$

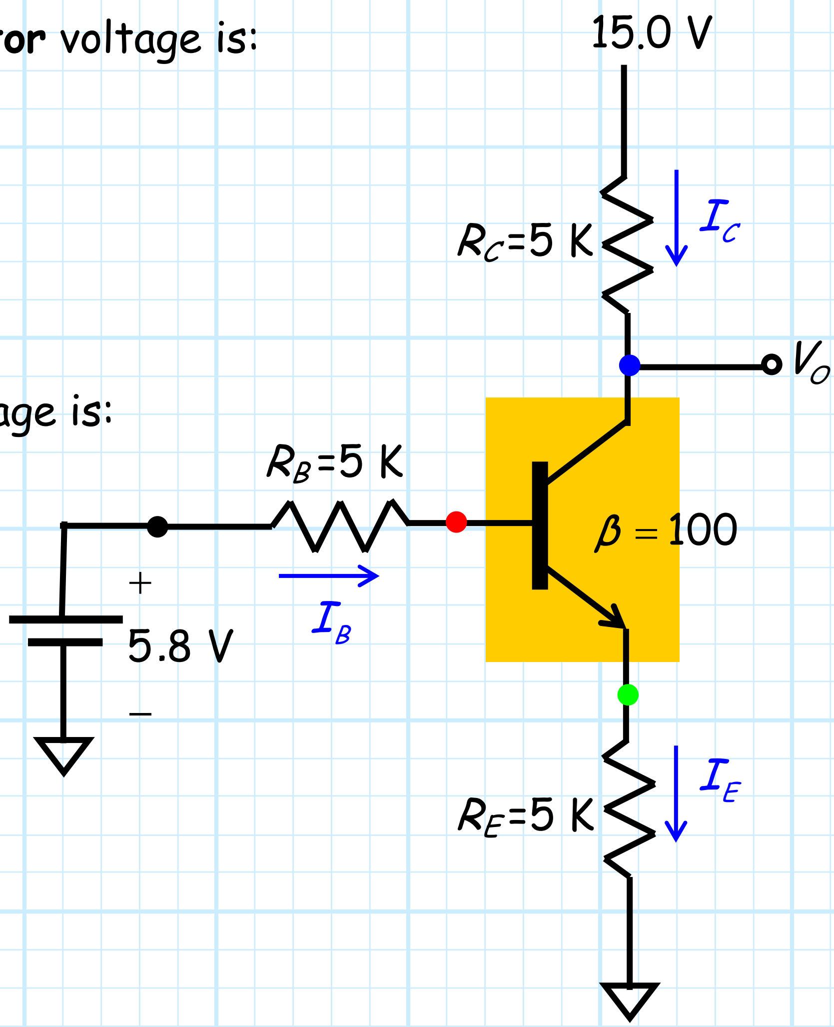
Q: *Since we know the DC bias currents, we have* ***all*** *the information we need to determine the* ***small-signal parameters****.*

Why don't we proceed ***directly*** *to step 2?*

A: Because we still need to CHECK our assumption! To do this, we must determine either V_{CE} or V_{CB}.

Note that the **Collector** voltage is:

$$\begin{aligned} V_C &= 15 - I_C R_C \\ &= 15 - (1.0)5 \\ &= 10.0 \text{ V} \end{aligned}$$



And the **Emitter** voltage is:

$$\begin{aligned} V_E &= I_E R_E \\ &= (1.01)5 \\ &= 5.05 \text{ V} \end{aligned}$$

Therefore, V_{CE} is:

$$\begin{aligned} V_{CE} &= V_C - V_E \\ &= 10.0 - 5.05 \\ &= 4.95 \text{ V} \end{aligned}$$

We now can complete our CHECK:

$$I_C = 1.0 \text{ mA} > 0$$

$$V_{CE} = 4.95 \text{ V} > 0.7$$

Time to move on to **step 2!**

Step 2: Calculate the **small-signal circuit parameters** for each BJT.

If we use the **Hybrid-Π** model, we need to determine g_m and r_π:

$$g_m = \frac{I_C}{V_T} = \frac{1.0\ mA}{0.025V} = 40\ \frac{mA}{V}$$

$$r_\pi = \frac{V_T}{I_B} = \frac{0.025\text{ V}}{0.01\text{ mA}} = 2.5\text{ K}$$

