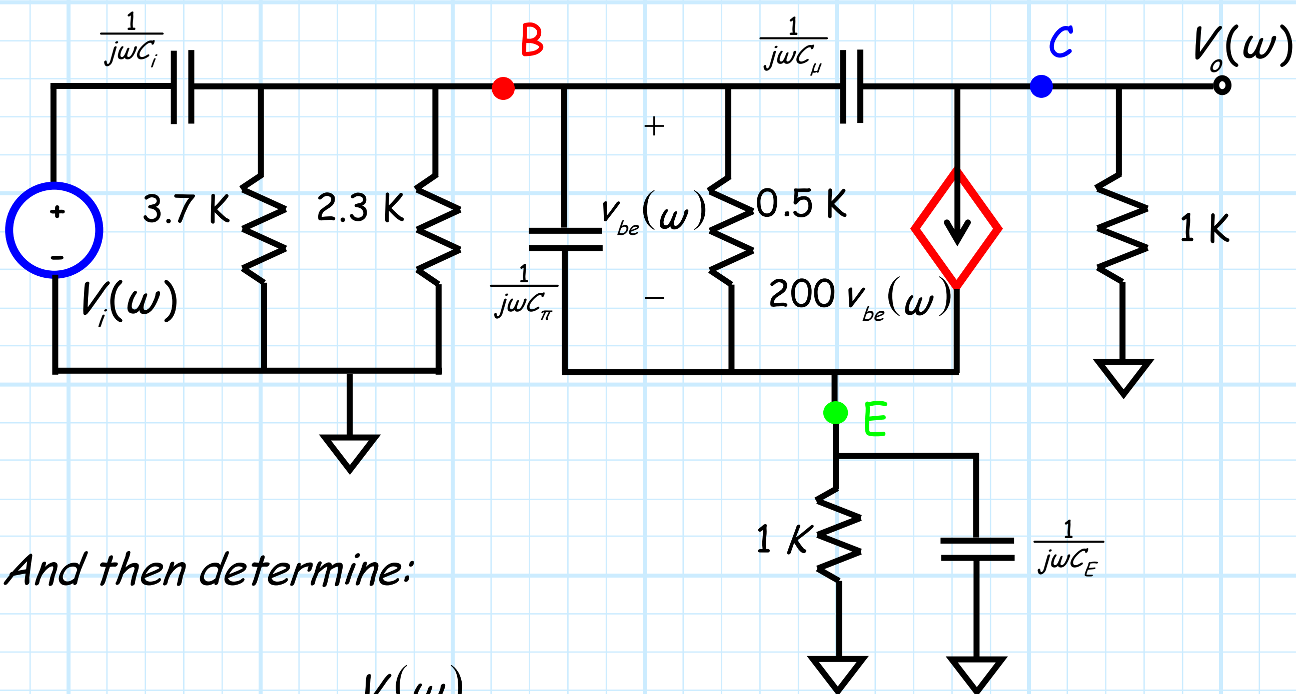
Low-Frequency Response

Q: *OK, I see how to determine mid-band gain, but what about determining amplifier* ***bandwidth****?*

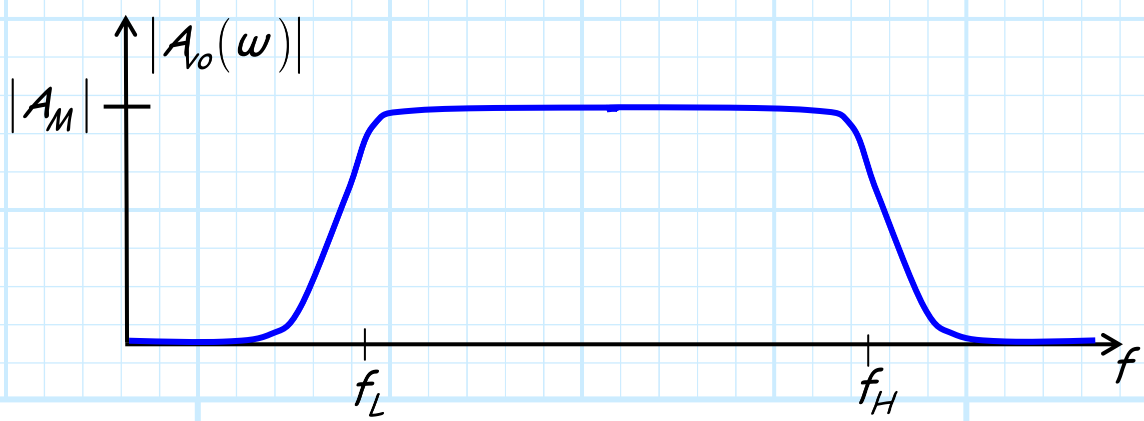
It seems like I have no alternative but to analyze the exact small-signal circuit (explicitly considering all capacitances):



