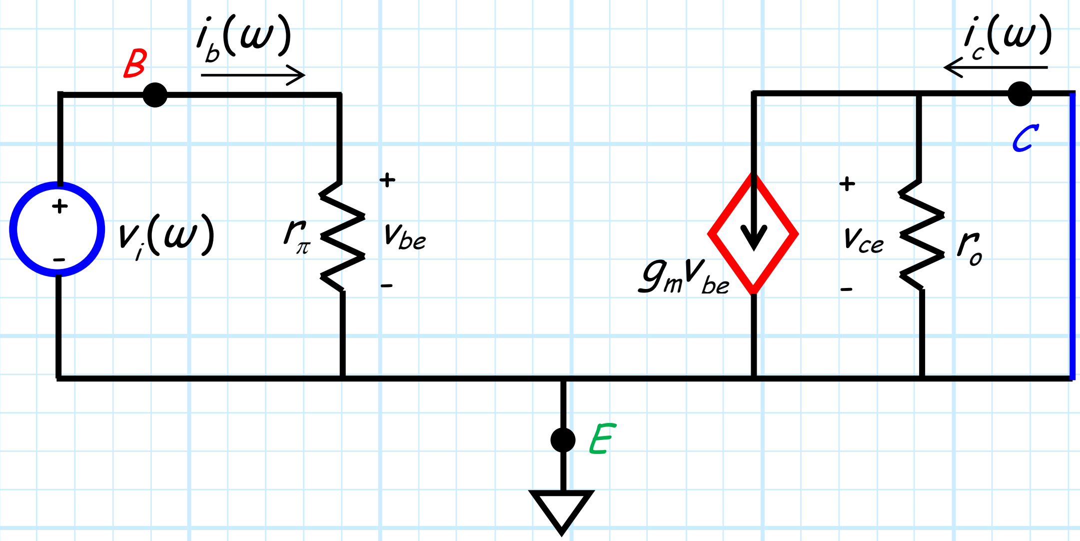
The Short-Circuit Current Gain h_{fe}

Consider the common emitter "low-frequency" small-signal model with its output **short-circuited**.



Boring! Tell me something I don't already know

In this case we find:

$$\begin{aligned} i_c(\omega) &= g_m v_{be}(\omega) \\ &= g_m r_\pi i_b(\omega) \end{aligned}$$

But we know that:

$$g_m r_\pi = \frac{I_C}{V_T}\frac{V_T}{I_B} = \frac{I_C}{I_B} = \beta$$

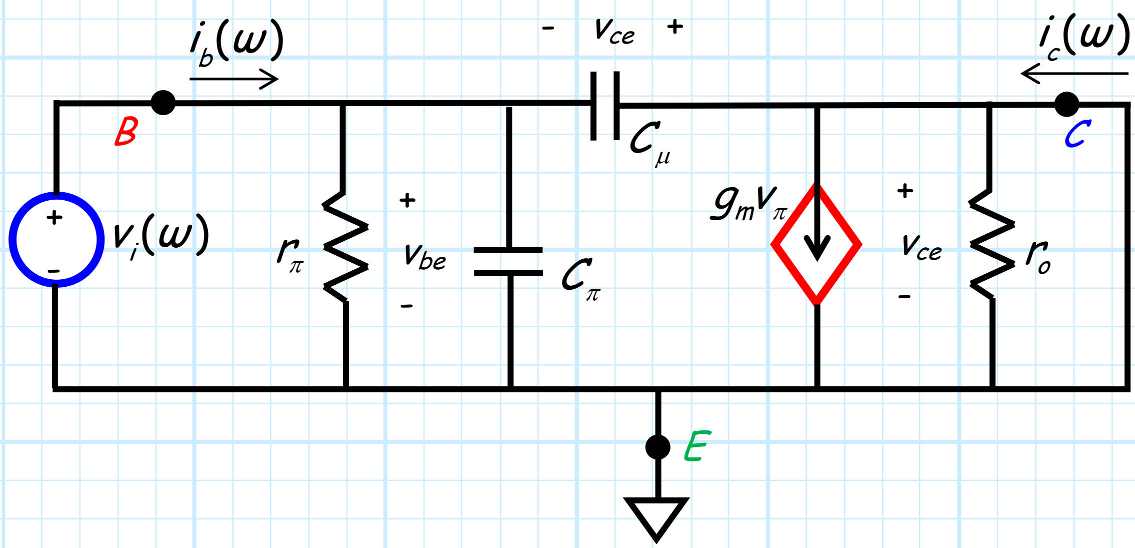
Therefore:

$$\frac{i_c(\omega)}{i_b(\omega)} = \beta$$

Just as we **expected**!

When the input signal is changing rapidly

Now, contrast this with the results using the **high frequency** model:



Evaluating this circuit, it is evident that the small-signal **base** current is:

$$i_b(\omega) = v_\pi(\omega)\left(\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)\right)$$

While the small-signal **collector** current is:

$$i_c(\omega) = v_\pi(\omega)\left(g_m - j\omega C_\mu\right)$$

Here's something you did not know

Therefore, the **ratio** of small-signal collector current to small-signal base current is:

$$\frac{i_c(\omega)}{i_b(\omega)} = \frac{r_\pi \, g_m - j\omega \, r_\pi \, C_\mu}{1 + j\omega \, r_\pi \left(C_\mu + C_\pi\right)}$$

Typically, we find that $g_m \gg \omega C_\mu$, so that we find:

$$\frac{i_c(\omega)}{i_b(\omega)} \approx \frac{r_\pi \, g_m}{1 + j\omega \, r_\pi \left(C_\mu + C_\pi\right)}$$

and again we know:

$$r_\pi \, g_m = \frac{I_C}{V_T} \frac{V_T}{I_B} = \frac{I_C}{I_B} = \beta$$

Therefore:

$$\frac{i_c(\omega)}{i_b(\omega)} \approx \frac{\beta}{1 + j\omega \, r_\pi \left(C_\mu + C_\pi\right)}$$

Your BJT is frequency dependent!

