5.8 BJT Internal Capacitances

Reading **Assignment**: 485-490

BJT's exhibit **capacitance** between each of its terminals (i.e., base, emitter, collector). These capacitances ultimately limit amplifier **bandwidth**.

Q: *Yikes! Who put these capacitors in the BJT? Why did they put them there? Why don't we just* ***remove*** *them?*

A: These capacitances are **parasitic** capacitances. Since the terminals are made of conducting materials (e.g., metal), there will **always** be some capacitance associated with any two terminals.

BJT designers and manufacturers work hard to **minimize** these capacitances—and indeed they are **very small**—but we **cannot** eliminate them entirely.

If the **signal frequency** gets **high** enough, these capacitances can affect amplifier performance.

HO: BJT INTERNAL CAPACITANCES

Now that we are aware of these internal capacitances, we must **modify** our small-signal circuit **models**.

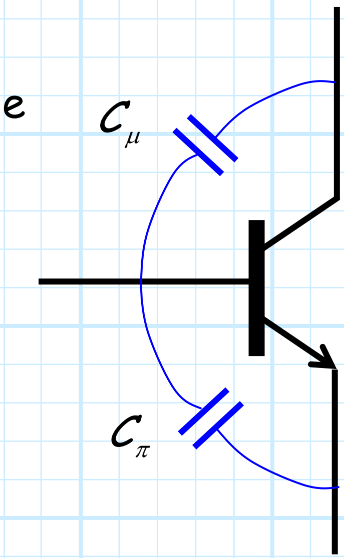
HO: The High-Frequency Hybrid Pi Model

The significance of the internal capacitances are typically specified in terms of a frequency-dependent BJT parameter called $h_{fe}(\omega)$. This parameter is often referred to as the "**short-circuit current gain**" of the BJT.

From this function $h_{fe}(\omega)$ we can extract a BJT parameter called the **unity-gain bandwidth**—a result very analogous to **op-amps**!

HO: The Short-Circuit Current Gain

BJT Internal Capacitances



There are **very small capacitances** in a BJT between the collector and the base, and the base and the emitter.

Since the capacitor values are very small, their impedance at **low** and moderate frequencies is large. I.E.:

$$Z_C = \frac{1}{j\omega C} \quad \text{is large if } \omega C \ll 1$$

In other words, at low and moderate frequencies, these capacitor impedances are approximately **open** circuits, and thus they can be **ignored**.

However, at **high** frequencies, the capacitor impedance can drop to **moderate** values (e.g., $K\Omega s$).

In this case, we can **no longer** ignore these capacitances, but instead must incorporate them into our **small-signal model**!

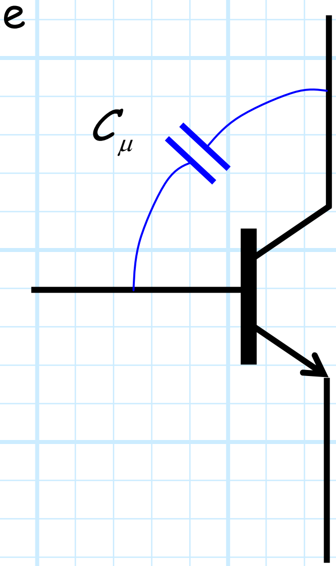
The capacitance between base and collector

Q: *What are C_μ and C_π?*

A: See below!

C_μ

C_μ is a parasitic capacitance between the **collector** and the **base**.



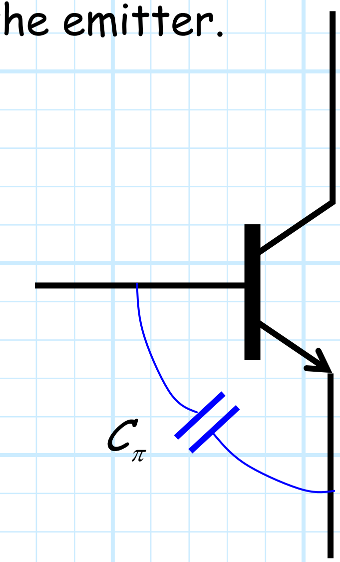
This capacitance is due to the *pn* **junction** (between collector and base).

Typical values of C_μ are a few picofarads or **less**.

The capacitance between base and emitter

C_π

C_π is a parasitic (i.e., small) capacitance between the base and the emitter.