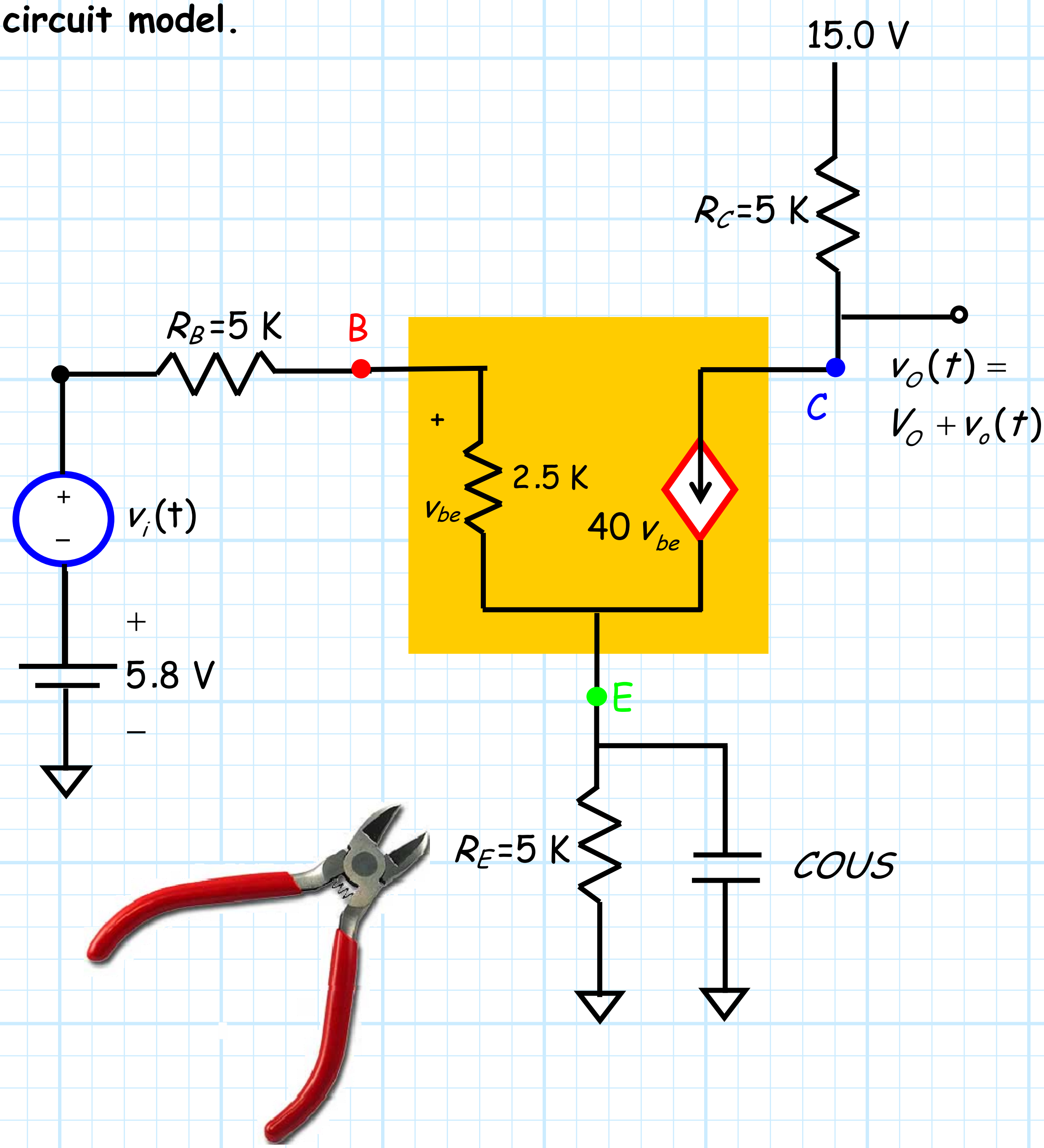
If we were to use the **T-model** we would likewise need to determine the emitter resistance:

$$r_e = \frac{V_T}{I_B} = \frac{0.025\text{ V}}{1.01\text{ mA}} = 24.7\ \Omega$$

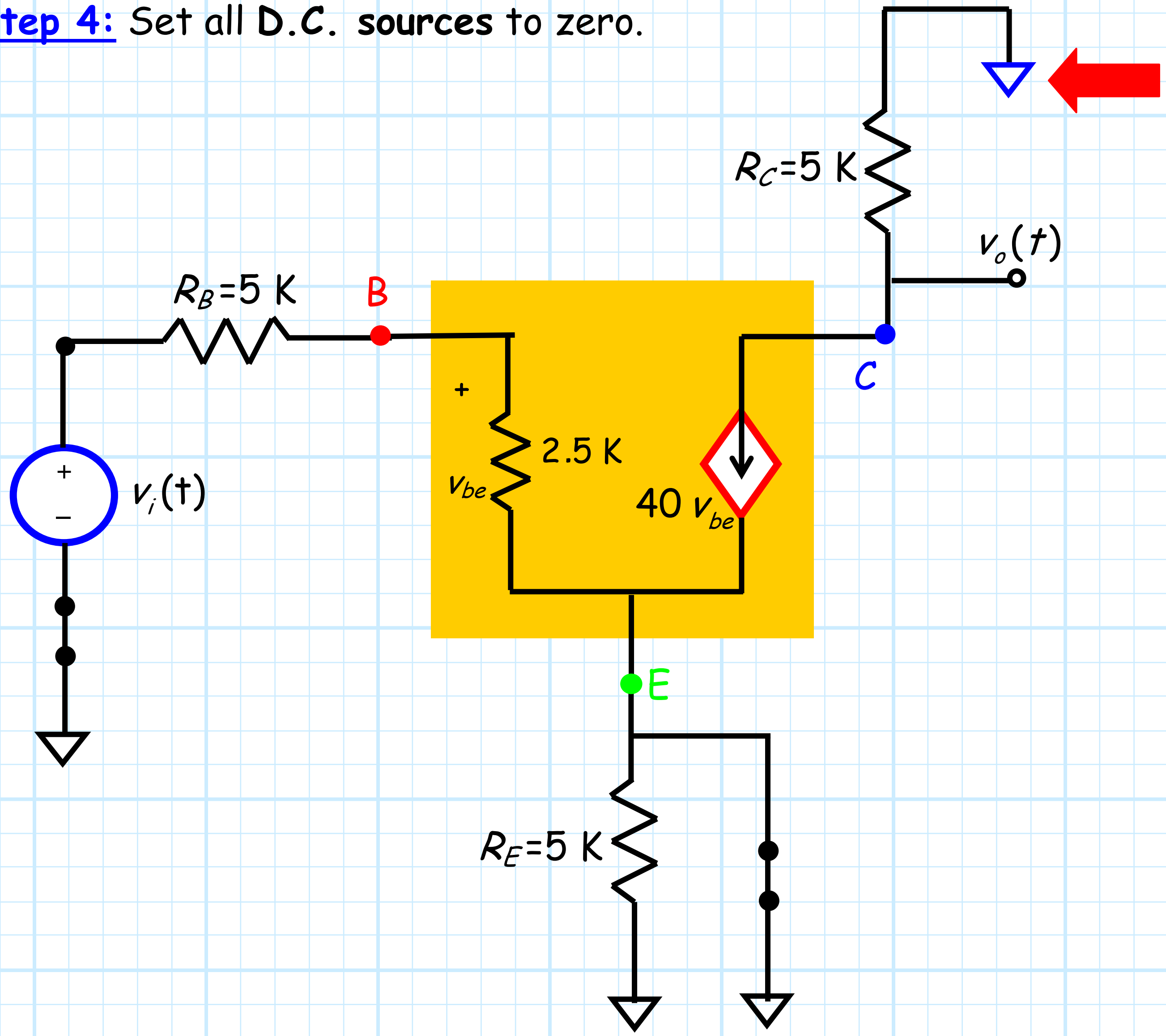
The **Early voltage** V_A of this BJT is unknown, so we will **neglect** the Early effect in our analysis.

