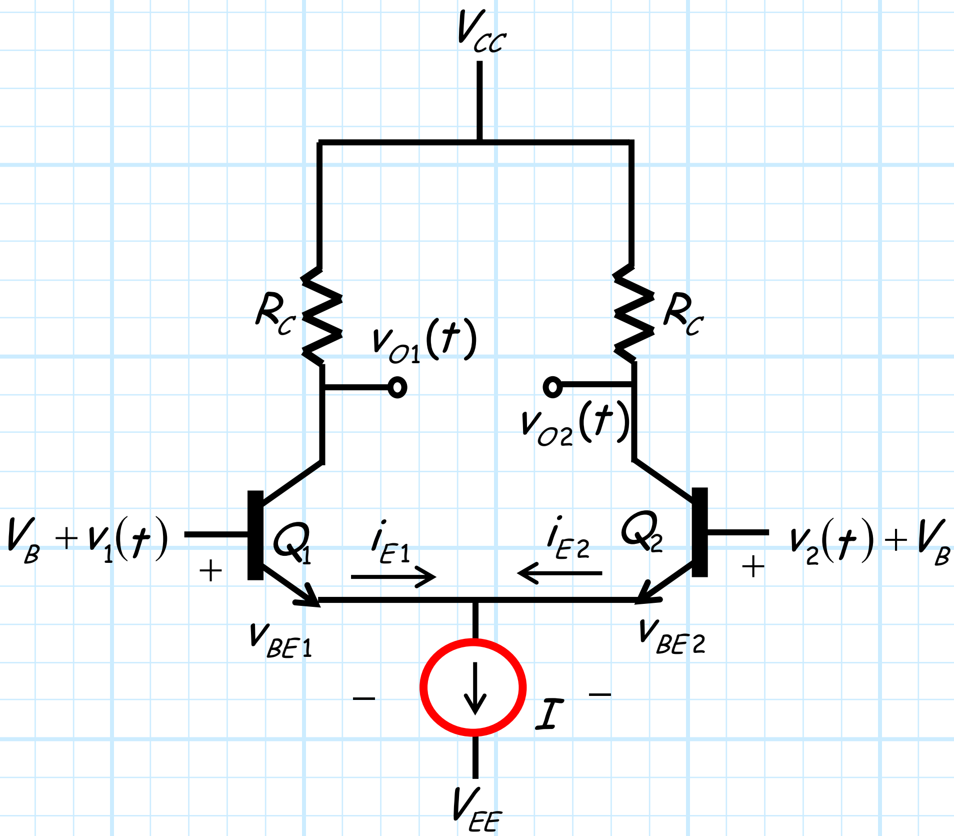
Small-Signal Analysis of BJT Differential Pairs

Now lets consider the case where each input of the differential pair consists of an **identical DC bias** term V_B, and also an AC small-signal component (i.e., $v_1(t)$ and $v_2(t)$)



As a result, the open-circuit output voltages will likewise have a **DC** and **small-signal** component.

Recall that we can alternatively express these two small-signal components in terms of their average (**common-mode**):

$$v_{cm}(t) \doteq \frac{v_1(t)+v_2(t)}{2}$$

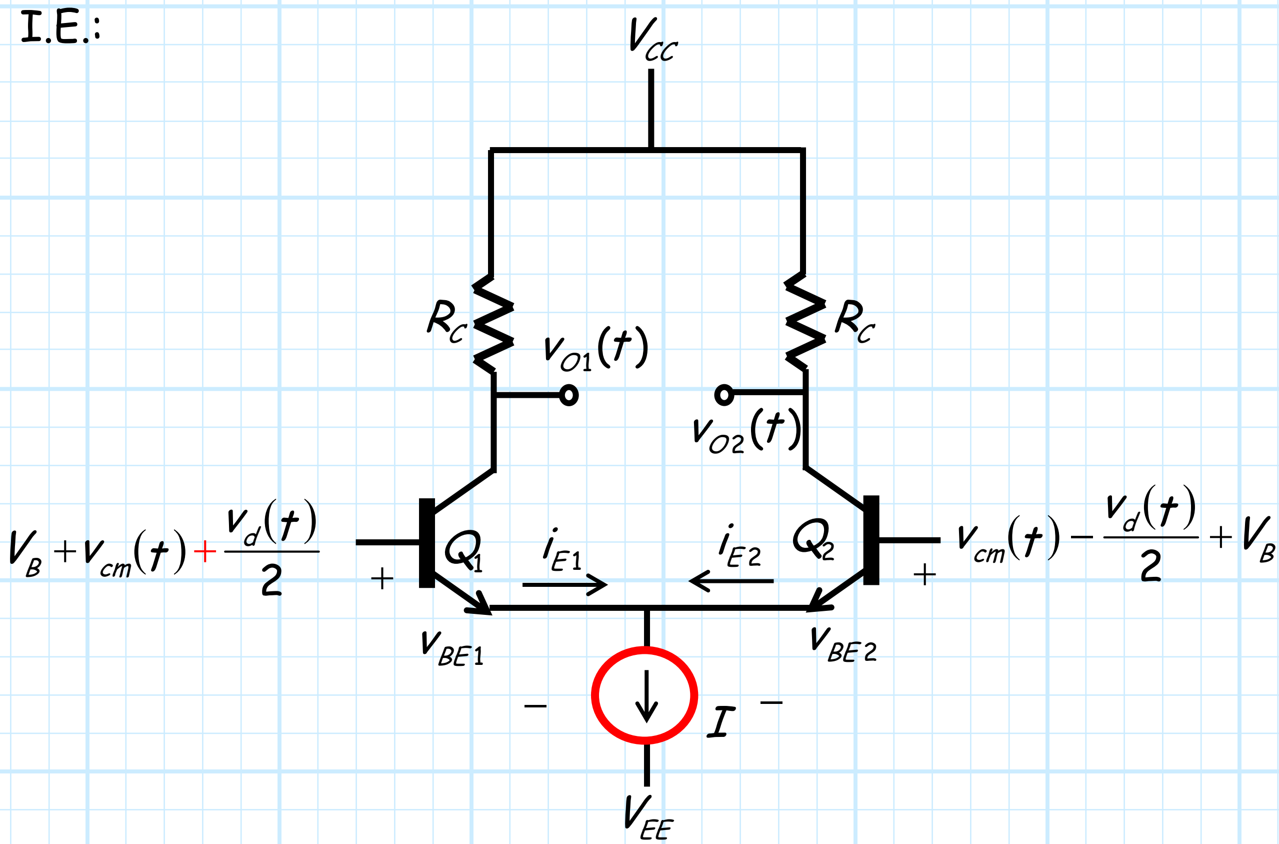
and their **differential mode**:

$$v_d(t) \doteq v_1(t)-v_2(t)$$

Such that:

$$v_1(t) = v_{cm}(t) + \frac{v_d(t)}{2} \qquad\qquad v_2(t) = v_{cm}(t) - \frac{v_d(t)}{2}$$

I.E.:



Now, let's determine the **small-signal voltage gain** of this amplifier!