This capacitance actually consists of **two** parts:

$$C_\pi = C_{je} + C_{de}$$

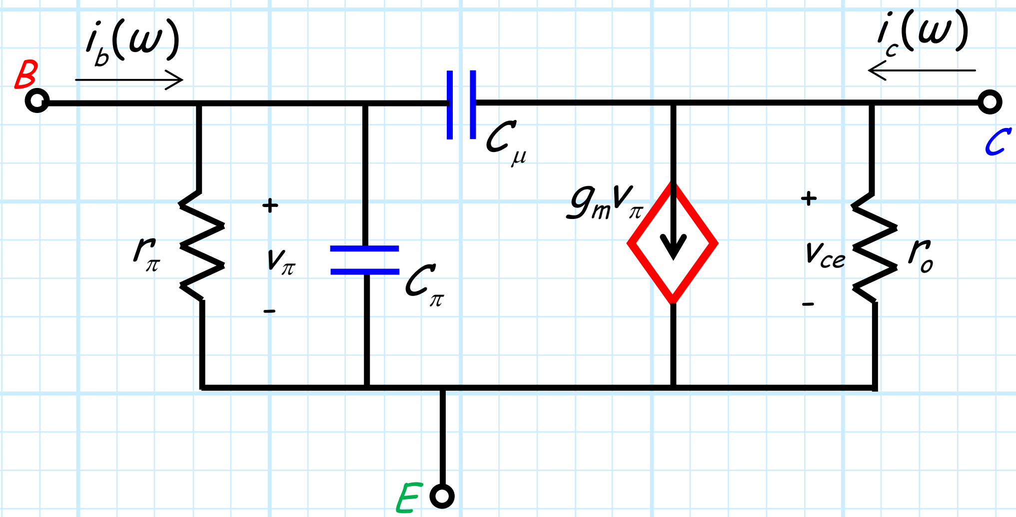
where:

$$\left.\begin{aligned} C_{de} &= \text{diffusion capacitance} \\ C_{je} &= \text{junction capacitance} \end{aligned}\right\} \textit{pn} \text{ junction capacitance}$$

Typically, C_π is a **few picofarads**.

The High-Frequency Hybrid-π Model

Combine the internal capacitances and lead resistance in a **modified** Hybrid-π model.



* Therefore use this model to construct small-signal circuit when v_i is operating at **high** frequency.

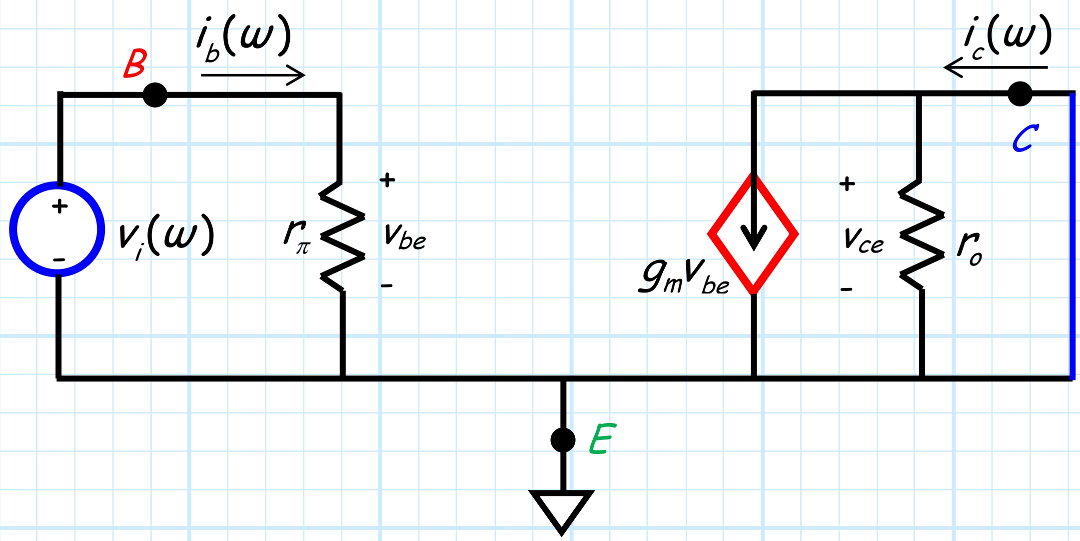
* Note since $Z_C = 1/jwC$, all currents and voltages will be **dependent on operating frequency** ω.

* Note the voltage across r_π is v_π, but $v_\pi \neq v_{be}$!!!

* Note at low-frequencies, the model reverts to the **original** Hybrid-π model.

The Short-Circuit Current Gain h_{fe}

Consider the common emitter "low-frequency" small-signal model with its output **short-circuited**.