As such, we assume that the output resistance is **infinite** ($r_o = \infty$).

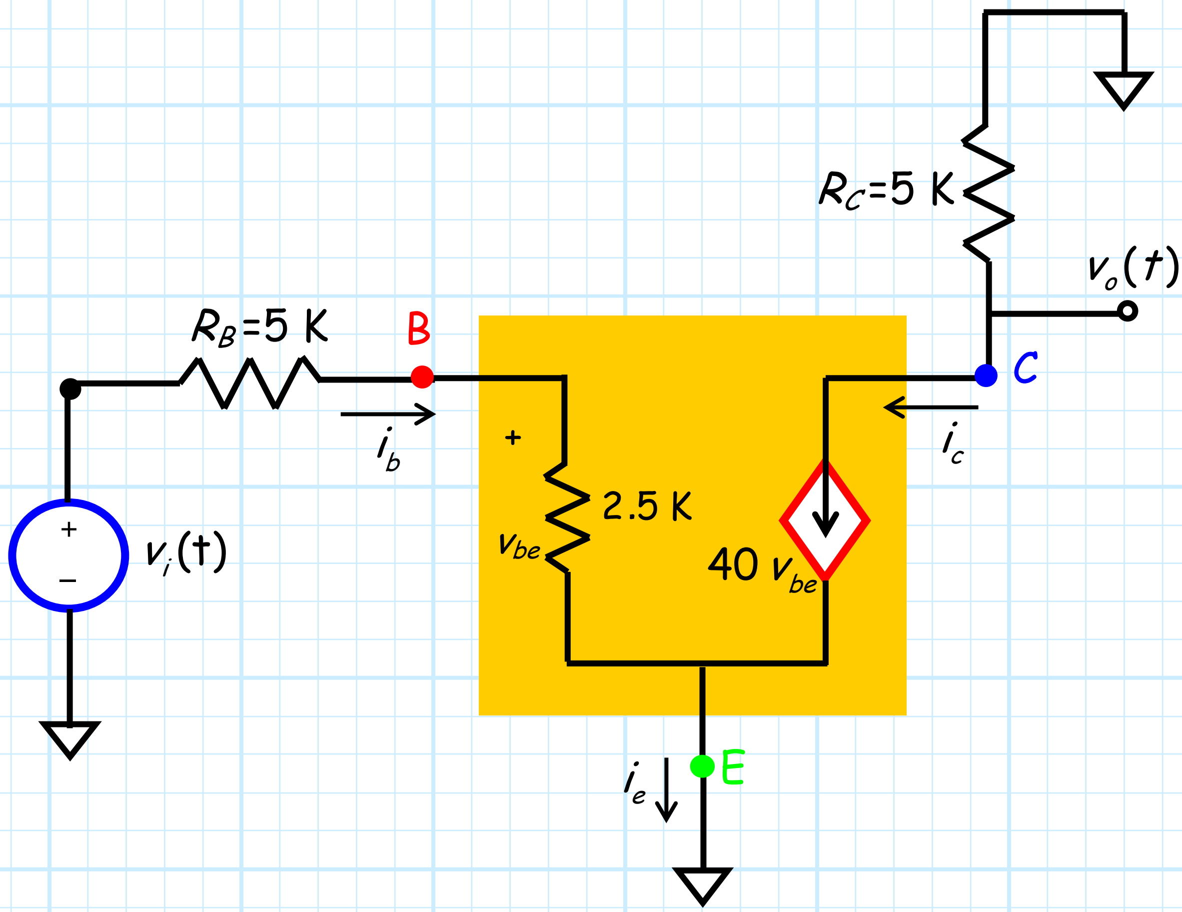
Step 3: Carefully replace all BJTs with their **small-signal circuit model.**



Step 4: Set all **D.C. sources** to zero.

