Consider now the MOSFET differential pair:

Note:

\[ V_{S1} = V_{S2} = V_S \]

and

\[ i_{D1} + i_{D2} = I \]

If each of the transistors are in saturation, then each drain current must be:

\[ i_{D1} = K (V_{GS1} - V_t)^2 \quad \text{and} \quad i_{D2} = K (V_{GS2} - V_t)^2 \]

where

\[ V_{GS1} = V_1 - V_S \quad \text{and} \quad V_{GS2} = V_2 - V_S \]
Therefore, we define the **differential input voltage** as:

\[ v_{GS1} - v_{GS2} = V_1 - V_2 = V_D \]

Now, doing just a bunch of algebra, we can combine all of the above equations to find the **drain current** for each transistor:

\[ i_{D1} = \frac{I}{2} + \sqrt{\frac{K}{I}} V_D \sqrt{1 - \frac{K}{2I} V_D^2} \]

\[ i_{D2} = \frac{I}{2} - \sqrt{\frac{K}{I}} V_D \sqrt{1 - \frac{K}{2I} V_D^2} \]

Note this result shows that the drain current of each transistor is dependent only on the **differential signal** \( V_D = V_1 - V_2 \) — the drain currents are independent of the **common-mode** (i.e., average) input signal!
Note then, that if:

\[ \nu_D = 0 \]

**Both** transistors will be in saturation and will share the available current 
**evenly:**

\[ i_{D1} = i_{D2} = \frac{I}{2} \]

\[ \nu_D < -\frac{\sqrt{2I}}{\sqrt{K}} \]

For this case, \( Q_1 \) is in **cutoff** and we find that:

\[ i_{D1} = 0 \]

whereas transistor \( Q_2 \) remains in **saturation** and “takes” all the available current:

\[ i_{D2} = I \]

\[ \nu_D > +\frac{\sqrt{2I}}{\sqrt{K}} \]

For this case, \( Q_2 \) is now in **cutoff** and we find that:

\[ i_{D2} = 0 \]
whereas $Q_1$ is the transistor that remains in saturation and “takes” all the available current:

$$i_{D1} = I$$

$$-\sqrt{\frac{2I}{K}} < V_D < +\sqrt{\frac{2I}{K}}$$

In this region, both transistors are in saturation, and the drain current through each transistor is described using the two equations given earlier. Plotting these equations, we find:

![Figure 7.6](image.png)

*Figure 7.6* Normalized plots of the currents in a MOSFET differential pair.

Note that the derivatives (i.e., slopes) of these curves are approximately constant in the center region of the plot (i.e., the region where $|V_D|$ is small).

This should not surprise us! If we look at the two equations for drain current:
\[ i_{d1} = \frac{I}{2} + \sqrt{\frac{KI}{2}} v_b \sqrt{1 - \frac{K}{2I} v_b^2} \]

\[ i_{d2} = \frac{I}{2} - \sqrt{\frac{KI}{2}} v_b \sqrt{1 - \frac{K}{2I} v_b^2} \]

we find that the last product term will approximately equal one:

\[ \sqrt{1 - \frac{K}{2I} v_b^2} \approx 1 \quad \text{when} \quad |v_b| \ll \sqrt{\frac{2I}{K}} \]

Thus, we can approximate the drain currents for the case when the differential voltage \(|v_b| \ll \sqrt{2I/K}\) as:

\[ i_{d1} \approx \frac{I}{2} + \sqrt{\frac{KI}{2}} v_b \]

\[ i_{d2} \approx \frac{I}{2} - \sqrt{\frac{KI}{2}} v_b \]

Now we can find the slope of the curves above by taking the derivative with respect to differential voltage \(v_b\):

\[ \frac{d}{dv_b} i_{d1} \approx +\sqrt{\frac{KI}{2}} \]

\[ \frac{d}{dv_b} i_{d2} \approx -\sqrt{\frac{KI}{2}} \]
The slopes (derivatives) are approximately constant for small $v_D$—just as we observed they were in the plots above!

**Q:** These derivative values likewise tell us something very important about the behavior of the MOSFET differential pair, do you know what these derivatives tell us?

**A:**