And then determine:

$$A_{vo}(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

and then ***plot*** *the magnitude:*



And then from the plot determine the amplifier bandwidth (i.e., determine f_L and f_H)?

A: You **could** do all that, but there is an easier way.

An amplifier frequency response $A_{vo}(\omega)$ (i.e., its eigen value!) can generally be expressed as the product of **three** distinct terms:

$$A_{vo}(\omega) = F_L(\omega)\, A_M\, F_H(\omega)$$

The middle term is the of course the **mid-band gain**—a number that is not frequency dependent.

The function $F_L(\omega)$ describes the **low-frequency response** of the amplifier—from it we can determine the lower cutoff frequency f_L.

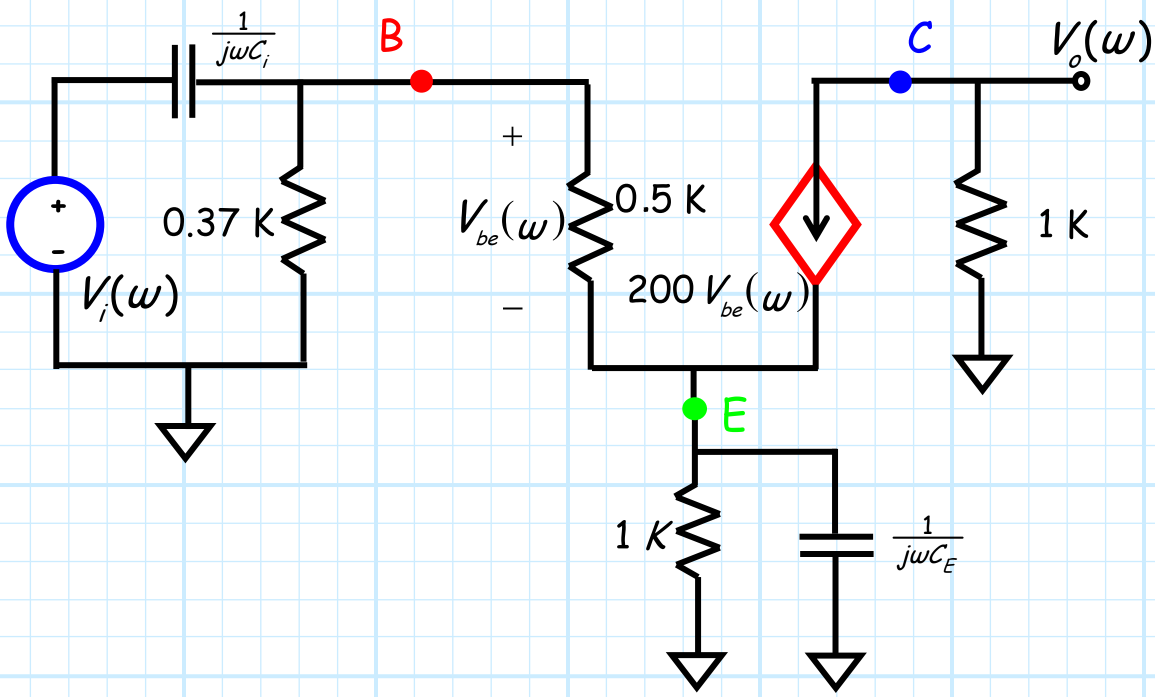
Conversely, the function $F_H(\omega)$ describes the **high-frequency** response of the amplifier—from it we can determine the upper cutoff frequency f_H.