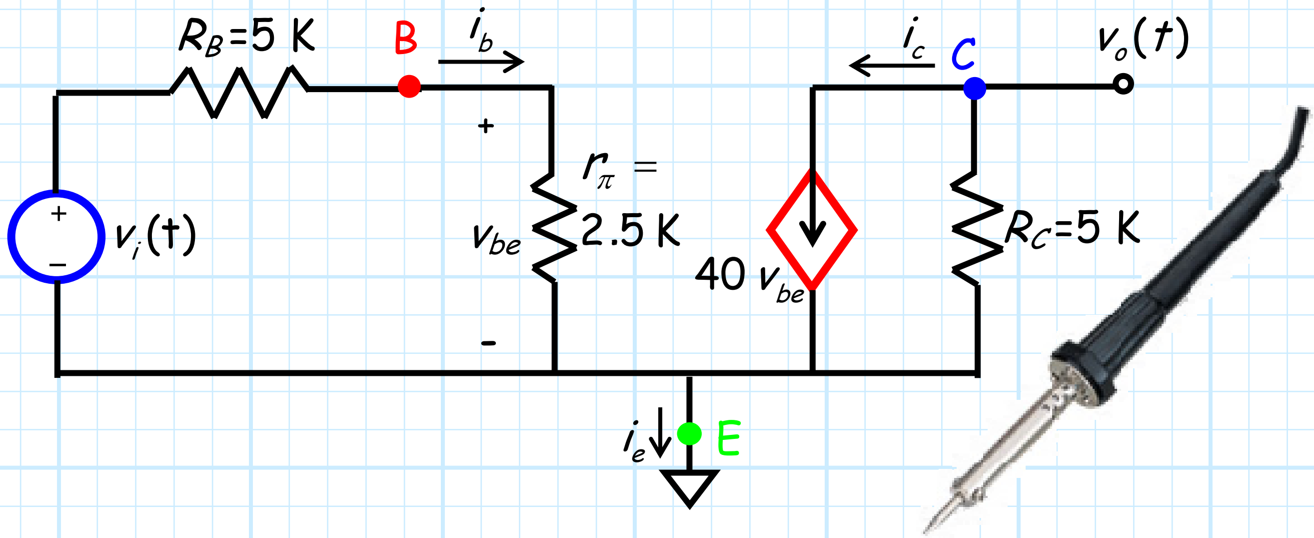


We likewise notice that the **large capacitor** (COUS) is an approximate ***AC* short**, and thus we can further simplify the schematic by replacing it with a short circuit.



We notice that one terminal of the small-signal voltage source, the emitter terminal, and one terminal of the collector resistor R_C are all connected to **ground**—thus they are **all** collected to **each other**!

We can use this fact to **simplify** the small-signal schematic.



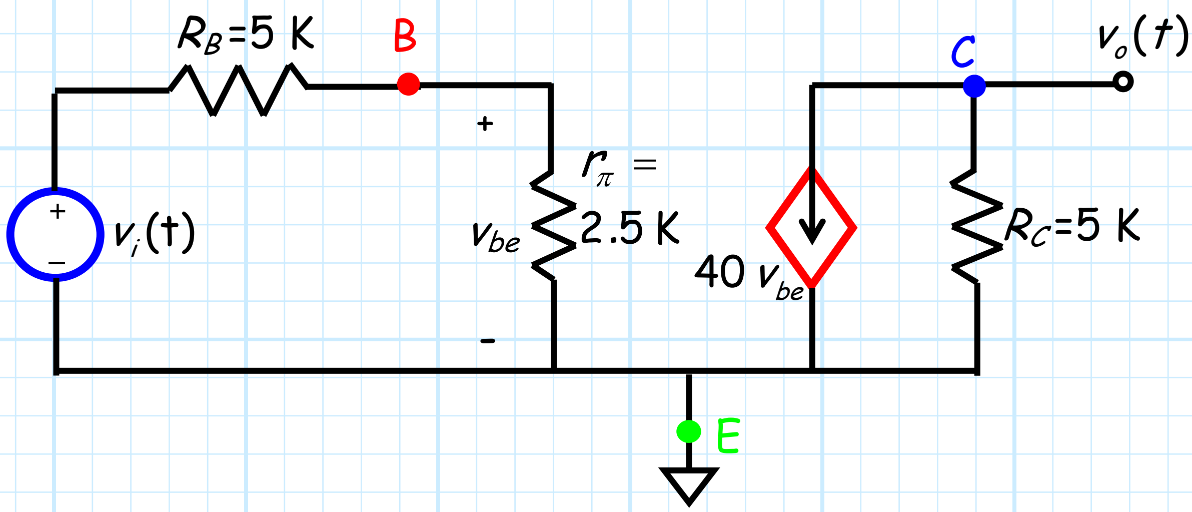
The schematic above is the **small-signal circuit** of this amplifier. We are ready to continue to **step 5**!

Step 5: Analyze **small-signal circuit**.

This is just a simple **EECS 211** problem! The **left** side of the circuit provides the **voltage divider** equation:

$$
\begin{aligned}
v_{be} &= \frac{r_\pi}{R_B + r_\pi} v_i \\
&= \frac{2.5}{5.0 + 2.5} v_i \\
&= \frac{v_i}{3}
\end{aligned}
$$

a result that relates the **input** signal to the base-emitter voltage.