Q: *What do you mean by **gain**? Is it:*

$$A_{vo} \doteq \frac{v_{o1}}{v_1} \quad \text{or} \quad A_{vo} \doteq \frac{v_{o2}}{v_2} \quad \text{or} \quad A_{vo} \doteq \frac{v_{o1}}{v_2} \quad \text{or} \quad A_{vo} \doteq \frac{v_{o2}}{v_1} \;???$$

A: Actually, **none** of those definitions!

This is a **differential** amplifier, so we typically define gain in terms of its **common-mode** (A_{cm}) and **differential** (A_d) gains:

$$A_{cm} \doteq \frac{v_{o1}}{v_{cm}} = \frac{v_{o2}}{v_{cm}} \qquad \text{and} \qquad A_d \doteq \frac{v_{o1}}{v_d} = -\frac{v_{o2}}{v_d}$$

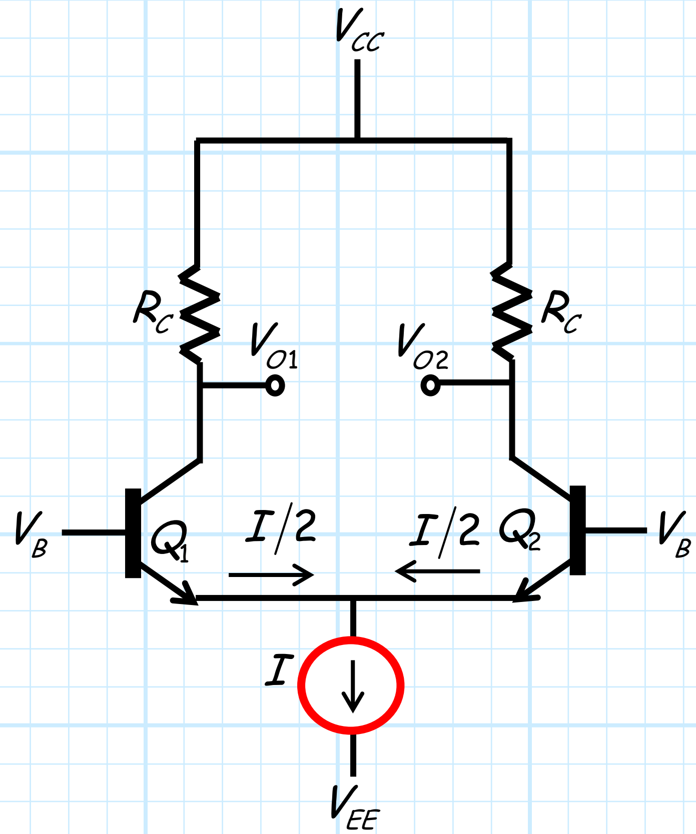
So that:

$$v_{o1}(t) = A_{cm}\, v_{cm}(t) + A_d\, v_d(t)$$

$$v_{o2}(t) = A_{cm}\, v_{cm}(t) - A_d\, v_d(t)$$

Q: *So how do we determine the differential **and** common-mode gains?*

A: The first step—of course—is to accomplish a **DC analysis**; turn **off** the small-signal sources!



This DC analysis is quite **simple**!

1. Since the DC base voltage V_B is the same for each transistor, we know the two emitter currents will each be:

$$I_{E1} = I_{E2} = \frac{I}{2}$$

We know **one** current, we know em' **all**!

$$I_{C1} = I_{C2} = \alpha \frac{I}{2}$$

$$I_{B1} = I_{B2} = \frac{I}{2} \frac{1}{(\beta + 1)}$$

Likewise, for the BJTs to be in **active** mode, we know that:

$$V_{CB} > 0 \qquad \Rightarrow \qquad V_C > V_B .$$

From KVL, the **collector voltage** is:

$$V_C = V_{CC} - R_C I_C = V_{CC} - \alpha \frac{I}{2} R_C$$

Therefore, in order for the BJTs to be in the **active** mode:

$$V_{CC} - \alpha \frac{I}{2} R_C > V_B \qquad \Rightarrow \qquad I < 2\frac{V_{CC} - V_B}{\alpha R_C}$$

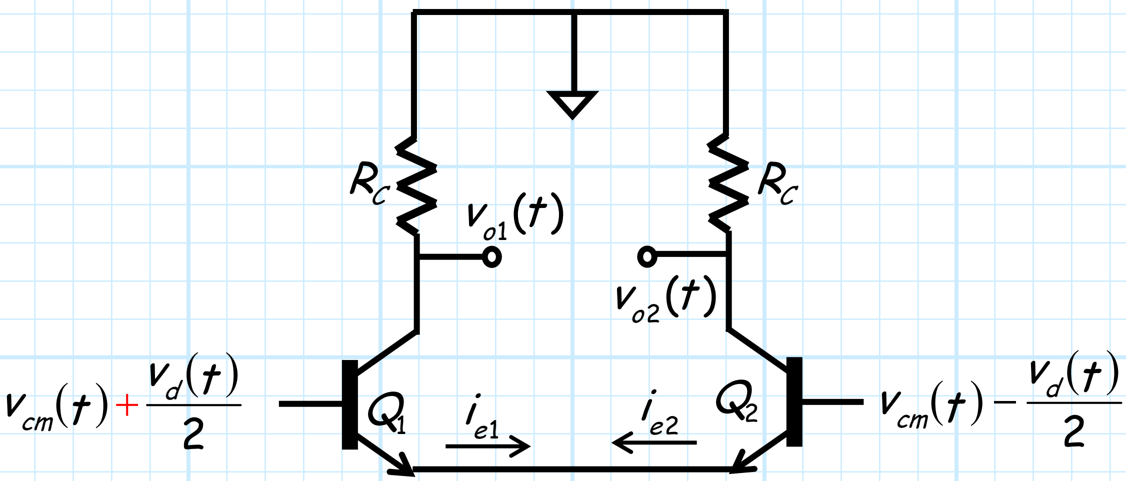
2. Now, we determine the **small-signal parameters** of each transistor:

$$g_{m1} = g_{m2} = \frac{I_C}{V_T} = \frac{\alpha}{2}\frac{I}{V_T}$$

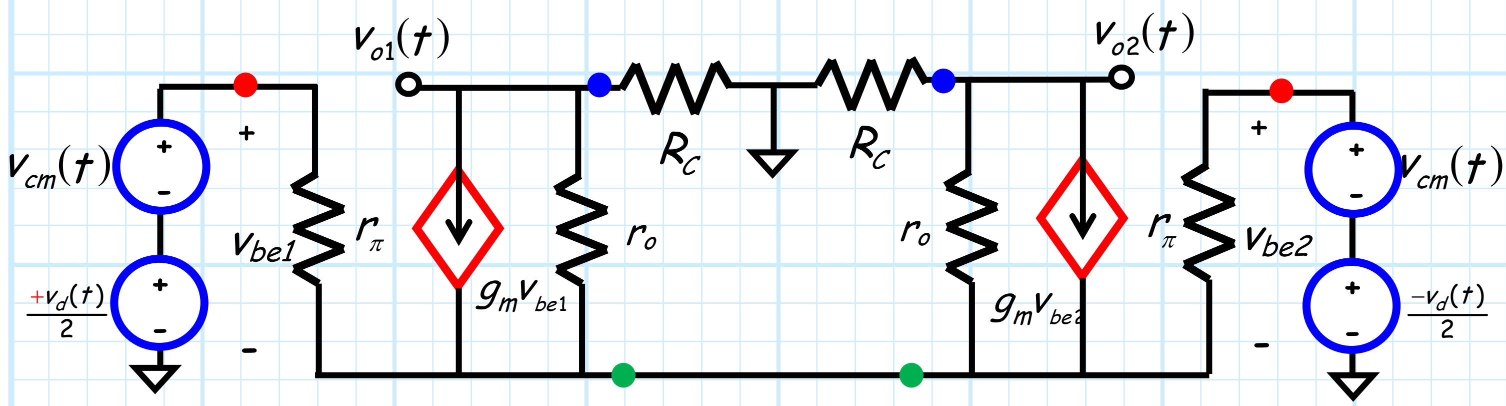
$$r_{\pi 1} = r_{\pi 2} = \frac{I_B}{V_T} = \frac{1}{2(\beta + 1)}\frac{I}{V_T}$$

$$r_{o1} = r_{o2} = \frac{2}{\alpha}\frac{V_A}{I}$$

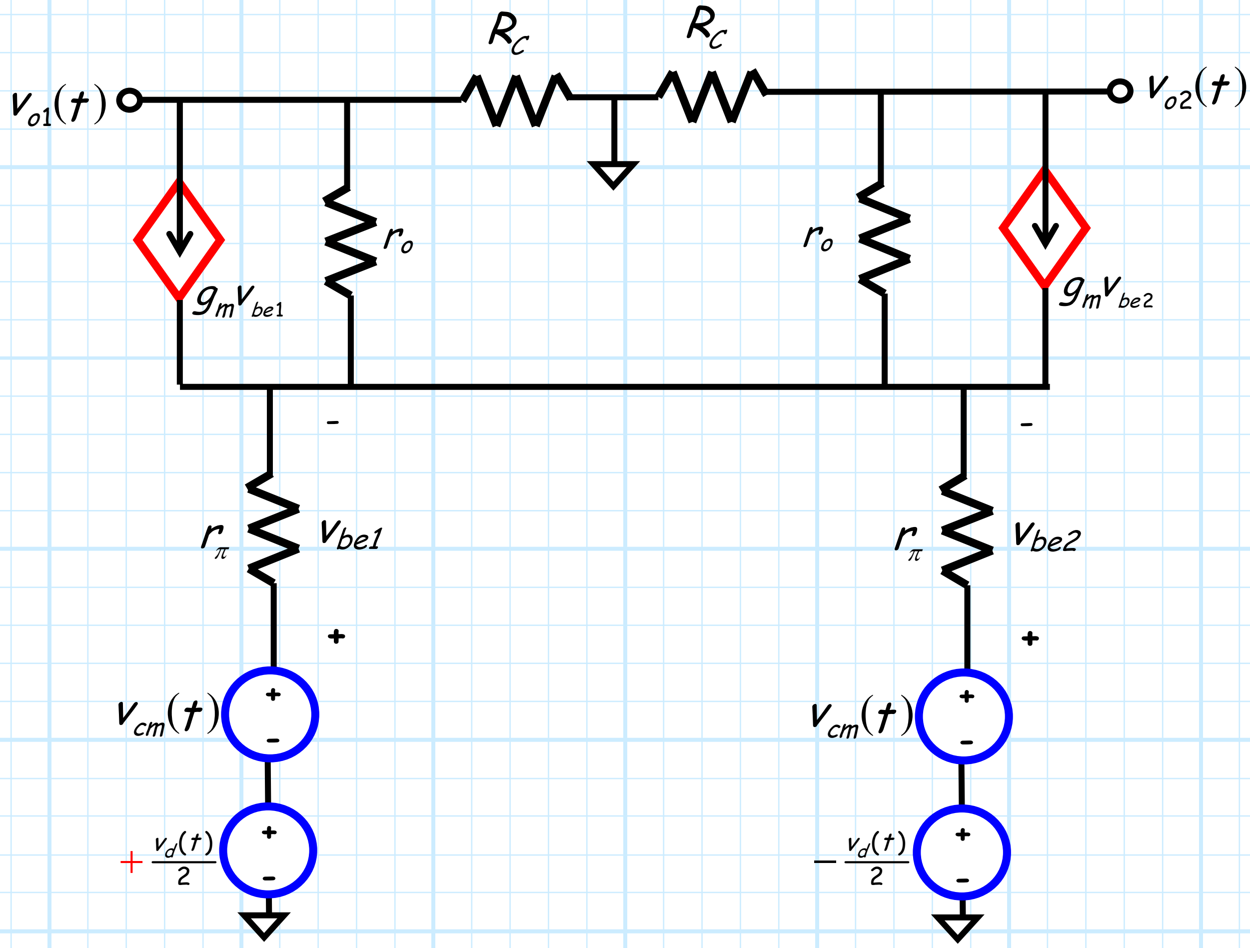
3. Turning **off** the DC sources:



And now inserting the **hybrid-pi** BJT model:



Now, **tidying** this schematic up a bit:



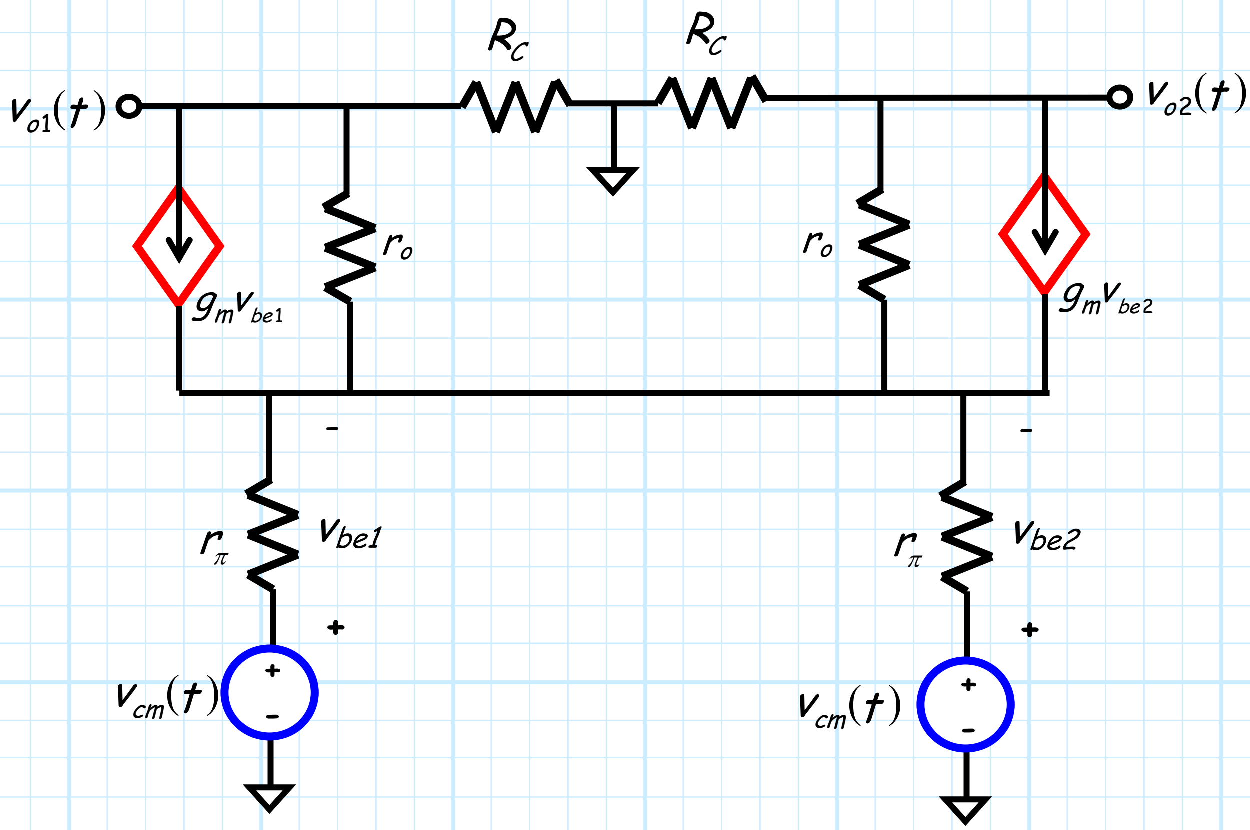
Q: *Yikes! How do we analyze this **mess**?*

A: In a word, **superposition**!

Q: *I see, we turn off three sources and analyze the circuit with the one remaining source on. We then move to the next source, until we have **four** separate analysis—then we add the results together, **right**?*

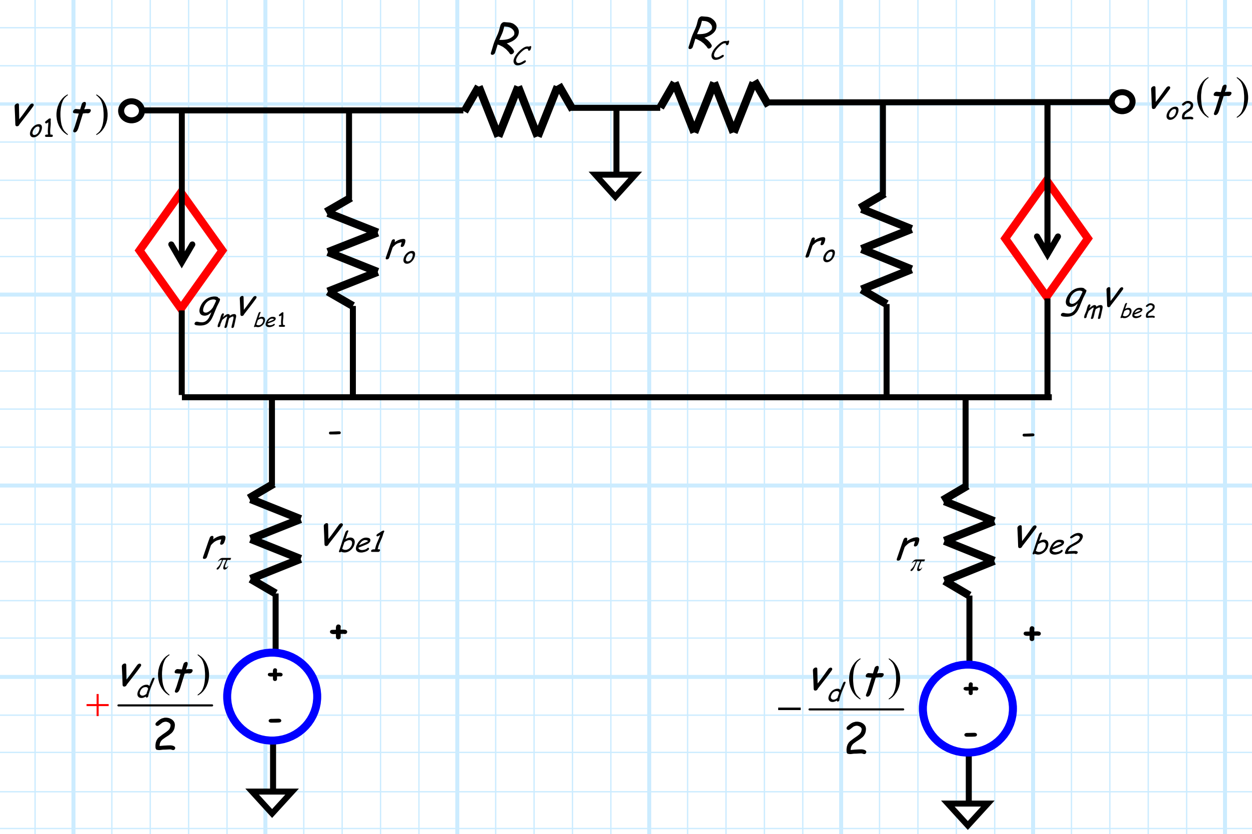
A: It's actually much **easier** than that!

