5.7 Single Stage BJT Amplifiers

Reading Assignment: 460-485

Small signal BJT amplifiers typically can be classified as one of three types.

Each type has its own specific characteristics, and thus each type has its own specific uses!

First, we consider the common-emitter amplifier:

**HO: THE Emitter CAPACITOR: WHAT’S UP WITH THAT?**

Next, the common collector amplifier—otherwise known as the emitter follower.

**HO: THE COMMON-Collector AMPLIFIER**

Finally, the common-base amplifier:

**HO: THE COMMON-BASE AMPLIFIER**
The Emitter Capacitor: What's up with that?

Note that in a previous amplifier example, there is a mysterious capacitor attached to the emitter:

A: Let's do a small-signal analysis and see why we place this large capacitor at the emitter.

Q: Why is this big capacitor here? Is it really required?
Let's analyze this amplifier!

Step 1 - DC Analysis

This is already completed! Recall that we designed the single supply DC bias circuit such that:

\[ I_C = 5 \text{ mA} \]

and

\[ V_{CE} = 5.0 > 0.7 \quad \checkmark \]

Step 2 - Calculate the BJT small-signal parameters

If we apply the hybrid-\( \pi \) model, we will require the small signal parameters:

\[ g_m = \frac{I_C}{V_T} = \frac{5 \text{ mA}}{0.025V} = 200 \text{ mA/V} \]

\[ r_n = \frac{V_T}{I_B} = \frac{\beta V_T}{I_C} = \frac{100(0.025)}{5.0} = 0.5 \text{ K} \]
This is step 3…

Steps 3 and 4 - Replace the BJT with its small-signal equivalent circuit, and turn off all DC sources.

Tuning off the DC sources, and replacing the Capacitors Of Unusual Size with short circuits, we find that the circuit becomes:

\[ \beta = 100 \]
...and this is step 4

Now carefully replace the BJT with its small-signal model:

Note that $\frac{3.7}{2.3} \cdot 0.5 = 370 \, \Omega$, therefore our small-signal circuit is equivalently:
A hefty gain

Step 5 - Analyze the small-signal circuit.

Since for this circuit $v_{be} = v_i$ and $v_o = -(1)200v_{be}$, the open-circuit, small-signal voltage gain of this amplifier is:

$$A_v = \frac{v_o}{v_i} = \frac{-200v_{be}}{v_{be}} = -200$$

Likewise, we can find that the small-signal input and output resistances are:

$$R_{in} = 370\Omega$$

and

$$R_{out} = 1.0 \, K$$

Note that the gain in this case is fairly large—46 dB.
Still, what's up with the capacitor?

Q: I still don't understand why the emitter capacitor is required.

Sure, our amplifier has large voltage gain, but I don't see how a capacitor could be responsible for that.

A: To see why the emitter capacitor is important, we need to compare these results to those obtained if the emitter capacitor is removed.

Note that if we remove the emitter capacitor, the first two steps of the small-signal analysis remains the same—the DC operating point is the same, and thus the small-signal parameters remain unchanged.

However, this does not mean that our resulting small signal circuit is left unchanged!
The emitter resistor is not “shorted out”!

* Recall that large capacitors (COUS) are approximated as AC shorts in the small-signal circuit.

* The emitter capacitor thus “shorts out” the emitter resistor in the small-signal circuit—the BJT emitter is connected to small-signal ground.

* If we remove the emitter capacitor, the emitter resistor is no longer shorted, and thus the BJT emitter is no longer connected to ground!
A horse of an entirely different color

The small-signal circuit in this case is:

![Circuit Diagram]

Note that the resistors $R_1 = 3.7 \, \text{K}$ and $R_2 = 2.3 \, \text{K}$ are no longer in parallel with base resistance $r_\pi = 0.5 \, \text{K}$!

As a result, we find that small signal voltage $v_{be}$ is not equal to small signal input voltage $v_i$. 
This circuit—it’s harder

Note also that the collector resistor is **not** connected in parallel with the dependent current source!

![Circuit Diagram]

Analyzing this small-signal circuit is not so easy!