The **right** side of the schematic allows us to determine the **output** voltage in terms of the base-emitter voltage:

$$\begin{aligned} v_o &= -i_c R_C \\ &= -(g_m v_{be}) R_C \\ &= -40(5) v_{be} \\ &= -200\, v_{be} \end{aligned}$$

Combining these two equations, we find:

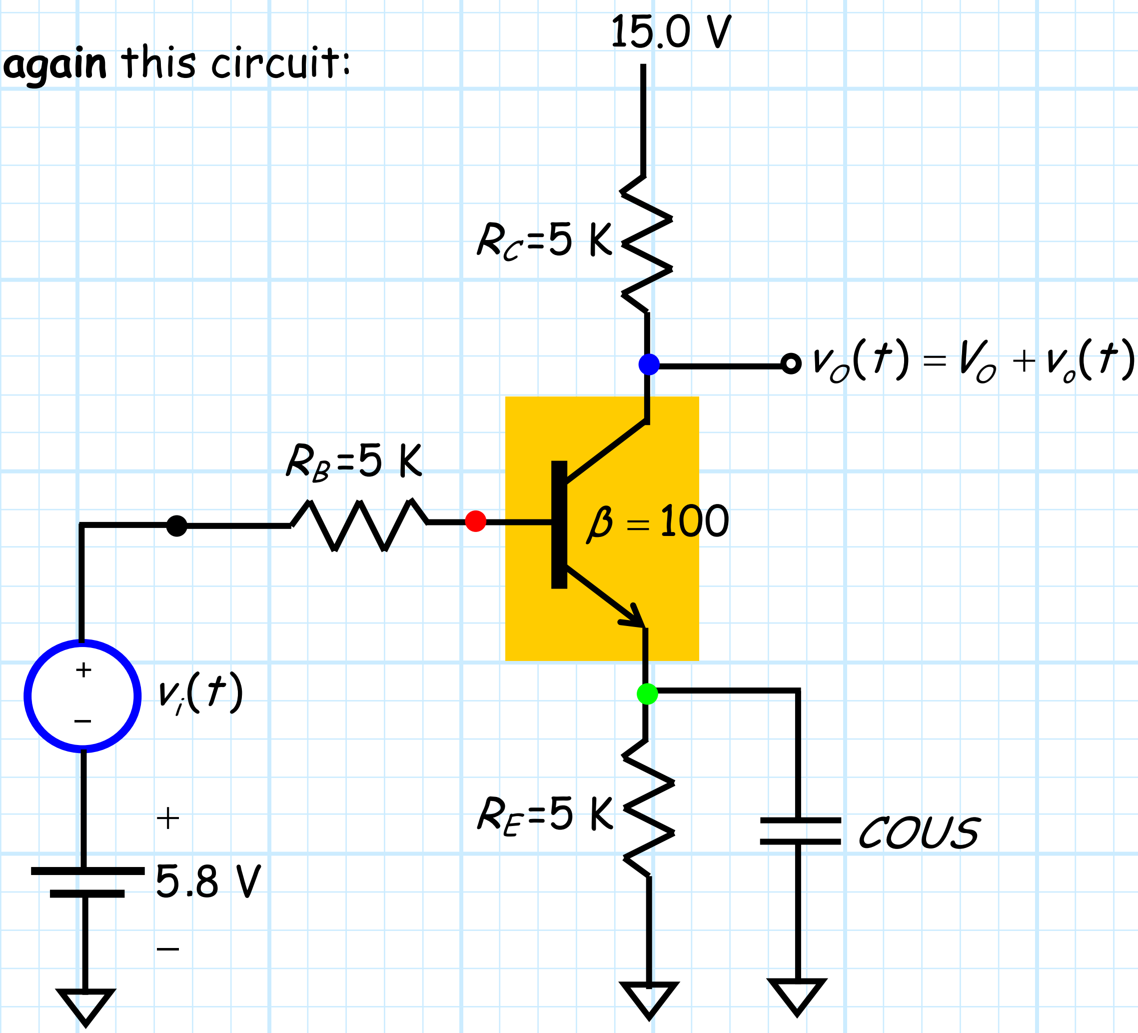
$$\begin{aligned} v_o &= -200\, v_{be} \\ &= -200\, \frac{v_i}{3} \\ &= -66.7\, v_i \end{aligned}$$

The **open-circuit**, small-signal **voltage gain** of this amplifier gain is therefore:

$$A_{vo} = \frac{v_o}{v_i} = -66.7$$

Example: Small-Signal Input and Output Resistances

Consider **again** this circuit:

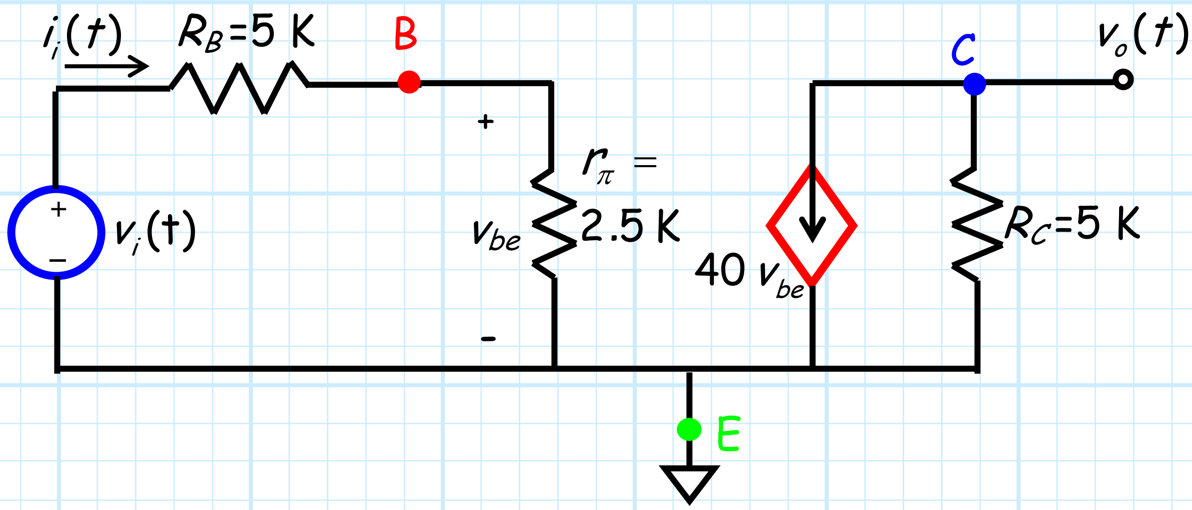


Recall we **earlier** determined the open-circuit **voltage gain** A_{vo} of this amplifier. But, recall also that voltage gain alone is **not** sufficient to **characterize** an amplifier—we likewise require the amplifier's input and output **resistances**!

Q: *But how do we **determine** the small-signal input and output resistances of this BJT amplifier?*

A: The same way we always have, only now we apply the procedures to the **small-signal circuit**.

Recall that **small-signal circuit** for this amplifier was determined to be:



The **input resistance** of an amplifier is defined as:

$$R_{in} = \frac{v_i}{i_i}$$

For this amplifier, it is evident that the **input current** is:

$$i_i = \frac{v_i}{R_B + r_\pi} = \frac{v_i}{5 + 2.5} = \frac{v_i}{7.5}$$

and thus the **input resistance** of this amplifier is:

$$R_{in} = \frac{v_i}{i_i} = 7.5 \text{ K}$$

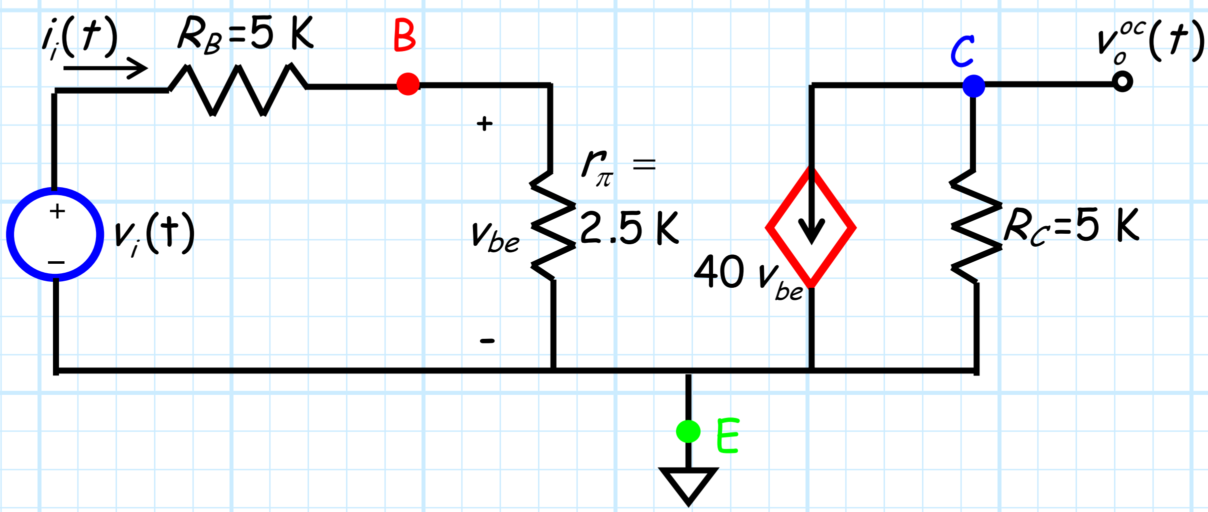
Now for the **output resistance**. Recall that determining the output resistance is much more **complex** than determining the input resistance.

The output resistance of an amplifier is the ratio of the amplifier's **open-circuit** output voltage and its **short-circuit** output current:

$$R_{out} = \frac{v_o^{oc}}{i_o^{sc}}$$

Again, we determine these values by analyzing the **small-signal** amplifier circuit.

