Amplifier Bandwidth

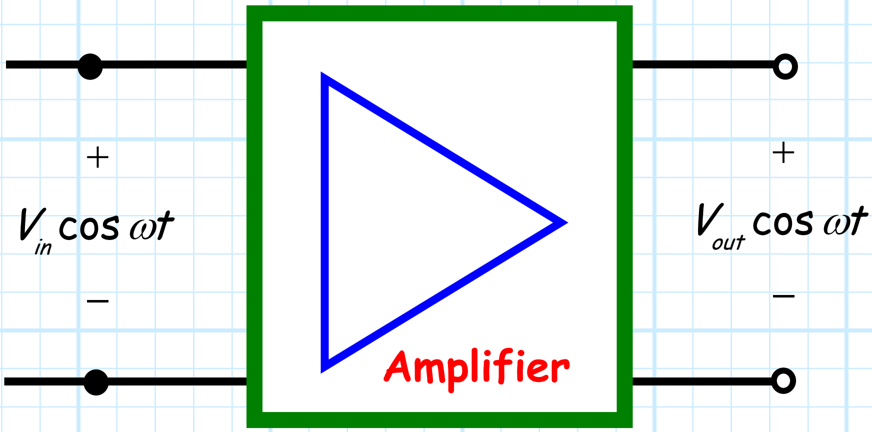
BJT amplifiers are band-limited devices—in other words, they exhibit a **finite bandwidth**.

Q: *???*

A: Say the input to a BJT small-signal amplifier is **the** eigen function of **linear**, time-invariant system:

$$V_{in} \cos \omega t = V_{in} \operatorname{Re}\left\{ e^{-j\omega t} \right\}$$

Since the small-signal BJT amp is (approximately) a **linear** system, the output will likewise be **the** eigen function—an **undistorted** sinusoidal function of precisely the same frequency ω as the input!



Q: *Of course that's true! We know that:*

$$v_{out}(t) = A_{vo}\, v_{in}(t)$$

Therefore the magnitudes of the input and output sinusoids are related as:

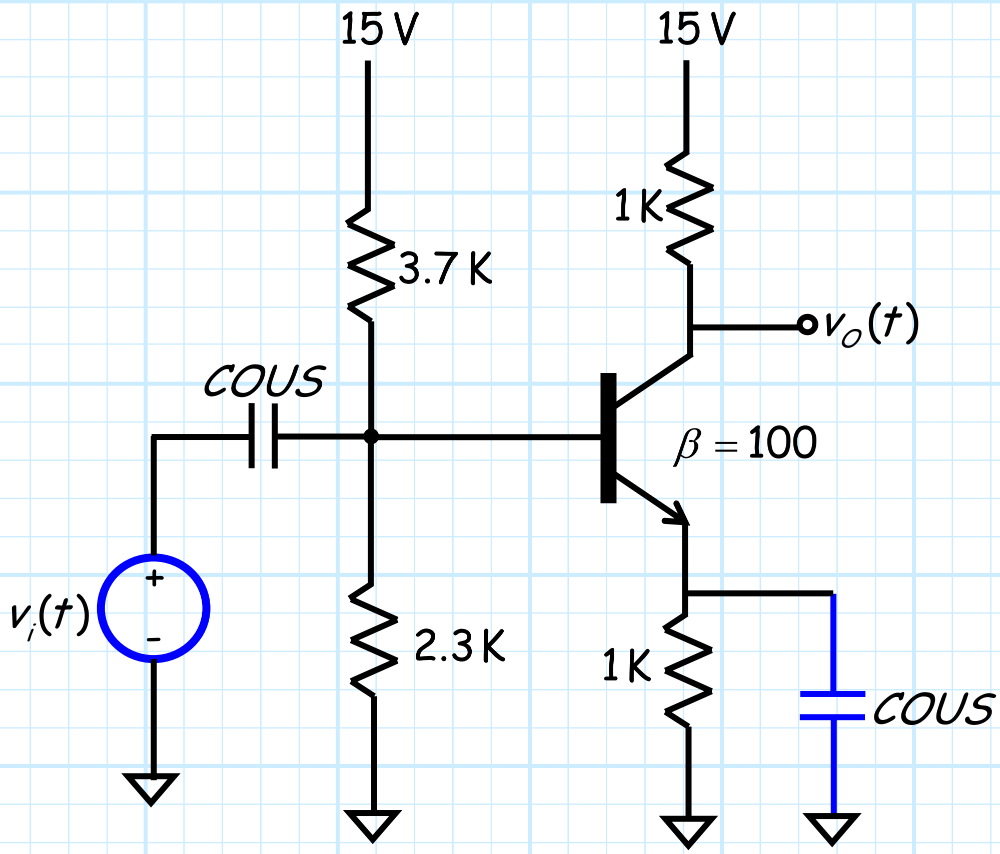
$$V_{out} = A_{vo}\, V_{in}$$

Right?