We first need to determine the small signal **base-emitter** voltage $v_{be}$ in terms of input voltage $v_i$. 
Start with KCL

From KCL, we know that:

\[ i_e = i_b + i_c \]

Where:

\[ i_e = \frac{v_e}{R_E} = \frac{v_e}{1} = v_e \]

\[ i_b = \frac{v_{be}}{r_\pi} = \frac{v_{be}}{0.5} = 2.0 \, v_{be} \]

\[ i_c = 200 \, v_{be} \]

Therefore:

\[ v_e = 2.0 \, v_{be} + 200 \, v_{be} = 202 \, v_{be} \]
And now for KVL

Likewise, from KVL:

\[ 0 + v_i - v_{be} - v_e = 0 \]

\[ \Rightarrow v_{be} = v_i - v_e \]
This is NOT voltage division!

Inserting this into the first KCL result:

\[ v_e = 202 \, v_{be} \]
\[ = 202 \, v_i - 202 \, v_e \]

And now solving for small-signal emitter voltage:

\[ v_e = \frac{202}{203} \, v_i \]

Note that the small-signal base voltage is not related to the small signal input voltage by voltage division, i.e.:

\[ v_e \neq \frac{R_E}{r_i + R_E} \, v_i = \frac{1}{1.5} \, v_i \]
\( v_{be} \) is really small!

Therefore, we can finally determine \( v_{be} \) in terms of input voltage \( v_i \):

\[
v_{be} = v_i - v_e = v_i - \frac{202}{203} v_i = \left(1 - \frac{202}{203}\right) v_i = \frac{v_i}{203}
\]

Note then that not only is \( v_{BE} \neq v_i \), the small-signal base-emitter voltage is much smaller than input voltage \( v_i \)!

This of course is evident from the relationship:

\[
v_e = \frac{202}{203} v_b = \frac{202}{203} v_i
\]

which states that the emitter voltage is \textit{approximately equal} to the base (input) voltage \( v_b \) (\( v_i \)).
Now for the output voltage

This result will have a **profound** impact on amplifier performance!

To determine the output voltage, we begin with **KCL**:

\[ i_r = -i_c = -200v_{be} \]
What a wimpy gain

Now applying Ohm's Law to $R_C$:

$$\frac{v_o - 0}{R_C} = \frac{v_o}{1} = i_r = -200v_{be} \implies v_o = -200v_{be}$$

But recall that:

$$v_{be} = \frac{v_i}{203}$$

so we find that the small-signal output voltage is:

$$v_o = -200v_{be} = -\frac{200}{203}v_i$$

And thus the open-circuit voltage gain of this amplifier is:

$$A_v = \frac{v_o}{v_i} = -\frac{200}{203} \approx -1.0$$
See, the emitter capacitor is important

Yikes! Removing the emitter capacitor cause the voltage gain to change from -200 (i.e., 46 dB) to approximately -1.0 (i.e., 0dB)—a 46 dB reduction!

That emitter capacitor makes a big difference!

We can likewise finish the analysis and find that the small-signal input and output resistances are:

\[ R_{in} \approx R_1 \parallel R_2 = 3.7 \parallel 2.3 = 1.42 \text{ K} \]

\[ R_{out} = 1.0 \text{ K} \]

Note that input resistance actually improved in this case, increasing in value from 370 \( \Omega \) to 1.42 K \( \Omega \).

However, the decrease in voltage gain makes this amplifier (without a emitter capacitor) almost completely useless.
He only knows this because your TA explained it to him

The amplifier in this case (with the emitter capacitor) is an example of a design known as a common-emitter amplifier.

There are an infinite number of common-emitter designs, but they all share one thing in common—the emitter of the BJT is always connected directly to small-signal ground.

Common-emitter amplifier, such as the one examined here, typically result in large small-signal voltage gain (this is good!).

However, another characteristic of common emitter amplifiers is a typically low small-signal input resistance and high small-signal output resistance (this is bad!).
Make sure you can answer this question

One way to construct a common-emitter amplifier \textbf{without} using an emitter capacitor is simply to connect the BJT emitter directly to ground:

In this case, the emitter is at \textbf{both} AC (small-signal) ground and DC ground!

Q: \textit{Why is this common-emitter design \textbf{seldom} used??}

A:
The Common-Collector Amplifier

The common-collector amplifier: the BJT collector is at small-signal ground!
Examples of this type of amplifier include:

Do you see why each of these four circuits is a gul-durn common-collector amplifier?
Make dang sure that you do!
We’ll use the T-model

Let’s consider circuit (a).