Boring! Tell me something I don't already know

In this case we find:

$$\begin{aligned} i_c(\omega) &= g_m \, v_{be}(\omega) \\ &= g_m \, r_\pi \, i_b(\omega) \end{aligned}$$

But we know that:

$$g_m \, r_\pi = \frac{I_C}{V_T} \frac{V_T}{I_B} = \frac{I_C}{I_B} = \beta$$

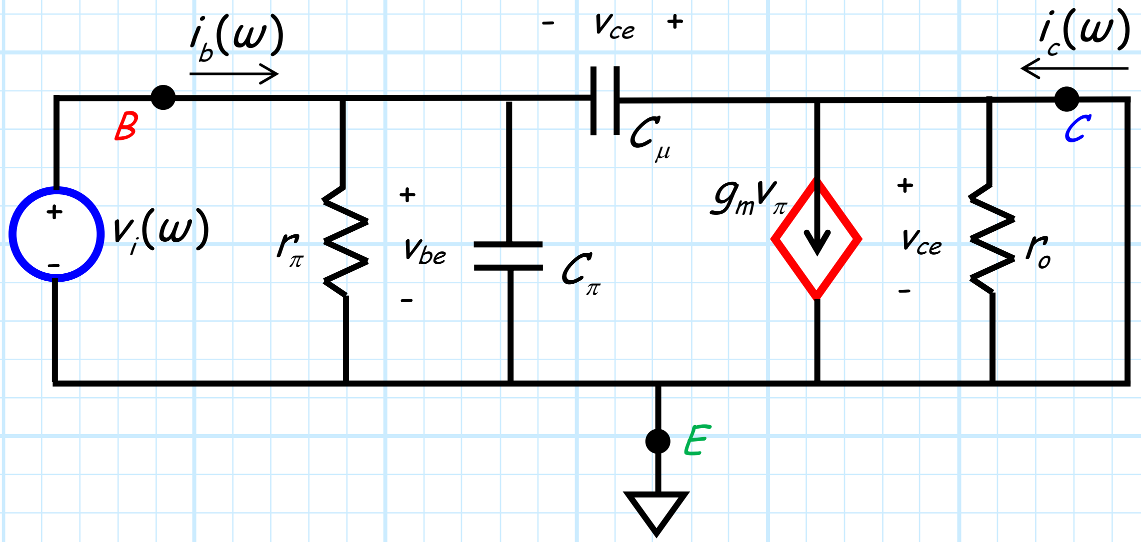
Therefore:

$$\frac{i_c(\omega)}{i_b(\omega)} = \beta$$

Just as we **expected**!

When the input signal is changing rapidly

Now, contrast this with the results using the **high frequency** model:



Evaluating this circuit, it is evident that the small-signal **base** current is:

$$i_b(\omega) = v_\pi(\omega)\left(\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)\right)$$

While the small-signal **collector** current is:

$$i_c(\omega) = v_\pi(\omega)\left(g_m - j\omega C_\mu\right)$$

Here's something you did not know

Therefore, the **ratio** of small-signal collector current to small-signal base current is:

$$\frac{i_c(\omega)}{i_b(\omega)} = \frac{r_\pi\, g_m - j\omega\, r_\pi\, C_\mu}{1 + j\omega\, r_\pi \left(C_\mu + C_\pi\right)}$$

Typically, we find that $g_m \gg \omega C_\mu$, so that we find:

$$\frac{i_c(\omega)}{i_b(\omega)} \approx \frac{r_\pi\, g_m}{1 + j\omega\, r_\pi \left(C_\mu + C_\pi\right)}$$

and again we know:

$$r_\pi\, g_m = \frac{I_C}{V_T}\frac{V_T}{I_B} = \frac{I_C}{I_B} = \beta$$

Therefore:

$$\frac{i_c(\omega)}{i_b(\omega)} \approx \frac{\beta}{1 + j\omega\, r_\pi \left(C_\mu + C_\pi\right)}$$

Your BJT is frequency dependent!