Q: *So just how do we determine these functions $F_L(\omega)$ and $F_H(\omega)$??*

A: The low-frequency response $F_L(\omega)$ is dependent **only** on the large capacitors (COUS) in the amplifier circuit. In other words the parasitic capacitances have **no affect** on the low-frequency response.

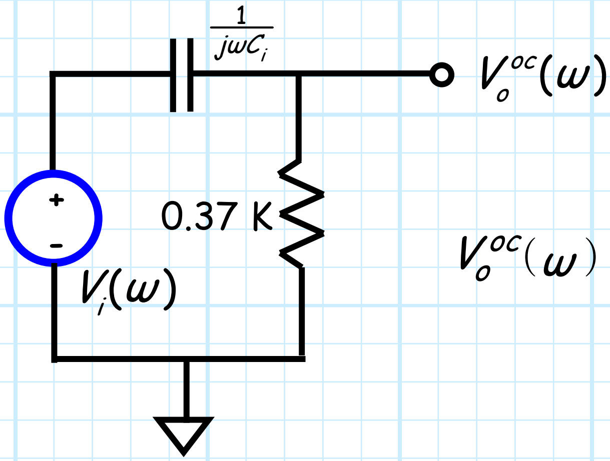
Thus, we simply "ignore" the **parasitic** capacitances when determining $F_L(\omega)$!

For example, say we include the COUS in our common-emitter example, but **ignore** C_μ and C_π. The resulting small-signal circuit is:



To simplify this analysis, we first determine the **Thevenin's** equivalent circuit of the portion of the circuit connected to the base.

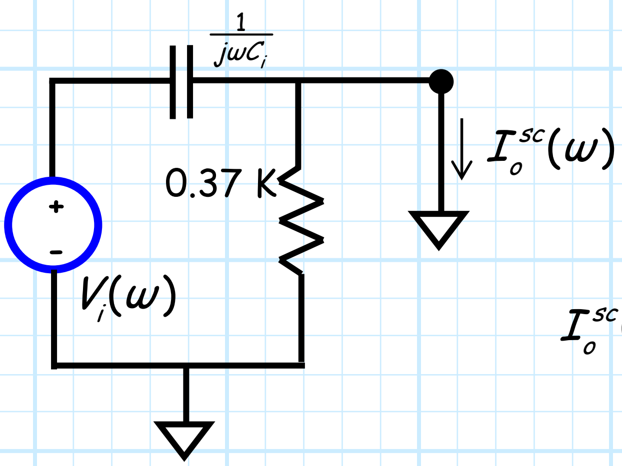
We start by finding the **open-circuit voltage**:



$$V_o^{oc}(\omega) = V_i(\omega)\left(\frac{0.37}{0.37 + \frac{1}{j\omega C_i}}\right)$$

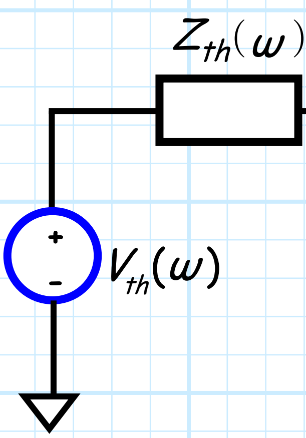
$$= V_i(\omega)\left(\frac{j\omega(0.37)C_i}{1 + j\omega(0.37)C_i}\right)$$

And the **short-circuit** output current is:



$$I_o^{sc}(\omega) = \frac{V_i(\omega)}{\frac{1}{j\omega C_i}} = V_i(\omega)\left(j\omega C_i\right)$$