We first turn off the **two** differential-mode sources, and analyze the circuit with only the two **remaining** (equal valued) **common-mode** sources.



From this analysis, we can determine things like the **common-mode gain** and input resistance!

We then turn **off** the two common-mode sources, and analyze the circuit with only the two (equal but opposite valued) **differential-mode** sources.

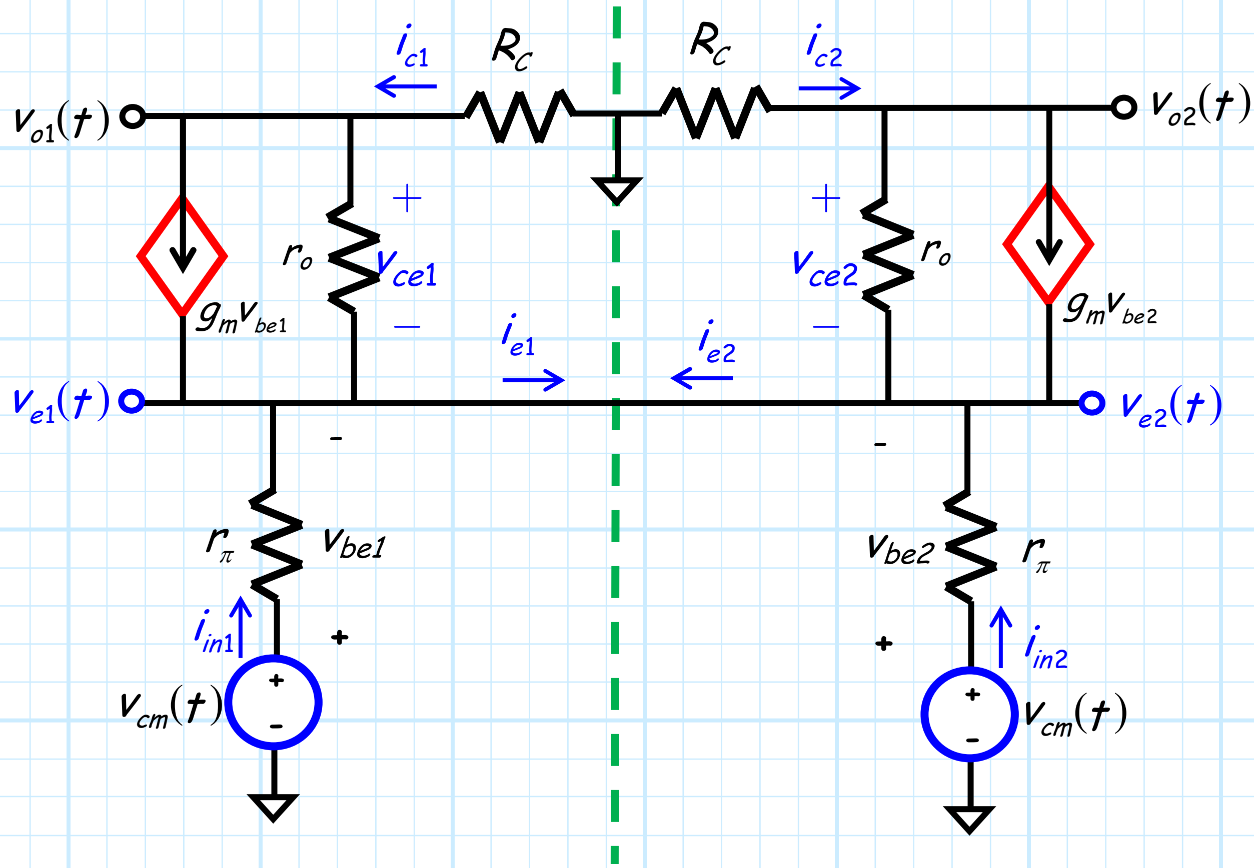


From this analysis, we can determine things like the **differential mode gain** and input resistance!

Q: *This still looks very **difficult!** How do we analyze these "differential" and "common-mode" circuits?*

A: The key is **circuit symmetry**.

We notice that the common-mode circuit has a perfect plane of **reflection** (i.e., bilateral) **symmetry**:



The left and right side of the circuit above are **mirror images** of each other (including the sources with **equal** value v_{cm}).

The two sides of the circuit a perfectly and precisely **equivalent**, and so the currents and voltages on each side of the circuit must likewise be perfectly and precisely **equal**!

