The Short-Circuit Current Gain $h_{fe}$

Consider the common emitter “low-frequency” small-signal model with its output short-circuited.
**Boring! Tell me something I don’t already know**

In this case we find:

\[ i_c(\omega) = g_m v_{be}(\omega) = g_m r_\pi i_b(\omega) \]

But we know that:

\[ g_m r_\pi = \frac{I_C}{V_T} \frac{V_T}{I_B} = \frac{I_C}{I_B} = \beta \]

Therefore:

\[ \frac{i_c(\omega)}{i_b(\omega)} = \beta \]

Just as we expected!
When the input signal is changing rapidly

Now, contrast this with the results using the high frequency model:

\[ i_b(\omega) = \frac{1}{r_\pi} + j\omega (C_\pi + C_\mu) \]

\[ i_c(\omega) = v_\pi(\omega) \left( g_m - j\omega C_\mu \right) \]
Here's something you did not know

Therefore, the ratio of small-signal collector current to small-signal base current is:

\[
\frac{i_c(\omega)}{i_b(\omega)} = \frac{r_\pi g_m - j\omega r_\pi C_\mu}{1 + j\omega r_\pi (C_\mu + C_\pi)}
\]

Typically, we find that \( g_m \gg \omega C_\mu \), so that we find:

\[
\frac{i_c(\omega)}{i_b(\omega)} \approx \frac{r_\pi g_m}{1 + j\omega r_\pi (C_\mu + C_\pi)}
\]

and again we know:

\[
r_\pi g_m = \frac{I_C}{V_T} \frac{V_T}{I_B} = \frac{I_C}{I_B} = \beta
\]

Therefore:

\[
\frac{i_c(\omega)}{i_b(\omega)} \approx \frac{\beta}{1 + j\omega r_\pi (C_\mu + C_\pi)}
\]
Your BJT is frequency dependent!

We define this ratio as the small-signal BJT (short-circuit) current gain, $h_{fe}(\omega)$:

$$h_{fe}(\omega) = \frac{i_{c}(\omega)}{i_{b}(\omega)} \approx \frac{\beta}{1 + j\omega r_{\pi} \left( C_{\mu} + C_{\pi} \right)}$$

Note this function is a low-pass function, where we can define a 3dB break frequency as:

$$\omega_{\beta} = \frac{1}{r_{\pi} \left( C_{\pi} + C_{\mu} \right)}$$

Therefore:

$$h_{fe}(\omega) = \frac{i_{c}(\omega)}{i_{b}(\omega)} \approx \frac{\beta}{1 + j\omega/\omega_{\beta}}$$
This is how we define BJT bandwidth

Plotting the magnitude of $h_{fe}(\omega)$ (i.e., $|h_{fe}(\omega)|^2$) we find that:

We see that for frequencies less than the 3 dB break frequency, the value of $h_{fe}(\omega)$ is approximately equal to $\beta$:

$$h_{fe}(\omega) \approx \beta \quad \omega < \omega_\beta$$
This should SO remind you of op-amps

Note then for frequencies greater than this break frequency:

\[
h_{fe}(\omega) = \frac{\beta}{1 + j\frac{\omega}{\omega_v}}
\]

\[
\approx j\frac{\beta \omega_v}{\omega} \quad \omega > \omega_v
\]

Note then that \(|h_{fe}(\omega)| = 1\) when \(\omega = \beta \omega_v\).

We can thus define this frequency as \(\omega_T\), the unity-gain frequency:

\[
\omega_T = \beta \omega_v
\]

so that:

\[
|h_{fe}(\omega = \omega_T)| = 1.0
\]
Déjà vu; all over again

We can therefore state that:

$$|h_{fe}(\omega)| \approx \frac{\beta \omega_{\beta}}{\omega} = \frac{\omega_r}{\omega} \quad \omega > \omega_{\beta}$$

and also that:

$$|h_{fe}(\omega)| \approx \beta \quad \omega < \omega_{\beta}$$