And thus the **Thevenin's equivalent** source is:



$$V_{th}(\omega) = V_o^{oc}(\omega)$$

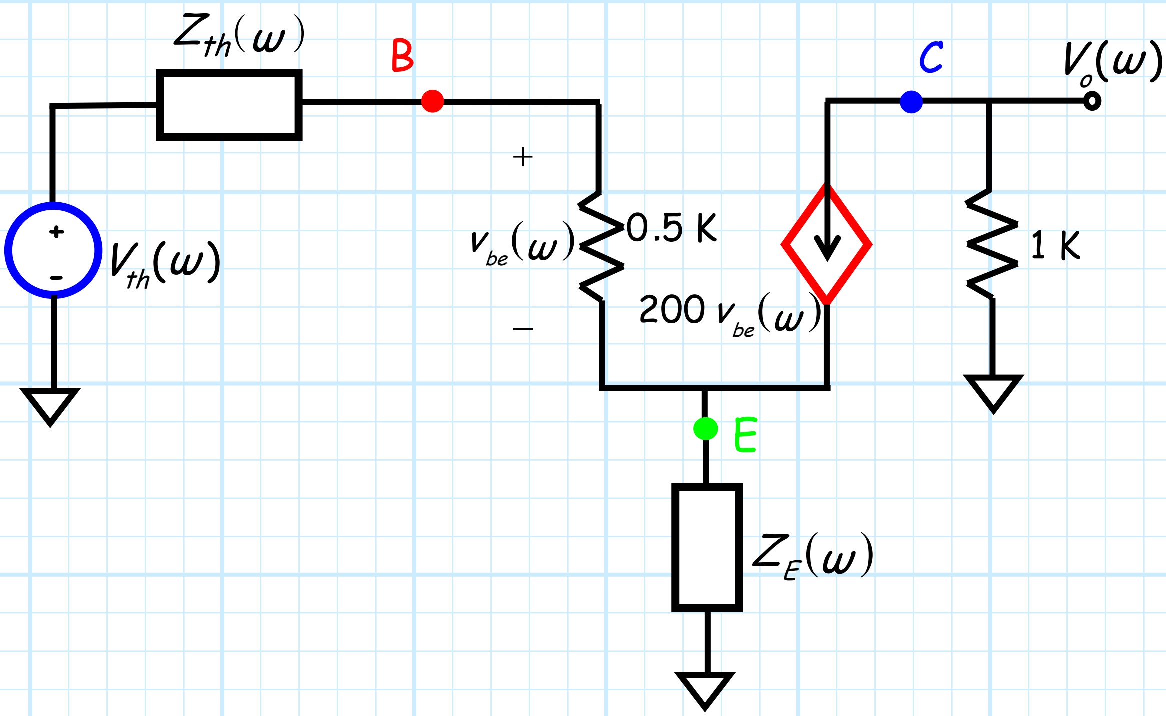
$$= V_i(\omega)\left(\frac{j\omega(0.37)C_i}{1 + j\omega(0.37)C_i}\right)$$

$$Z_{th}(\omega) = \frac{V_o^{oc}(\omega)}{I_o^{sc}(\omega)}$$
$$= \left(\frac{j\omega(0.37)C_i}{1+j\omega(0.37)C_i}\right)\left(\frac{1}{j\omega C_i}\right)$$
$$= \frac{(0.37)}{1+j\omega(0.37)C_i}$$

Likewise, the two parallel elements on the emitter terminal can be combined:

$$Z_E(\omega) = \frac{\frac{1}{j\omega C_E}}{1+\frac{1}{j\omega C_E}} = \frac{1}{1+j\omega C_E}$$

Thus, the small-signal circuit is **now**:



From KVL:

$$0 + V_{th} - I_b\left(Z_{th} + 0.5\right) - (\beta + 1) I_b Z_E = 0$$

$$\Rightarrow \qquad I_b = \frac{V_{th}}{Z_{th} + 0.5 + 101 Z_E}$$

From Ohm's Law:

$$V_{be} = 0.5 I_b$$

Therefore:

$$\begin{aligned} V_o(\omega) &= -200\, V_{be}\,(1) \\ &= -200(0.5)\frac{V_{th}(\omega)}{Z_{th} + 0.5 + 101 Z_E} \\ &= V_{th}(\omega)\left(\frac{-100}{Z_{th} + 0.5 + 101 Z_E}\right) \end{aligned}$$