For example:

$$v_{be1} = v_{be2} \qquad i_{in1} = i_{in2}$$
$$v_{o1} = v_{o2} \qquad \text{and} \qquad g_m v_{be1} = g_m v_{be2}$$
$$v_{ce1} = v_{ce2} \qquad i_{c1} = i_{c2}$$
$$v_{e1} = v_{e2} \qquad i_{e1} = i_{e2}$$

Q: *Wait! You say that—because of "circuit* ***symmetry****"—that:*

$$i_{e1} = i_{e2}.$$

But, just ***look*** *at the circuit; from* ***KCL*** *it is evident that:*

$$i_{e1} = -i_{e2}$$

How can ***both*** *statements be correct?*

A: **Both** statements **are** correct!

In fact, the statements (taken together) tell us what the small-signal emitter currents **must** be (for this common-mode circuit).

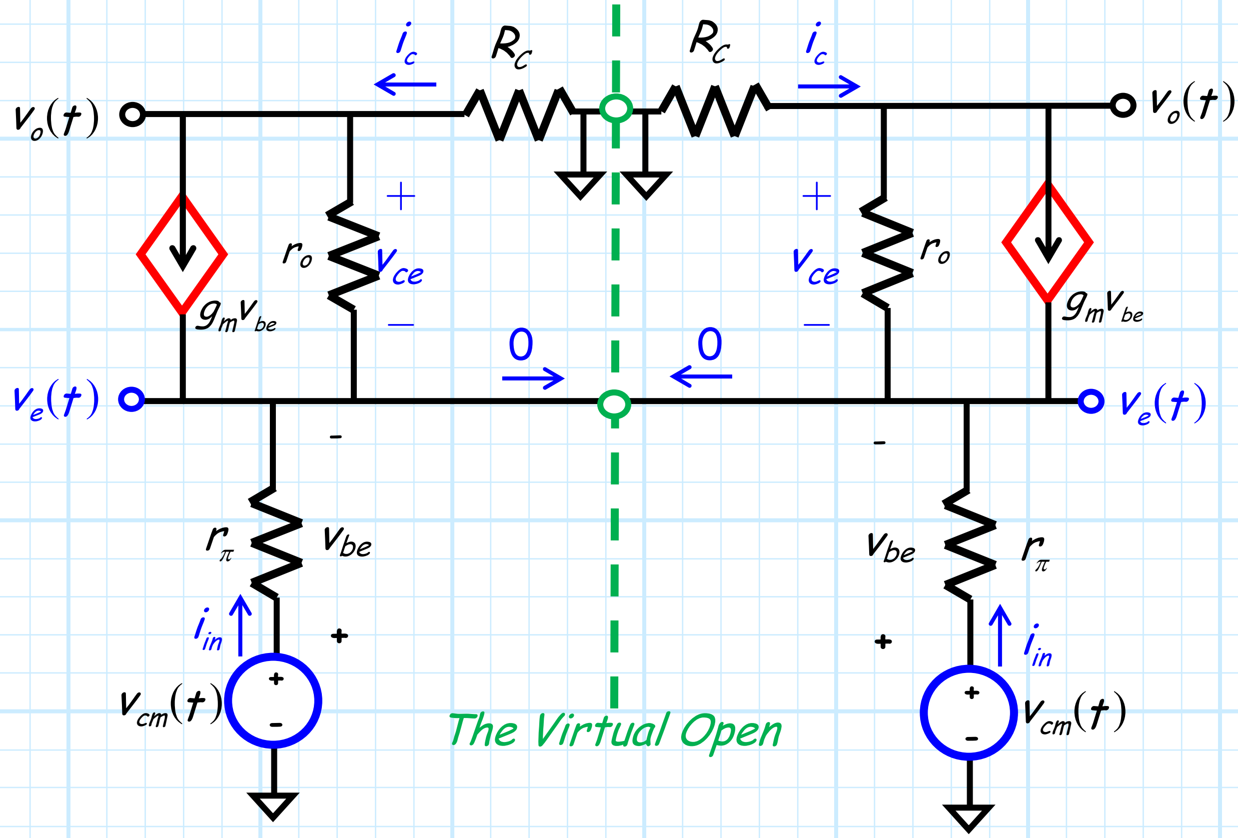
There is **only one possible solution** that satisfies the two equations—the common-mode, small-signal emitter currents must be equal to **zero**!

$$i_{e1} = i_{e2} = -i_{e2} = 0$$

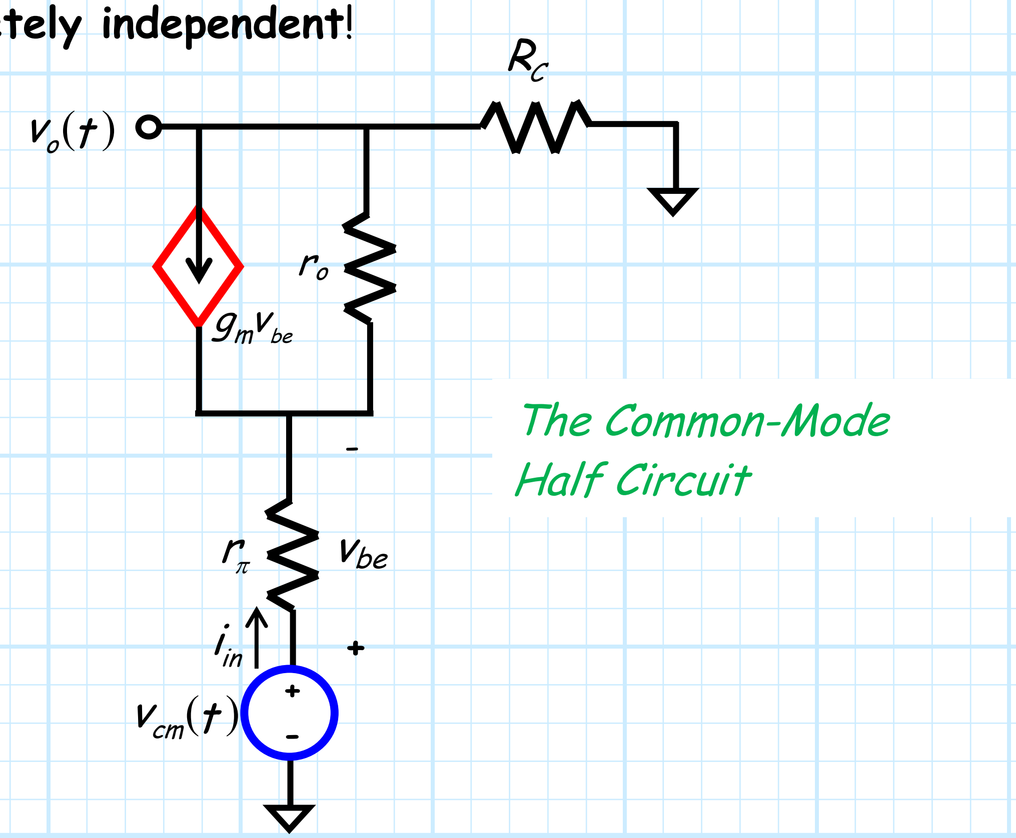
Hopefully this result is a bit obvious to **you**.