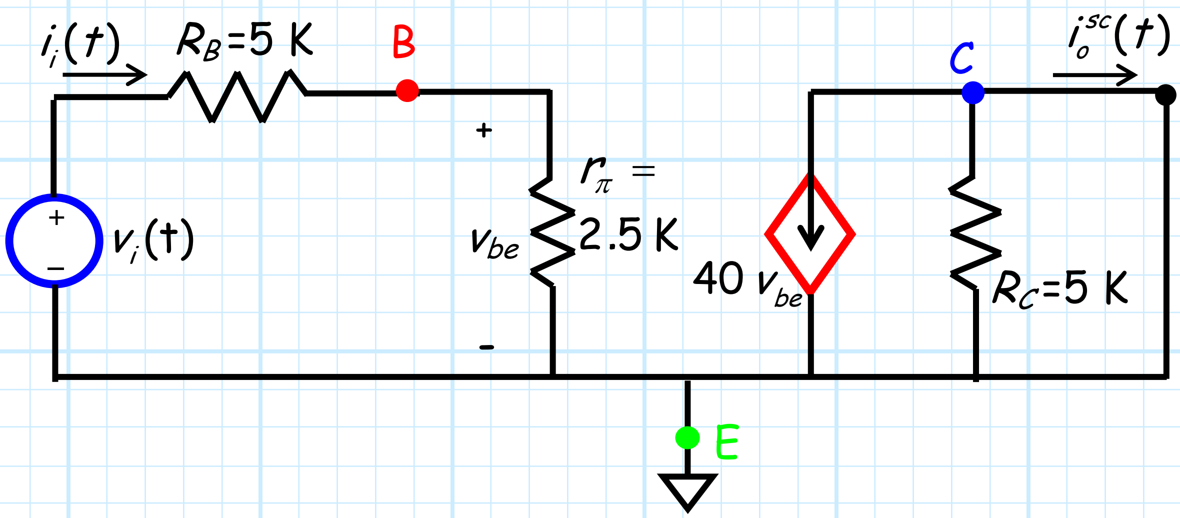
First, let's determine the open-circuit **output voltage**. This, of course, is the amplifier output voltage when the output terminal is **open-circuited**!



It is evident that the output voltage is simply the voltage across the **collector resistor** R_C:

$$v_o^{oc} = -\left(g_m v_{be}\right) R_C = -40\left(5\right) v_{be} = -200 v_{be} \text{ V}$$

Now, we must determine the short-circuit **output current** i_o^{sc}. This, of course, is the amplifier output current when the output terminal is **short-circuited**!



It is evident that the short-circuit output current is:

$$i_o^{sc} = -g_m v_{be} = -40 v_{be} \text{ mA}$$

and therefore the **output resistance** of this amplifier is:

$$R_{out} = \frac{v_o^{oc}}{i_o^{sc}} = \frac{-200 v_{be} \text{ V}}{-40 v_{be} \text{ mA}} = 5 \text{ K}\Omega$$

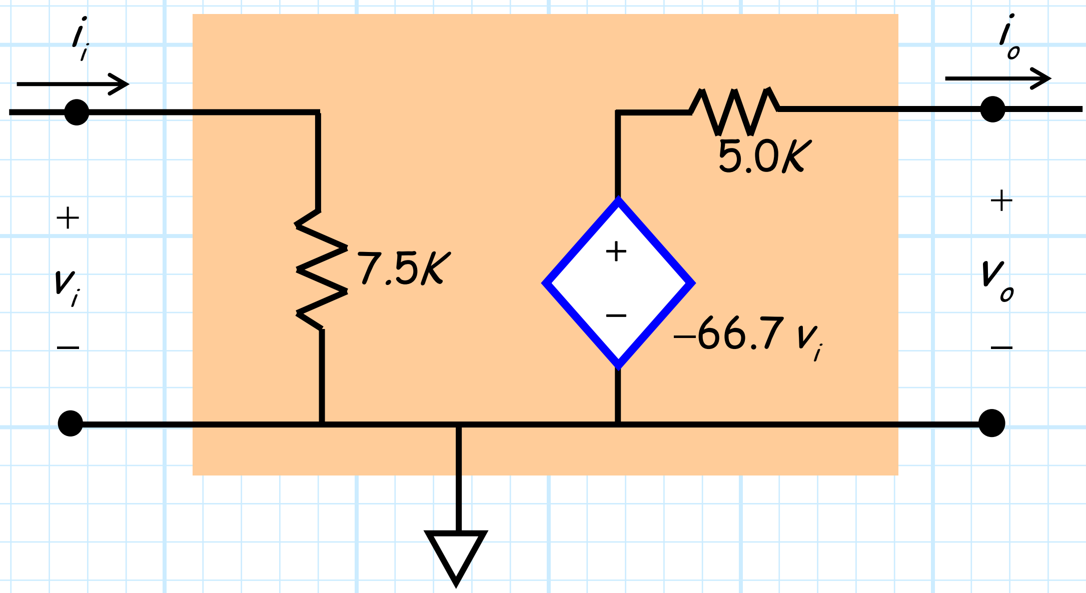
Now we know **all three** of the parameters required to characterize this amplifier!