A: Not **necessarily**!

The small-signal, open-circuit voltage **gain** of a BJT amplifier **depends** on the **frequency** ω of the input signal!

Q: *Huh!?! We determined earlier that the small-signal voltage gain of this amplifier:*



was:

$$A_{vo} = \frac{v_o}{v_i} = -200$$

So then if the small-signal input is:

$$v_i(t) = V_{in} \cos \omega t$$

isn't the small-signal output simply:

$$v_o(t) = -200\, V_{in} \cos \omega t \qquad ?????????$$

A: Maybe—or maybe **not**!

Again, the gain of the amplifier is **frequency dependent**. We find that if ω is **too high** (i.e., large) or **too low** (i.e., small), then the output might be much less than the 200 times larger than the input (e.g., only 127.63 times larger than the input—Doh!).

Now, the signal frequencies ω for which

$$v_o(t) = -200\, V_{in} \cos \omega t$$

is an **accurate** statement, are frequencies that are said to lie **within the bandwidth** of this amplifier (ω is just right!).

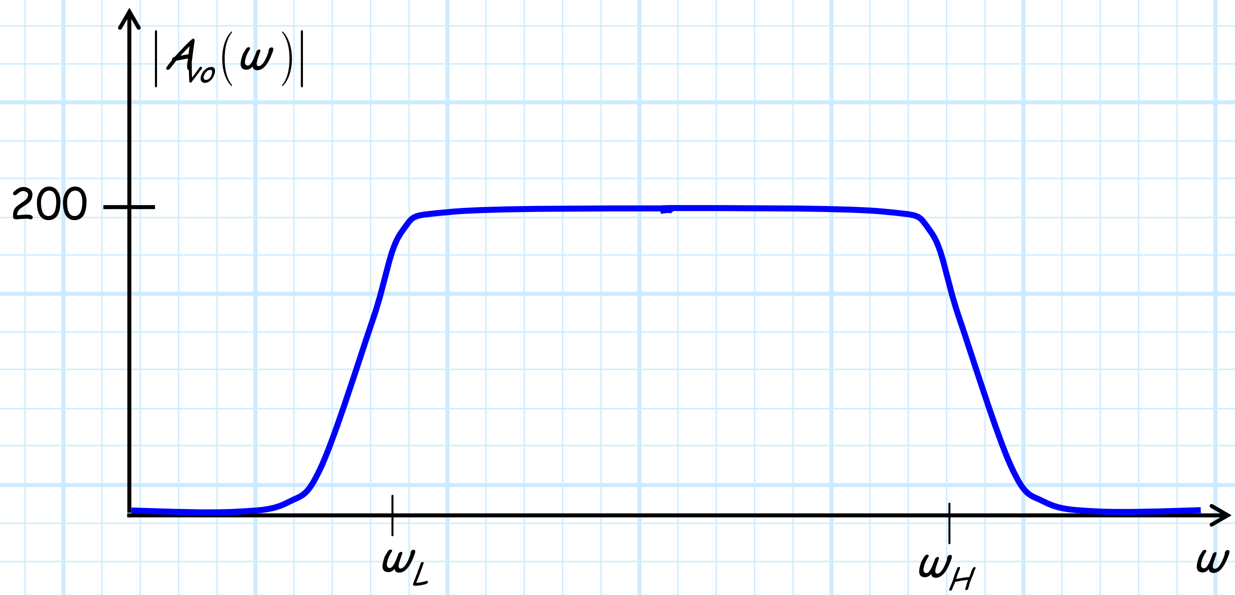
Conversely, frequencies ω for which:

$$v_o(t) \neq -200\, V_{in} \cos \omega t$$

are frequencies ω that lie **outside** this amplifier's bandwidth.

Fortunately, the frequencies that compose an amplifier's bandwidth typically form a **continuum**, such that the frequencies outside this bandwidth are either **higher** or **lower** than all frequencies within the bandwidth.

Perhaps a **plot** would help.