Inserting the expressions for the Thevenin's equivalent source, as well as Z_E.

$$\begin{aligned} V_o(\omega) &= V_{th}(\omega)\left(\frac{-100}{Z_{th} + 0.5 + 101 Z_E}\right) \\ &= V_i(\omega)\left(\frac{jw(0.37)C_i}{1 + jw(0.37)C_i}\right)\left(\frac{-200}{\dfrac{2(0.37)}{1 + jw(0.37)C_i} + 1 + \dfrac{202}{1 + jwC_E}}\right) \end{aligned}$$

Now, it can be shown that:

$$\frac{1}{\dfrac{2(0.37)}{1+jw(0.37)C_i}+1+\dfrac{202}{1+jwC_E}} \cong \frac{jw(C_E/203)}{1+jw(C_E/203)}$$

Therefore:

$$V_o(\omega)=V_i(\omega)\left(\frac{jw(0.37)C_i}{1+jw(0.37)C_i}\right)\left(\frac{jw(C_E/203)}{1+jw(C_E/203)}\right)(-200)$$

And so:

$$A_{vo}(\omega)=\left(\frac{jw(0.37)C_i}{1+jw(0.37)C_i}\right)\left(\frac{jw(C_E/203)}{1+jw(C_E/203)}\right)(-200)$$

Now, since we are **ignoring** the parasitic capacitances, the function $F_H(\omega)$ that describes the high frequency response is:

$$F_H(\omega)=1$$

And so:

$$A_{vo}(\omega)=F_L(\omega)\,A_M\,F_H(\omega)=F_L(\omega)\,A_M$$

By inspection, we see for this example:

$A_M = -200$ ← We knew this **already**!

And:

$$F_L(\omega) = \left(\frac{j\omega(0.37)C_i}{1+j\omega(0.37)C_i}\right)\left(\frac{j\omega(C_E/203)}{1+j\omega(C_E/203)}\right)$$

Now, let's define:

$$\omega_{P1} = \frac{1}{0.37C_i} = \frac{2.7}{C_i} \qquad \text{and} \qquad \omega_{P2} = \frac{203}{C_E}$$

Thus,

$$F_L(\omega) = \left(\frac{j(\omega/\omega_{P1})}{1+j(\omega/\omega_{P1})}\right)\left(\frac{j(\omega/\omega_{P2})}{1+j(\omega/\omega_{P2})}\right)$$