It turns out that for common-collector amplifiers, the T-model (as opposed to the hybrid-\(\pi\)) typically provides the easiest small-signal analysis.

Using the T-model, we find that the small-signal circuit for amplifier (a) is:

\[
\begin{align*}
V_{i}(t) & \quad (\text{input}) \\
R_{B} & \quad (\text{bias resistor}) \\
\beta & \quad (\text{current gain}) \\
R_{e} & \quad (\text{emitter resistor}) \\
V_{be} & \quad (\text{base-emitter voltage}) \\
V_{o}(t) & \quad (\text{output}) \\
R_{E} & \quad (\text{emitter resistor})
\end{align*}
\]
Let's analyze this amplifier!

Let's determine the open-circuit voltage gain of this small-signal amplifier:

\[ A_{vo} = \frac{V_{oc}}{v_i} \]

We therefore must determine the output voltage \( v_o \) in terms of input voltage \( v_i \).

From KVL, we find that:

\[ 0 + v_i - R_B i_b - v_{be} = v_o \]
Let's apply KCL!

And from KCL, we find:

\[ i_b = i_e - i_c = i_e - g_m v_{be} \]

Where from Ohm's Law:

\[ i_e = \frac{v_o - 0}{R_E} = \frac{v_o}{R_E} \]

So:

\[ i_b = i_e - g_m v_{be} = \frac{v_o}{R_E} - g_m v_{be} \]

Inserting this into the KVL equation above:

\[ v_o = v_i - R_B i_b - v_{be} \]

\[ = v_i - R_B \left( \frac{v_o}{R_E} - g_m v_{be} \right) - v_{be} \]

\[ = v_i - \frac{R_B}{R_E} v_o + g_m R_B v_{be} \]
Let's apply Ohm's Law!

Likewise using KCL and Ohm's Law:

\[ i_e = \frac{v_{be}}{r_e} = \frac{v_o}{R_E} \]

Or rearranging:

\[ v_{be} = \frac{r_e}{R_E} v_o \]

Inserting this result in the solution above:

\[
v_o = v_i - \left( \frac{R_B}{R_E} \right) v_o + g_m R_B v_{be} \\
= v_i - \left( \frac{R_B}{R_E} \right) v_o + g_m R_B \left( \frac{r_e}{R_E} \right) v_o \\
= v_i - \left( \frac{R_B}{R_E} \right) v_o + g_m r_e \left( \frac{R_B}{R_E} \right) v_o \\
= v_i + \left( g_m r_e - 1 \right) \left( \frac{R_B}{R_E} \right) v_o
\]
It’s the gain—but look closer!

From this result we can determine the small-signal output voltage:

\[ v_o = \left(1 + \left(1 - g_m r_e \right) \frac{R_B}{R_E}\right)^{-1} v_i \]

And so the open-circuit voltage gain is:

\[ A_v = \frac{v_o}{v_i} = \left(1 + \left(1 - g_m r_e \right) \frac{R_B}{R_E}\right)^{-1} \]

We now note that:

\[ g_m r_e = \frac{V_T}{I_E} \frac{I_C}{V_T} = \frac{I_C}{I_E} = \alpha \]

Therefore:

\[ 1 - g_m r_e = 1 - \alpha = 1 - \frac{\beta}{\beta + 1} = \frac{1}{\beta + 1} \]
The output is no bigger than the input!

And so the gain becomes:

\[ A_vo = \frac{v_o}{v_i} = \left( 1 + \frac{1}{\beta + 1 R_E} \right)^{-1} \]

We note here that:

\[ \frac{1}{\beta + 1} \ll 1 \]

We find therefore, that the small-signal gain of this common-collector amplifier is approximately:

\[ A_vo = \left( 1 + \frac{1}{\beta + 1 R_E} \right)^{-1} \approx (1 + 0)^{-1} = 1.0 \]

The gain is approximately one!
This doesn’t seem to be useful

Q: What!? The gain is equal to one? That’s just dog-gone silly!

What good is an amplifier with a gain of one?

A: Remember, the open-circuit voltage gain is just one of three fundamental amplifier parameters.

The other two are input resistance $R_{in}$ and output resistance $R_{out}$.