The frequencies **between** ω_L and ω_H thus lie within the **bandwidth** of the amplifier. The gain within the bandwidth is sometimes referred to as the **midband gain**.

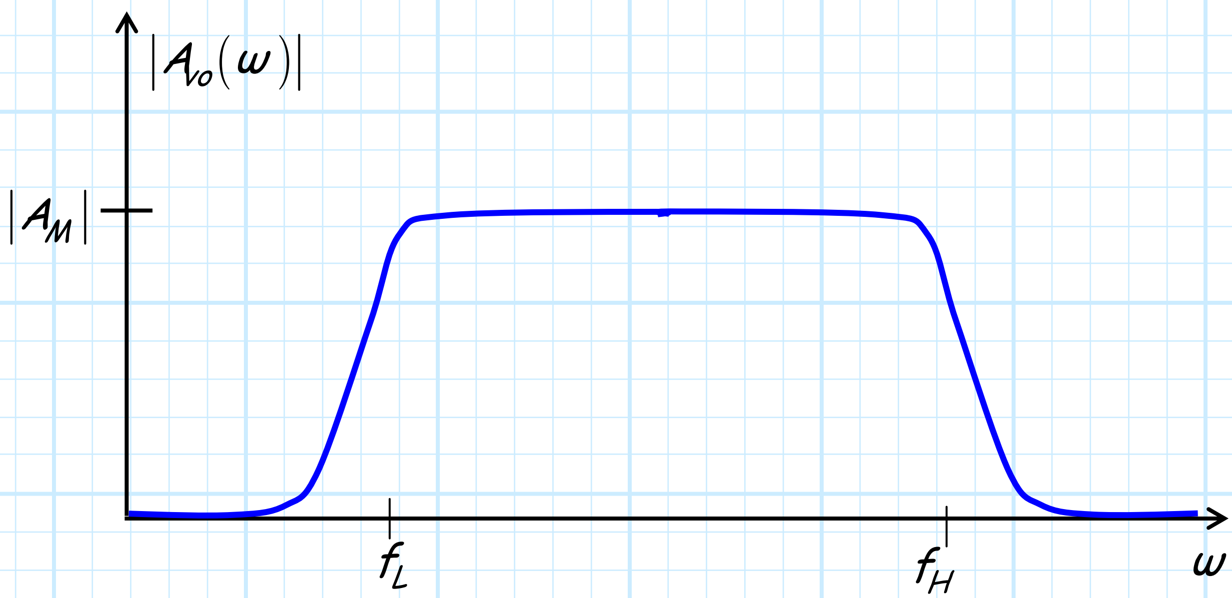
For signals with frequencies less than $\omega_L\,(f_L)$, the amplifier gain will be **less** than the midband gain—likewise for frequencies greater than $\omega_H\,(f_H)$.

Q: *So what then is the value:*

$$A_{vo} = \frac{v_o}{v_i} = -200$$

determined for the example amplifier? It doesn't seem to be a function of frequency!

A: The value -200 calculated for this amplifier is the **midband gain**—it's the gain exhibited for all signals that lie within the amplifier bandwidth. Your **book** at times uses the variable A_M to denote this value:



Q: *So it's actually the **midband gain** that we've been determining from our small-signal circuit analysis (e.g. $A_M = -200$)?*

A: That's **exactly** correct!

Q: *So how do we determine the* ***frequency dependent gain*** $A_{vo}(\omega)$*? More specifically, how do we determine midband gain* A_M*, along with* f_L *and* f_H *?*

A: The function $A_{vo}(\omega)$ is simply the **eigen value** of the **linear operator** relating the small-signal input and the small signal output:

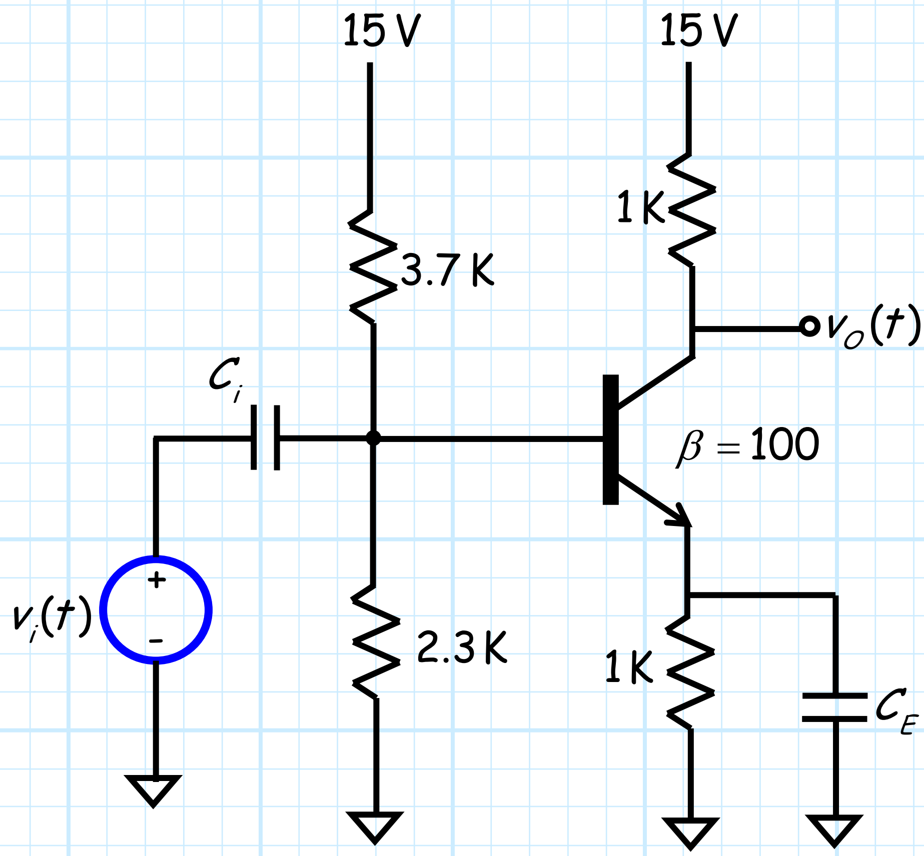
$$v_o(t) = \mathcal{L}\{v_i(t)\} \qquad \Rightarrow \qquad V_o(\omega) = A_{vo}(\omega)\, V_i(\omega)$$

Q: *Yikes! How do we determine the eigen value of this linear operator?*

A: We simply **analyze** the small-signal circuit, determining $V_o(\omega)$ in terms of $V_i(\omega)$.

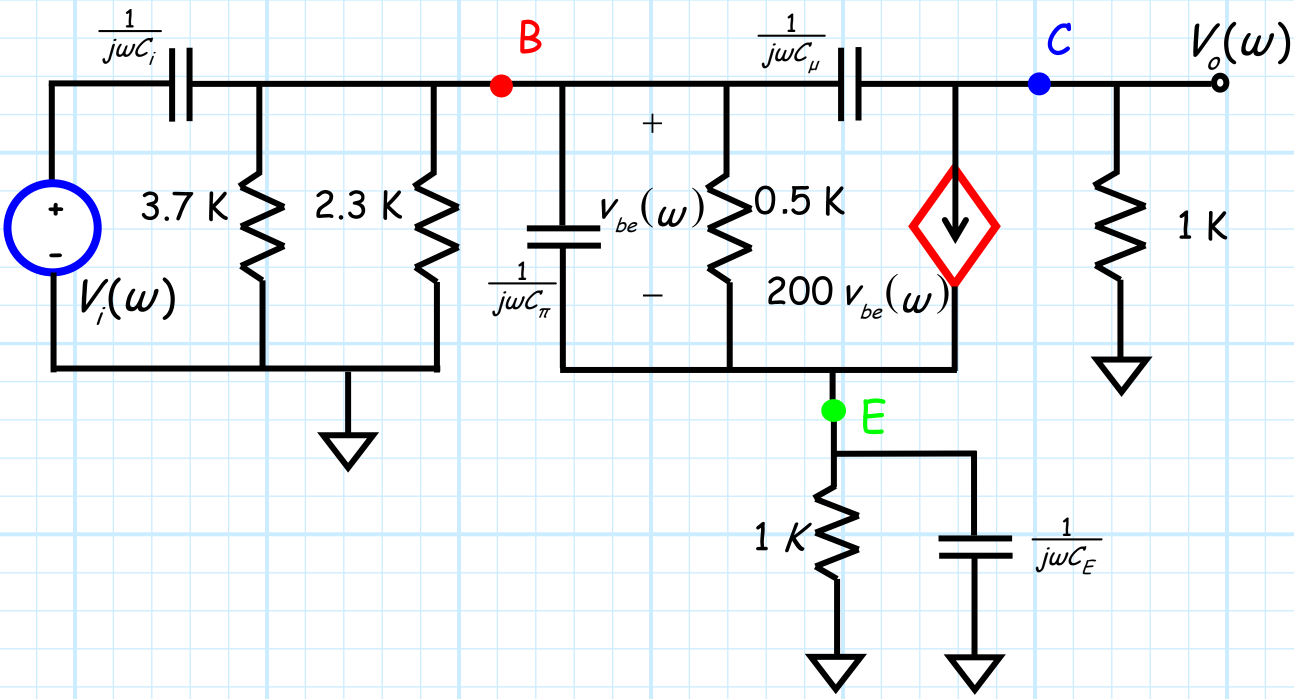
Specifically, we must e**xplicitly** consider the capacitance in the small-signal amplifier—**no longer** can we make **approximations**!