We **define** this ratio as the **small-signal BJT (short-circuit) current gain**, $h_{fe}(\omega)$:

$$h_{fe}(\omega) \doteq \frac{i_c(\omega)}{i_b(\omega)} \approx \frac{\beta}{1 + j\omega\, r_\pi \left(C_\mu + C_\pi\right)}$$

Note this function is a **low-pass** function, were we can define a 3dB **break frequency** as:

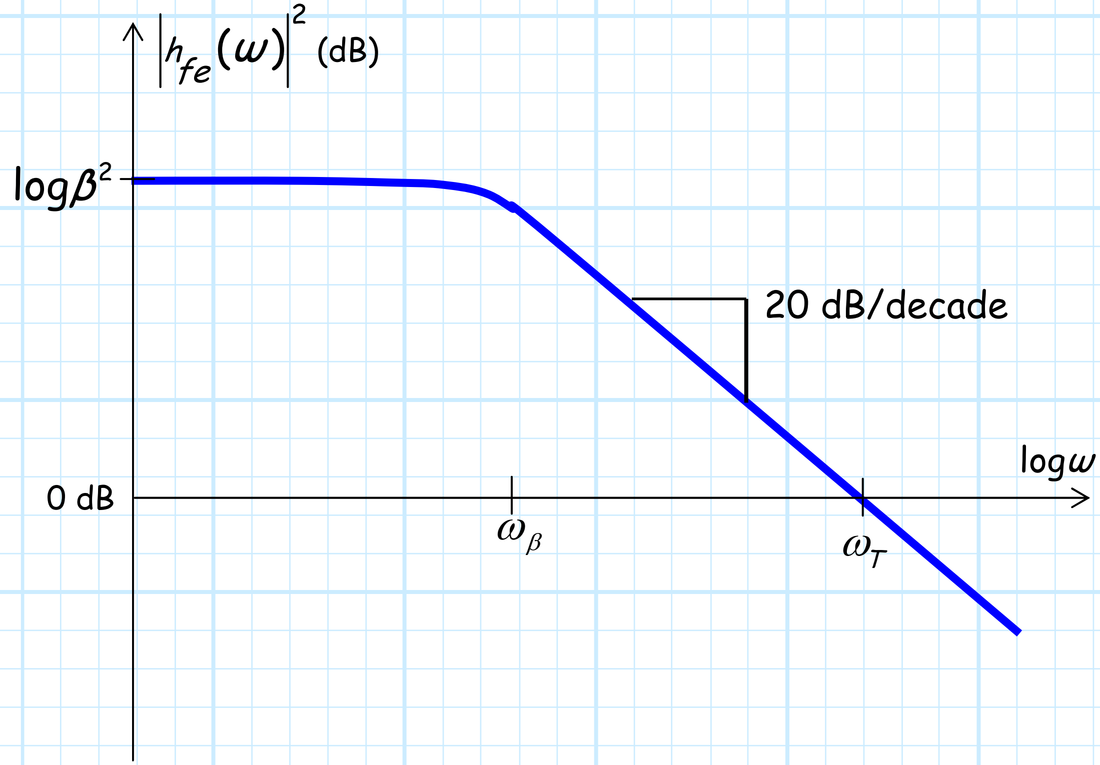
$$\omega_\beta = \frac{1}{r_\pi \left(C_\pi + C_\mu\right)}$$

Therefore:

$$h_{fe}(\omega) \doteq \frac{i_c(\omega)}{i_b(\omega)} \approx \frac{\beta}{1 + j\,\omega / \omega_\beta}$$

This is how we define BJT bandwidth

Plotting the magnitude of $h_{fe}(\omega)$ (i.e., $\left|h_{fe}(\omega)\right|^2$) we find that:



We see that for frequencies **less** than the 3 dB break frequency, the value of $h_{fe}(\omega)$ is approximately equal to **beta**:

$$h_{fe}(\omega) \approx \beta \qquad \omega < \omega_\beta$$

This should SO remind you of op-amps

Note then for frequencies greater than this break frequency:

$$h_{fe}(\omega) = \frac{\beta}{1 + j\,\omega / \omega_\beta}$$

$$\approx j\frac{\beta\,\omega_\beta}{\omega} \qquad \omega > \omega_\beta$$

Note then that $|h_{fe}(\omega)| = 1$ when $\omega = \beta\,\omega_\beta$.

We can thus define this frequency as ω_T, the **unity-gain** frequency:

$$\omega_T \doteq \beta\,\omega_\beta$$

so that:

$$|h_{fe}(\omega = \omega_T)| = 1.0$$

Déjà vu; all over again

We can therefore state that:

$$\left|h_{fe}(\omega)\right| \approx \frac{\beta\,\omega_\beta}{\omega} = \frac{\omega_T}{\omega} \qquad \omega > \omega_\beta$$

and also that:

$$\left|h_{fe}(\omega)\right| \approx \beta \qquad \omega < \omega_\beta$$