First, let’s examine the input resistance.
Let's determine the input resistance

Using the small-signal circuit, we find that:

\[ R_{\text{in}} = \frac{v_i}{i_i} = \frac{v_i}{i_b} \]

Using KVL,

\[ 0 + v_i - R_B i_b - (r_e + R_E) i_e = 0 \]

and adding the fact that \( i_e = (\beta + 1) i_b \), we find that the small-signal base current is:

\[ i_b = \frac{v_i}{R_B + (\beta + 1)(r_e + R_E)} \]
A large input resistance; it’s a very good thing

Combining these equations, we find that the input resistance for this common-collector amplifier is:

\[ R_{in} = \frac{v_i}{i_b} = R_B + (\beta + 1)(r_e + R_E) \]

Since beta is large, the input resistance is typically large—this is good!

Now, let’s consider the output resistance \( R_{out} \) of this particular common-collector amplifier.

Recall that the output resistance is defined as:

\[ R_{out} = \frac{v_o^{oc}}{i_o^{sc}} \]

where \( v_o^{oc} \) is the open-circuit output voltage and \( i_o^{sc} \) is the short-circuit output current.
We must find the short circuit output current

Using **KVL**,

\[ 0 + v_i - R_b i_b - r_e i_e = 0 \]

and adding the fact that \( i_b = (\beta + 1)^{-1} i_e \), we find that the small-signal **emitter** current is:

\[ i_e = \frac{v_i}{R_b (\beta + 1)^{-1} + r_e} \]

And from **KCL**, this emitter current is likewise the short circuit output current:

\[ i_{o\,sc} = i_e = \frac{v_i}{R_b (\beta + 1)^{-1} + r_e} \]
A small output resistance;
it's a very good thing as well

Of course, we already have determined that the open-circuit output voltage is approximately equal to the input voltage:

$$v_{o}^{oc} = v_{i} \quad \text{(i.e., } A_{v} \approx 1)$$

Therefore, we find that the output resistance will be:

$$R_{out} = \frac{v_{o}^{oc}}{i_{o}^{sc}} = R_{B} \left( \beta + 1 \right)^{-1} + r_{e}$$

Since the emitter resistance $r_{e}$ is typically small (e.g., $r_{e} = 2.5\Omega$ if $I_{E} = 10.0mA$), and $\beta$ is typically large, we find that the output resistance of this common-collector amplifier will typically be small!
The emitter follower is like a voltage follower—it’s a buffer!

Let’s summarize what we have learned about this common-collector amplifier:

1. The small-signal voltage gain is approximately equal to one.
2. The input resistance is typically very large.
3. The output resistance is typically very small.

This is just like the op-amp voltage follower!

The common-collector amplifier is alternatively referred to as an emitter follower (i.e., the output voltage follows the input voltage).
The emitter follower is a great output stage

The common-collector amplifier is typically used as an output stage, where it isolates a high gain amplifier with large output resistance (e.g. a common emitter) from an output load of small resistance (e.g. an audio speaker).
The Common-Base Amplifier

The final amplifier type is the **common-base** amplifier.

As with the other amplifier types, the name indicates that the **base** terminal is at small-signal ground.

For example, a COUS between base and ground make this a **common-base** amplifier.
Look at the base terminal

The small-signal circuit of this common-base amplifier is most easily analyzed using the T-model.

Clearly, the base is connected to small-signal ground!
The input resistance: very interesting

Rearranging this circuit:

The interesting feature of this amplifier is its input and output resistances.

It is apparent from the small-signal circuit that the input resistance is:

$$R_{in} = r_e \parallel R_E \cong r_e$$

And the output resistance is:

$$R_{out} = R_C$$
It's so darn small!

Recall that the small-signal emitter resistance:

\[ r_e = \frac{V_T}{I_e} \]

is typically very small.

For example, if \( I_e = 10 \text{ mA} \), then \( r_e = 2.5 \Omega \)!

Therefore, since the input resistance \( R_{in} \) of this common-base amplifier is equal to the small-signal emitter resistance \( r_e \), the input resistance of this common-base amplifier is likewise very small!
Recall the ideal current amplifier

**Q:** A small input resistance!? I thought a large input resistance is ideal.

**A:** Are large input resistance is desirable for an ideal voltage amplifier.

However, recall that a small input resistance is desirable for the ideal current amplifier!

Thus, common-base amplifiers are very useful as an input stage in a current amplifier.