We **define** this ratio as the **small-signal BJT (short-circuit) current gain**, $h_{fe}(\omega)$:

$$h_{fe}(\omega) \doteq \frac{i_c(\omega)}{i_b(\omega)} \approx \frac{\beta}{1 + j\omega\, r_\pi \left(C_\mu + C_\pi\right)}$$

Note this function is a **low-pass** function, were we can define a 3dB **break frequency** as:

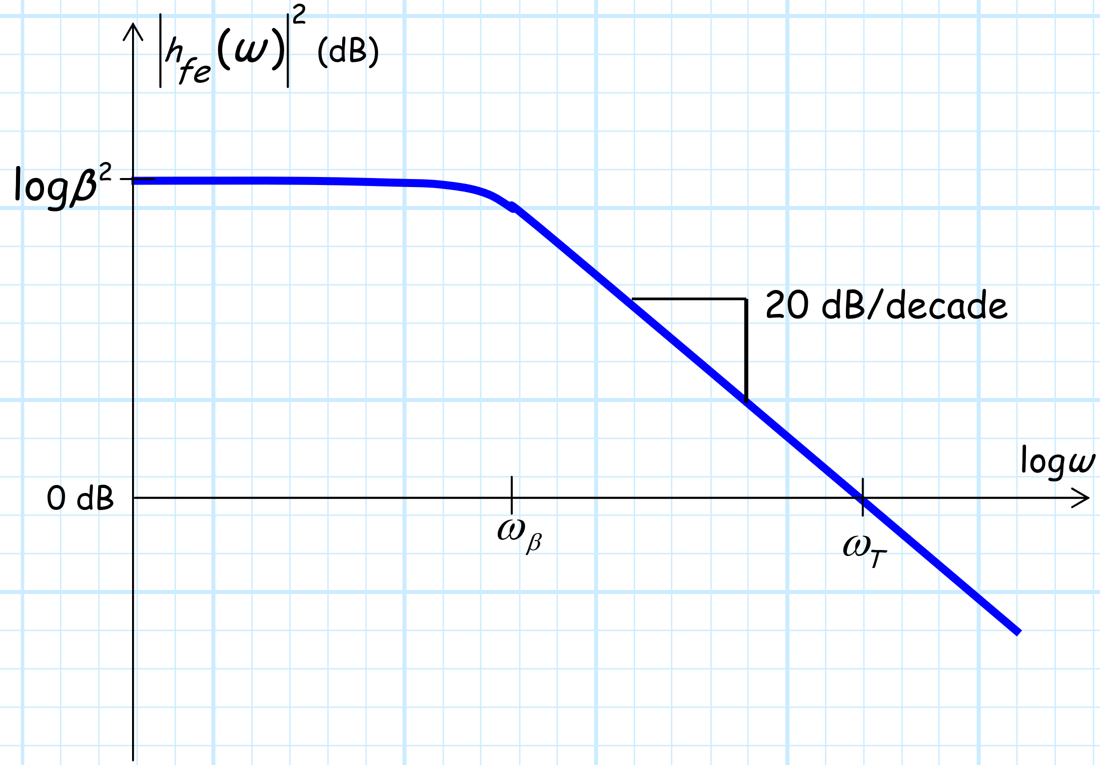
$$\omega_\beta = \frac{1}{r_\pi \left(C_\pi + C_\mu\right)}$$

Therefore:

$$h_{fe}(\omega) \doteq \frac{i_c(\omega)}{i_b(\omega)} \approx \frac{\beta}{1 + j\,\omega/\omega_\beta}$$

This is how we define BJT bandwidth

Plotting the magnitude of $h_{fe}(\omega)$ (i.e., $\left|h_{fe}(\omega)\right|^2$) we find that:



We see that for frequencies **less** than the 3 dB break frequency, the value of $h_{fe}(\omega)$ is approximately equal to **beta**:

$$h_{fe}(\omega) \approx \beta \qquad\qquad \omega < \omega_\beta$$

This should SO remind you of op-amps

Note then for frequencies greater than this break frequency:

$$h_{fe}(\omega) = \frac{\beta}{1 + j\,\omega / \omega_\beta}$$

$$\approx j\,\frac{\beta\,\omega_\beta}{\omega} \qquad \omega > \omega_\beta$$

Note then that $\left|h_{fe}(\omega)\right| = 1$ when $\omega = \beta\,\omega_\beta$.

We can thus define this frequency as ω_T, the **unity-gain** frequency:

$$\omega_T \doteq \beta\,\omega_\beta$$

so that:

$$\left|h_{fe}(\omega = \omega_T)\right| = 1.0$$

Déjà vu; all over again

We can therefore state that:

$$\boxed{\left|h_{fe}(\omega)\right| \approx \frac{\beta\,\omega_\beta}{\omega} = \frac{\omega_T}{\omega} \qquad \omega > \omega_\beta}$$

and also that:

$$\boxed{\left|h_{fe}(\omega)\right| \approx \beta \qquad \omega < \omega_\beta}$$