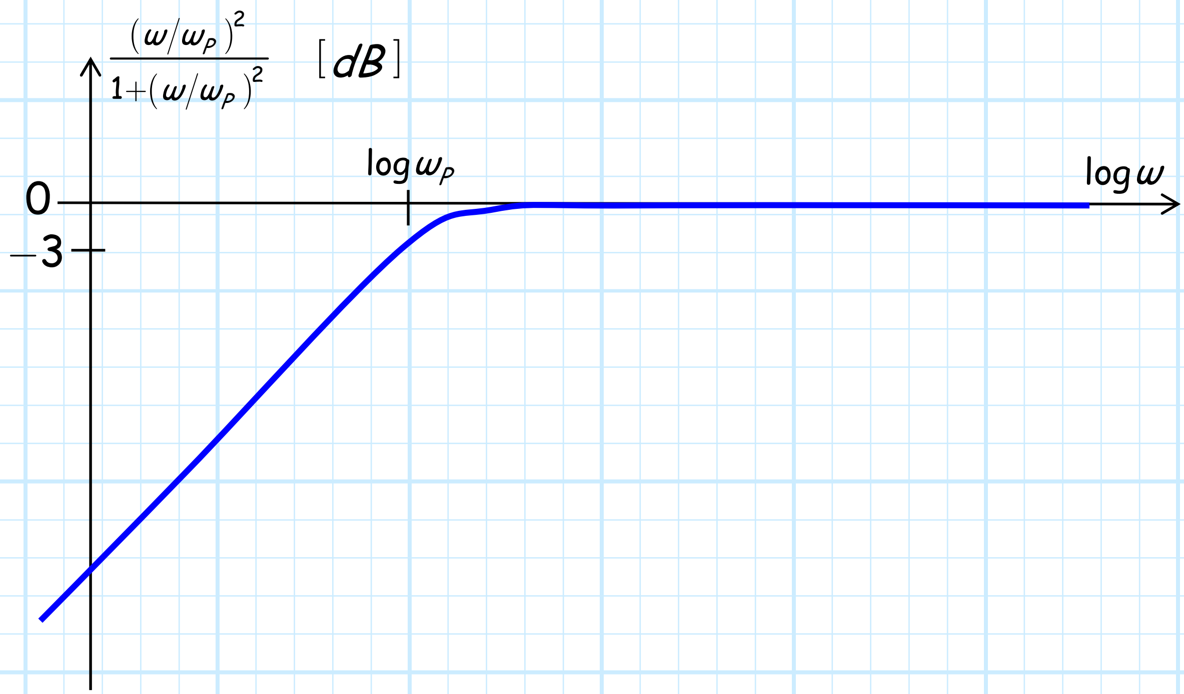
Now, functions of the type:

$$\left(\frac{j(\omega/\omega_P)}{1+j(\omega/\omega_P)}\right)$$

are **high-pass** functions:

$$\left|\frac{j(\omega/\omega_P)}{1+j(\omega/\omega_P)}\right|^2 = \frac{(\omega/\omega_P)^2}{1+(\omega/\omega_P)^2}$$

with a **3dB break frequency** of ω_P.



Thus:

$$\frac{(\omega/\omega_p)^2}{1+(\omega/\omega_p)^2} = \begin{cases} \cong 1.0 & \text{for } \omega > \omega_p \\ 0.5 & \text{for } \omega = \omega_p \\ 0 & \text{for } \omega = 0 \end{cases}$$

As a result, we find that the transfer function:

$$\begin{aligned} A_{vo}(\omega) &= F_L(\omega)\, A_M \\ &= \left(\frac{j(\omega/\omega_{p1})}{1+j(\omega/\omega_{p1})}\right)\left(\frac{j(\omega/\omega_{p2})}{1+j(\omega/\omega_{p2})}\right)(-200) \end{aligned}$$

will be **approximately** equal to the midband gain $A_M = -200$ for all frequencies ω that are greater than **both** ω_{p1} **and** ω_{p2}.

I.E.,:

$$A_{vo}(\omega) \cong A_M = -200 \qquad \text{if } \omega > \omega_{P1} \textbf{ and } \omega > \omega_{P2}$$

Hopefully, it is **now** apparent (please tell me it is!) that the lower end of the amplifier bandwidth—specified by frequency ω_L—is the determined by the **larger** of the two frequencies ω_{P1} and ω_{P2}!

The **larger** of the two frequencies is called the **dominant pole** of the transfer function $F_L(\omega)$.

For our example—comparing the two frequencies ω_{P1} and ω_{P2}:

$$\omega_{P1} = \frac{1}{0.37C_i} = \frac{2.7}{C_i} \qquad \text{and} \qquad \omega_{P2} = \frac{203}{C_E}$$

it is apparent that the **larger** of the two (the dominant pole!) is likely ω_{P2}—that darn **emitter capacitor** is the key!

Say we want the common-emitter amplifier in this circuit to have a bandwidth that extends down to $f_L = 100\ Hz$

The emitter capacitor **must** therefore be:

$$2\pi f_L > \omega_{P2} = \frac{203}{C_E}$$

$$\Rightarrow C_E > \frac{203}{2\pi f_L} = \frac{203}{2\pi(1000)} = 32{,}300\mu F \quad !!!!$$

This certainly **is** a **C**apacitor **O**f **U**nusual **S**ize !