$$A_{vo} = -66.7 \quad V/V$$

$$R_{in} = 7.5 \quad K\Omega$$

$$R_{out} = 5.0 \quad K\Omega$$

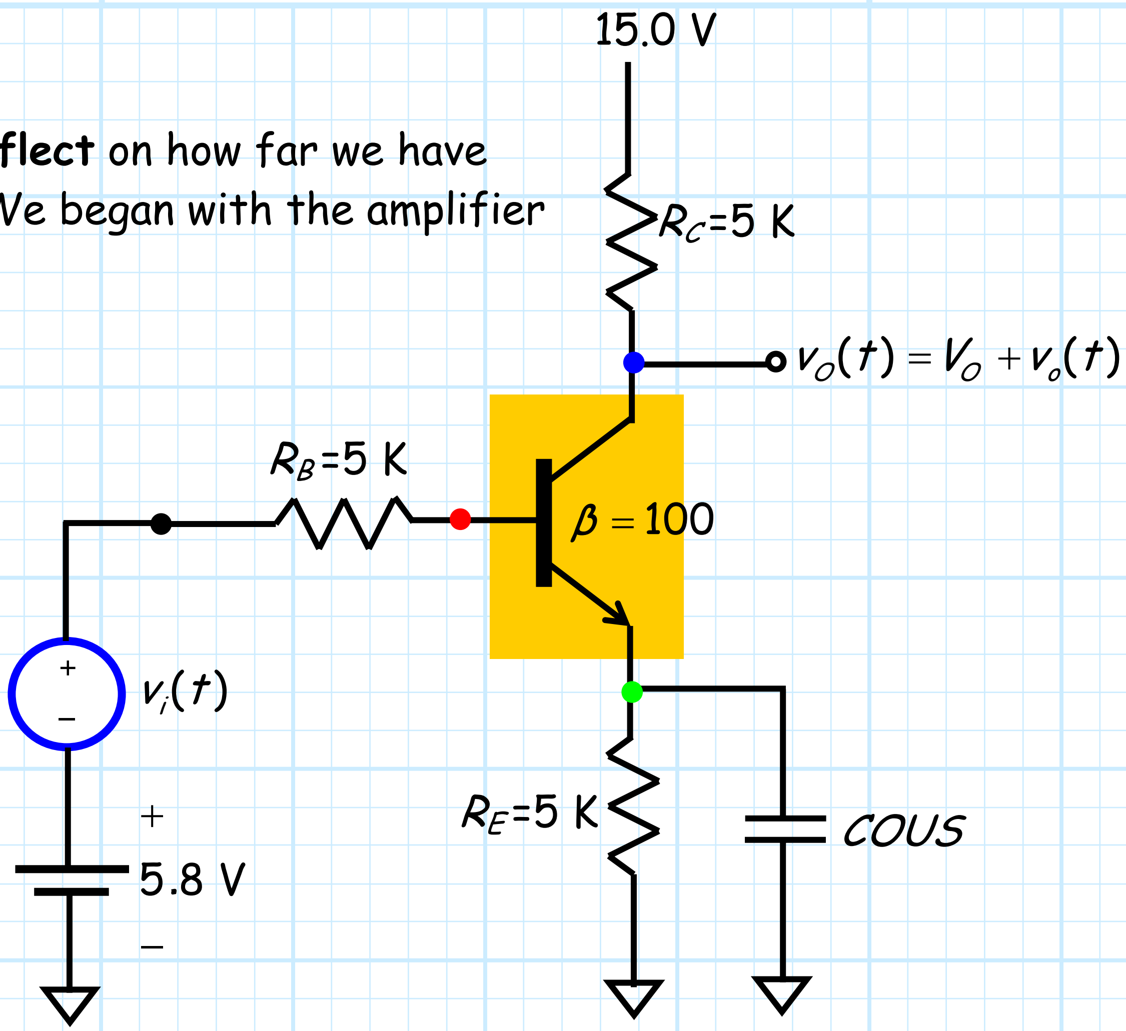
We can therefore write the **equivalent circuit model** for this amplifier:



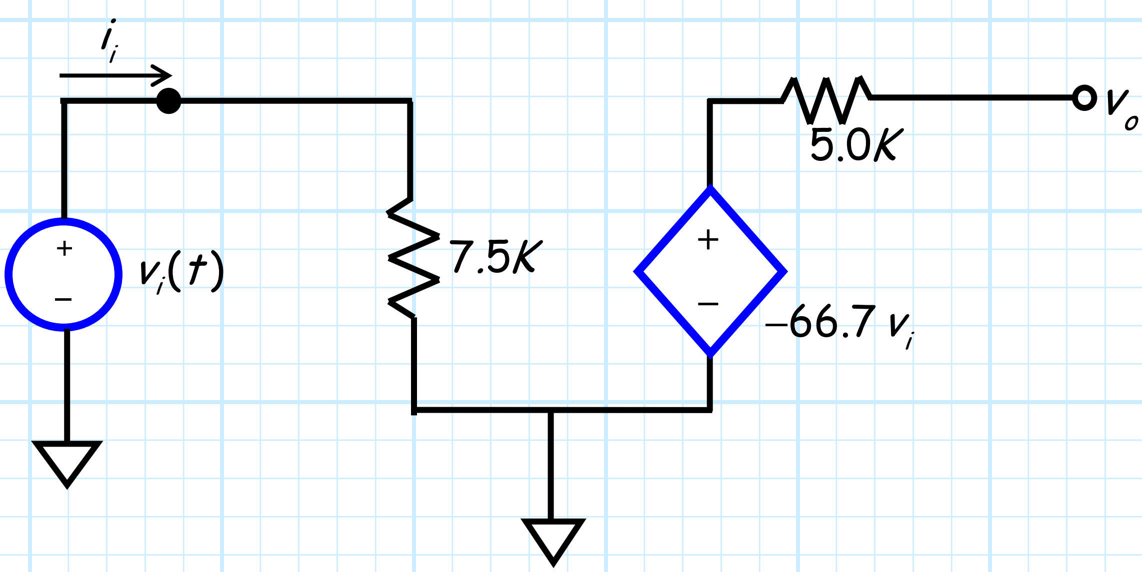
Note that the input resistance of this amplifier is **not** particularly large, and output resistance is **not** at all small.

→ This is **not** a particularly good voltage amplifier!

Now, **reflect** on how far we have come. We began with the amplifier circuit:



and now we have derived its **equivalent** small-signal circuit:



With respect to small signal input/output voltages and currents, these two circuits are **precisely** the same!