So, instead of **vaguely** labeling large capacitors as **C**apacitors **O**f **U**nusual **S**ize, let's explicitly consider the **exact values** of these large capacitors:



Likewise, we must consider the **parasitic capacitances** of the BJT—specifically C_μ and C_π.

The **small-signal circuit**—when we explicitly consider these capacitances—is thus:



We **analyze** this circuit to determine $V_o(\omega)$, and then the eigen value—the small-signal gain—is determined as:

$$A_{vo}(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

Q: *So what again is the meaning of $V_i(\omega)$ and $V_o(\omega)$?*

A: It's the **Fourier transform** of $v_i(t)$ and $v_o(t)$!

$$V_i(\omega) = \int_{-\infty}^{\infty} v_i(t)\, e^{j\omega t}\, dt \qquad \text{and} \qquad V_o(\omega) = \int_{-\infty}^{\infty} v_o(t)\, e^{j\omega t}\, dt$$

Q: So—I can't recall—what's the **relationship** between $v_i(t)$ and $v_o(t)$?

A: If:

$$V_o(\omega) = A_{vo}(\omega)\, V_i(\omega)$$

Then in the time domain, we find that the input and output are related by the **always enjoyable convolution** integral!!!

$$v_o(t) = \int_{-\infty}^{\infty} g(t-t')\, v_i(t')\, dt'$$

where the **impulse response** of the amplifier is of course:

$$g(t) = \int_{-\infty}^{\infty} A_{vo}(\omega)\, e^{-j\omega t}\, d\omega$$

Q: *What the heck? What happened to solutions like:*

$$v_o(t) = -200\, v_i(t) \quad ??$$

A: This result implies that the **impulse response** of the amplifier is:

$$g(t) = -200\, \delta(t)$$

Such that:

$$\begin{aligned} v_o(t) &= \int_{-\infty}^{\infty} g(t-t')\, v_i(t')\, dt' \\ &= -200 \int_{-\infty}^{\infty} \delta(t-t')\, v_i(t')\, dt' \\ &= -200\, v_i(t) \end{aligned}$$

Q: *You say that the result:*

$$v_o(t) = -200\, v_i(t)$$

***"implies"** that the impulse response of the amplifier is:*

$$g(t) = -200\, \delta(t)$$

*Are you saying the impulse response of the common-emitter example is **not** this function?*

A: It is definitely **not** that function. The impulse response

$$g(t) = -200\, \delta(t)$$

is **ideal**—the impulse response of an amplifier with an **infinite bandwidth**!

Q: *So **all** our small-signal analysis up to this point has been incorrect and **useless**???*

A: Not at all! The small-signal gain we have been evaluating up to this point (e.g., -200) is the amplifier **midband gain** A_M.

As long as the small-signal input $v_i(t)$ **resides completely** within the amplifier **bandwidth**, then the output will be:

$$v_o(t) = -200\, v_i(t)$$

The **problem** occurs when the input signal lies—at least partially—**outside** the amplifiers bandwidth.

In that case, we find that:

$$v_o(t) \neq -200\, v_i(t)$$

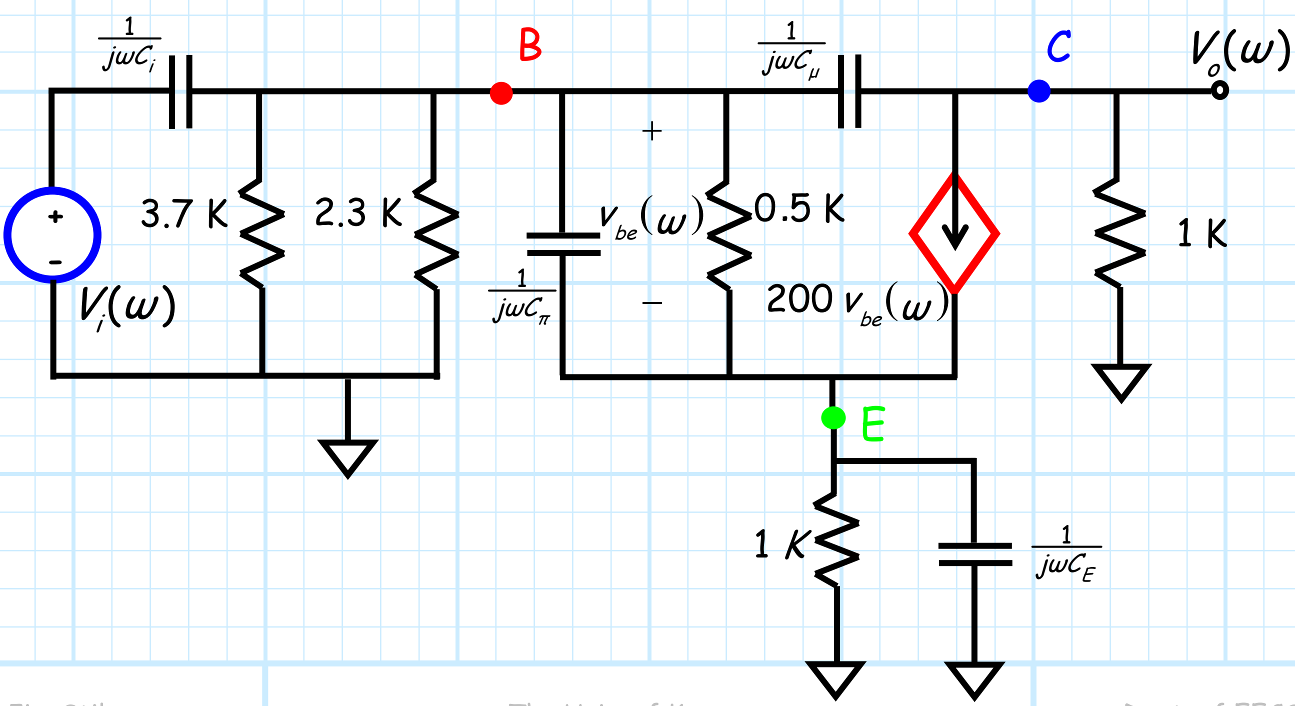
And instead:

$$v_o(t) = \int_{-\infty}^{\infty} g(t-t')\, v_i(t')\, dt'$$

where:

$$g(t) = \int_{-\infty}^{\infty} A_{vo}(\omega)\, e^{-j\omega t}\, d\omega$$

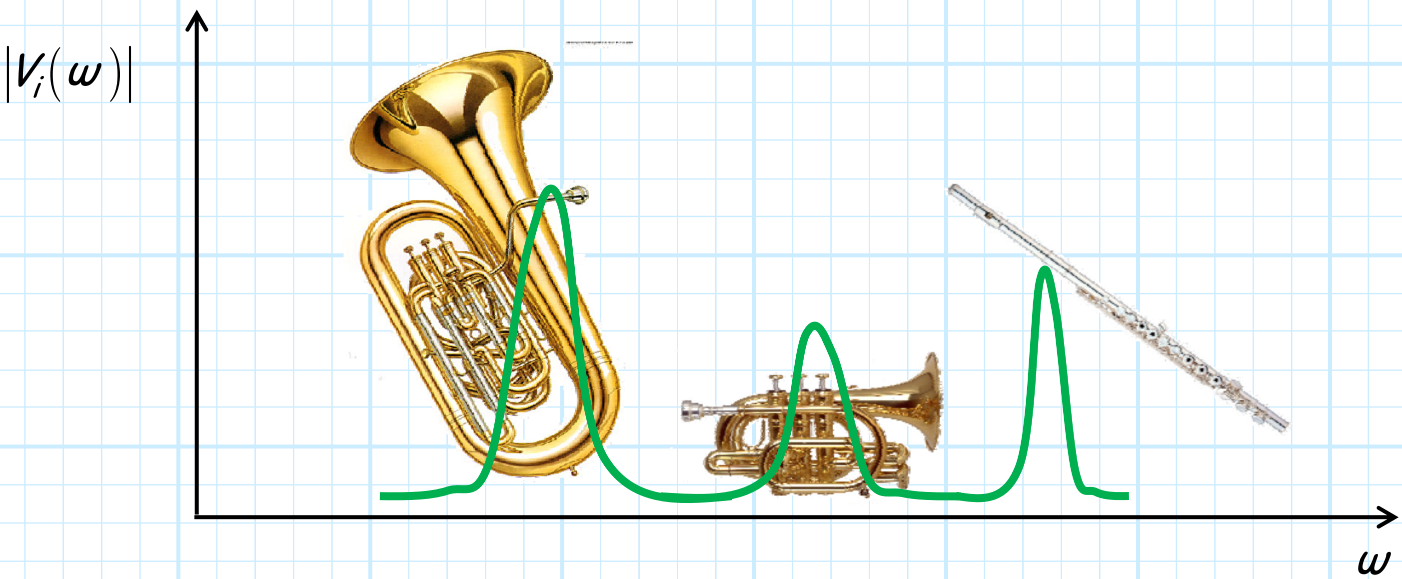
and the eigen value $A_{vo}(\omega) = V_o(\omega)/V_i(\omega)$ is determined by evaluating this small-signal circuit:



Q: *What do you mean when you say that a signal lies "**within** the amplifier bandwidth" or "**outside** the amplifier bandwidth? **How can we tell?***

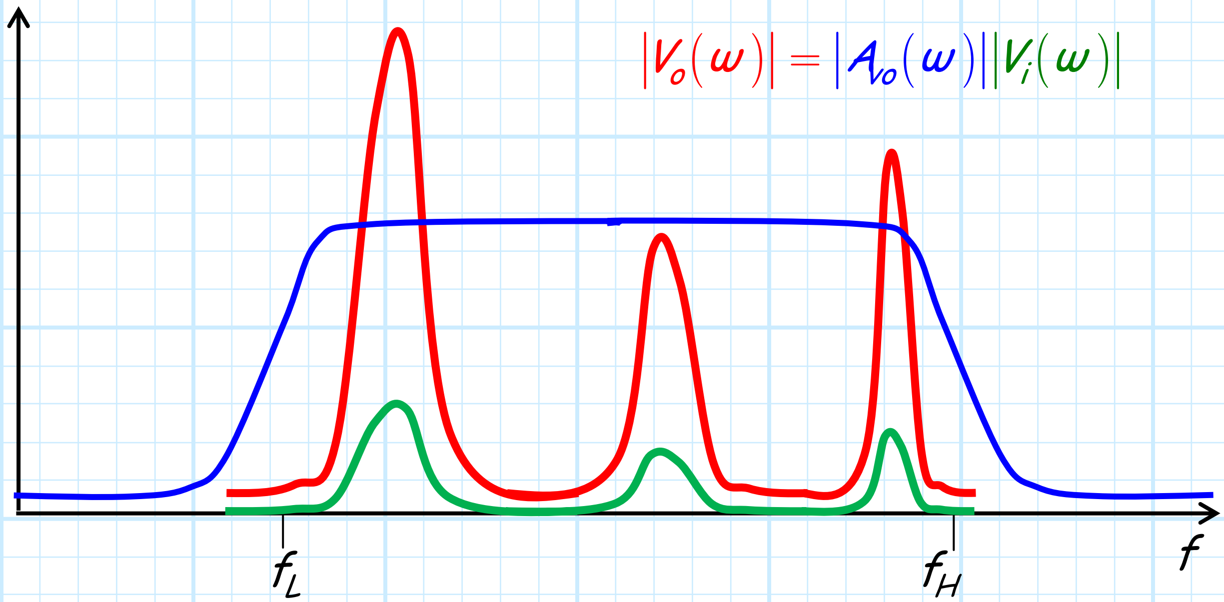
A: Use the Fourier Transform!

If we plot the magnitude of the Fourier Transform $V_i(\omega)$ of the input signal $v_i(t)$, we can see the **spectrum** of the input signal:



For example, if you are attempting to amplify a signal representing the audio of **symphonic music**, the spectrum $|V_i(\omega)|$ will include **low-frequency** signals (e.g., from the tubas), **mid-range** frequency signals (e.g., from the trumpets), and **high-frequency** signals (e.g., from the flutes).

Now, if it is your desire to reproduce **exactly** this music at the output of your amplifier, then the amplifier bandwidth must be **wide enough** to include **all** these spectral components !



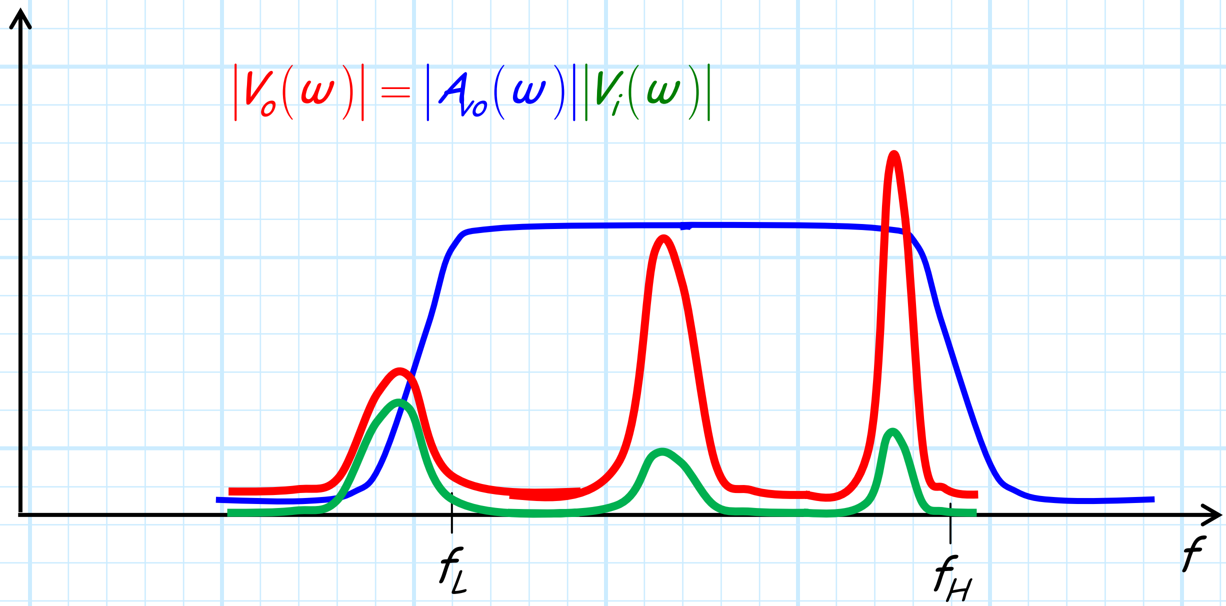
For the case above, the input signal resides **completely** within the bandwidth of the amplifier (i.e., between f_L and f_H), and so we find (for $A_M = -200$) that:

$$\left|V_o(\omega)\right| = (200)\left|V_i(\omega)\right|$$

and:

$$v_o(t) = -200\ v_i(t)$$

However, if the input spectrum resides (at least partially) **outside** the amplifier bandwidth, e.g.:



then we find that:

$$|V_o(\omega)| \neq (200)|V_i(\omega)|$$

and:

$$v_o(t) \neq -200\, v_i(t) \quad !!!!$$

Instead, we find the more general (and more **difficult!**) expressions:

$$V_o(\omega) = A_{vo}(\omega)\, V_i(\omega)$$

and:

$$v_o(t) = \int_{-\infty}^{\infty} g(t-t')\, v_i(t')\, dt'$$

where the **impulse response** of the amplifier is:

$$g(t) = \int_{-\infty}^{\infty} A_{vo}(\omega)\, e^{-j\omega t}\, d\omega$$

Q: *So just what* ***causes*** *this amplifier to have a finite bandwith?*

A: For mid-band frequencies f_m (i.e., between $f_L < f_m < f_H$), we will find that the Capacitors Of Unusual Size exhibit an impedance that is pretty small—approximately an **AC short** circuit:

$$\left|Z_{COUS}\left(\omega_m\right)\right| = \frac{1}{\omega_m C_{ous}} \cong 0$$

Likewise, the **tiny parasitic capacitances** C_μ and C_π exhibit an impedance that is very large for mid-band frequencies—approximately an **open** circuit:

$$\left|Z_{C_\pi}\left(\omega_m\right)\right| = \frac{1}{\omega_m C_\pi} \cong \infty$$

However, when the signal frequency ω drops too **low**, the COUS will **no longer** be a small-signal short.

The result is that the amplifier **gain is reduced**—the values of the COUS determine the **low-end amplifier bandwidth** f_L.

Likewise, when the signal frequency ω is **too high**, the parasitic caps will **no longer** be a small-signal **open**.

The result is that the amplifier **gain is reduced**—the values of the **parasitic capacitors** determine the **high-end amplifier bandwidth** f_H.