If a circuit possess a plane of perfect reflection (i.e., bilateral) symmetry, then **no current** will flow across the symmetric plane. If it **did**, then the symmetry would be **destroyed**!

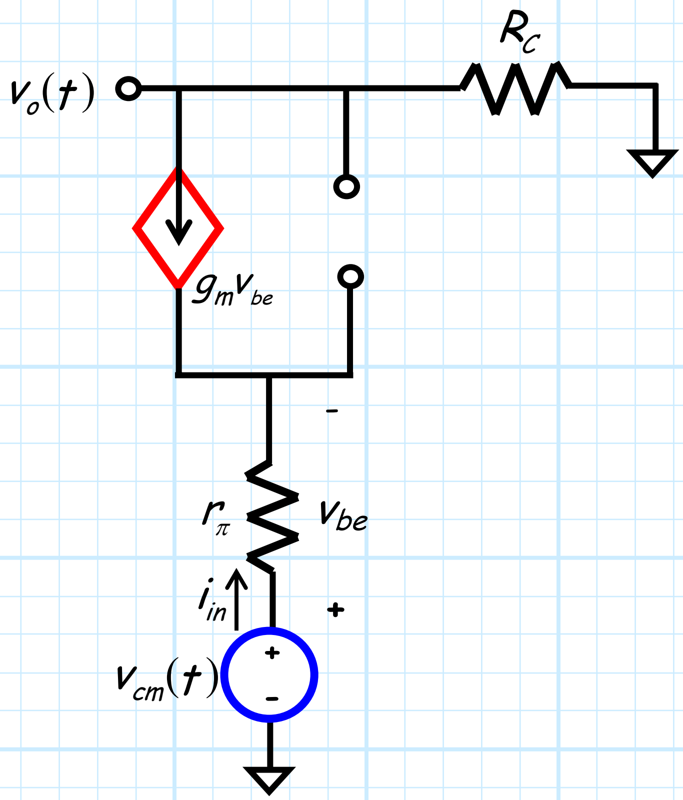
Thus, a plane of reflection symmetry in a circuit is known as **virtual open**—no current can flow across it!



Thus, we can take pair of scissors and cut this circuit into **two identical half-circuits**, without affecting **any** of the currents or voltages—the two circuits on either side of the virtual open are **completely independent**!



Now, since $r_o \gg r_\pi$ and $r_o \gg R_C$, we can simplify the circuit by approximating it as an open circuit:



Now, let's analyze this **half-circuit**!

From **Ohm's Law**:

$$v_{be} = r_\pi i_{in}$$

And from **KCL**:

$$i_{in} = -g_m v_{be}$$

Thus combining:

$$v_{be} = -(g_m r_\pi) v_{be} = -\beta v_{be}$$

Q: *Yikes! How can $v_{be} = -\beta v_{be}$?? The value β is not equal to -1!!*

A: You are right ($\beta \neq -1$)!

Instead, we **must** conclude from the equation:

$$v_{be} = -\beta v_{be}$$

that the small-signal voltage v_{be} must be equal to **zero** !

$$v_{be} = 0$$

Q: *No way! If $v_{be} = 0$, then $g_m v_{be} = 0$. **No current** is flowing, and so the **output voltage** v_o must likewise be equal to **zero**!*

A: That's precisely correct! The output voltage is approximately **zero**:

$$v_o(t) \cong 0$$

Q: *Why did you say "approximately" zero ??*

A: Remember, we **neglected** the output resistance r_o in our circuit analysis. **If** we had explicitly included it, we would find that the output voltage would be **very small**, but not exactly zero.

Q: *So what does this all mean?*

A: It means that the **common-mode gain** of a BJT differential pair is very small (**almost zero**!).

$$A_{cm} = \frac{v_o}{v_{cm}} \cong 0$$

Likewise, we find that:

$$i_{in} \cong 0$$

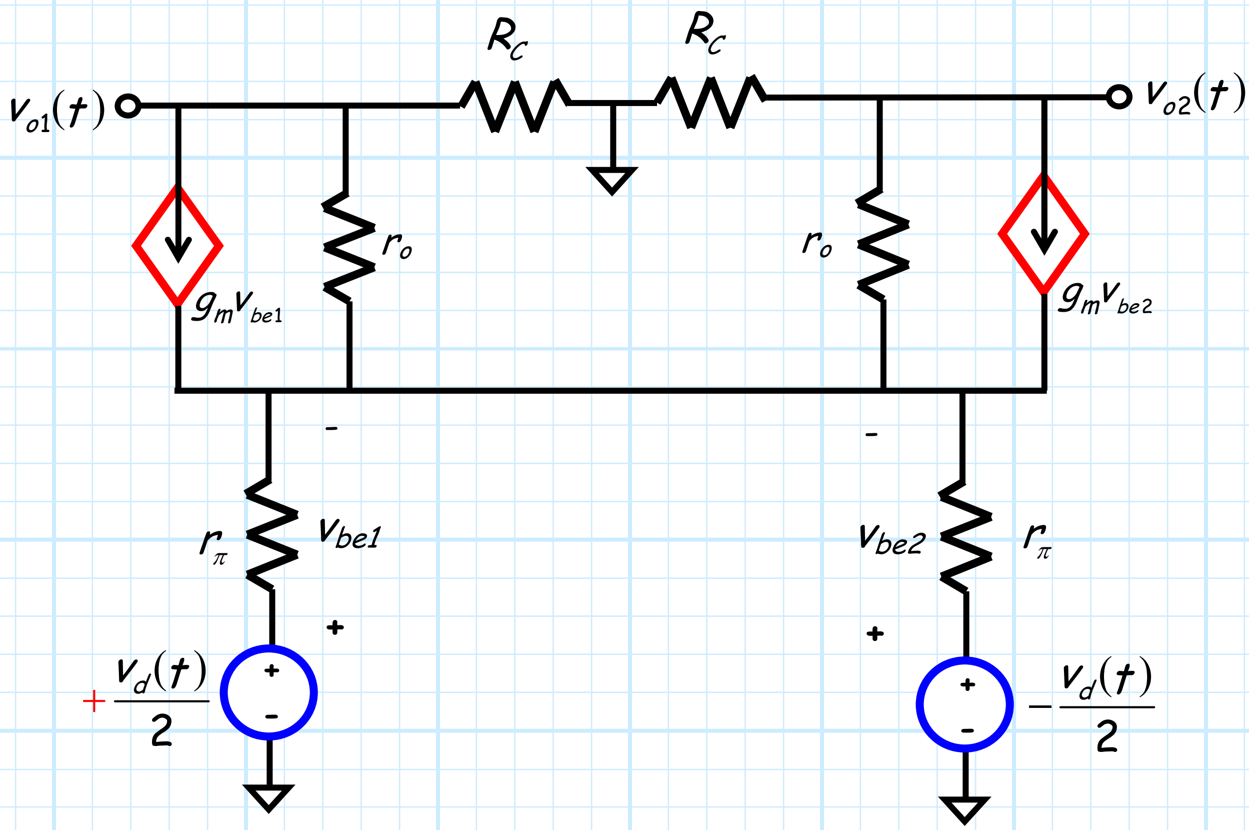
Such that the **common-mode input resistance** is really big:

$$R_{in}^{cm} \cong \infty \quad !!!$$

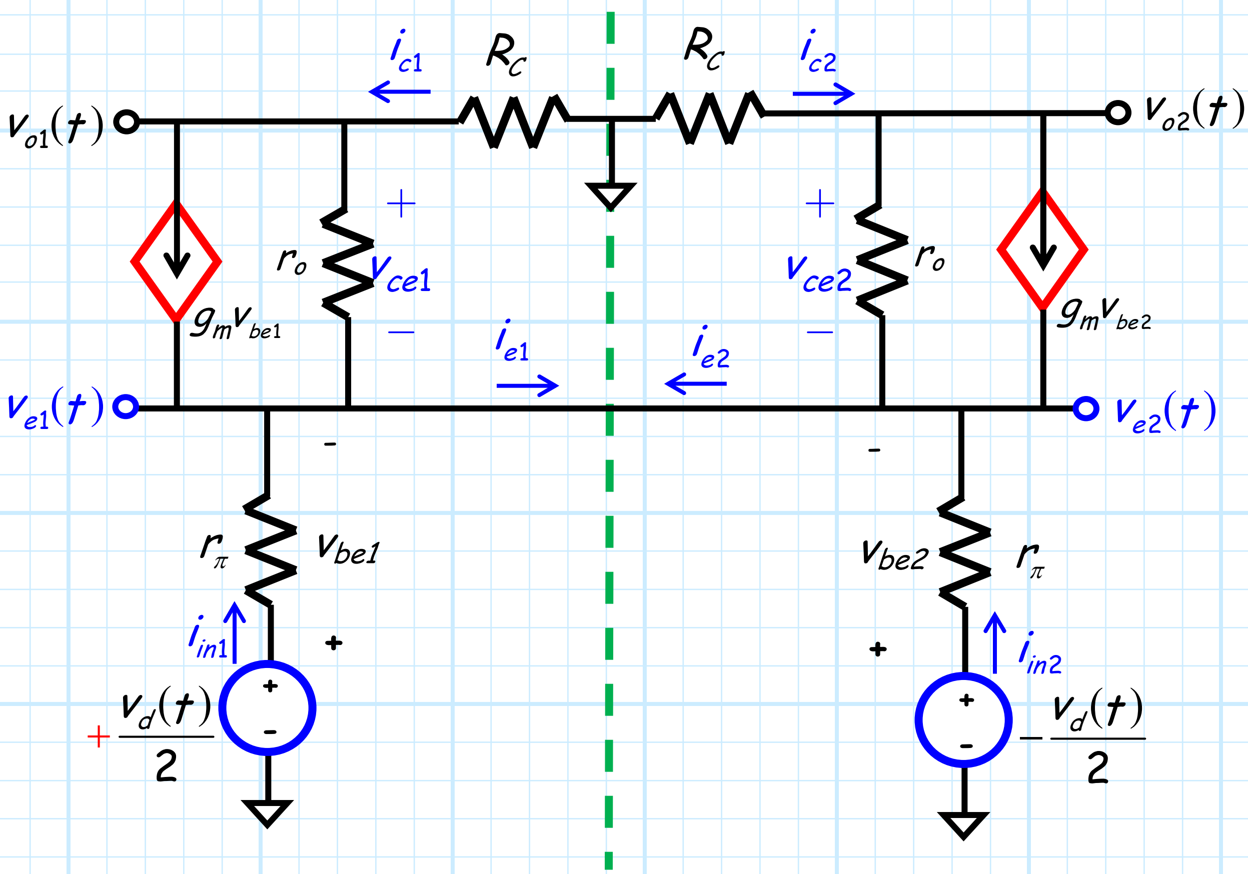
The **common-mode component** of inputs $v_1(t)$ and $v_2(t)$ have virtually **no effect** on a BJT differential pair!

Q: *So what about the **differential** mode?*

A: Let's complete our **superposition** and find out!



Q: *Hey, it looks like we have the **same** symmetric circuit as before—won't we get the **same** answers?*



A: Not so fast!

Look at the two-small signal sources—they are "equal but **opposite**". The fact that the two sources have **opposite "sign"** changes the symmetry of the circuit.

Instead of each current and voltage on either side of the symmetric plane being **equal** to the other, we find that each current and voltage must be "equal but **opposite**"!

For **example**:

$$\begin{aligned} v_{be1} &= -v_{be2} \\ v_{o1} &= -v_{o2} \\ v_{ce1} &= -v_{ce2} \\ v_{e1} &= -v_{e2} \end{aligned} \quad \text{and} \quad \begin{aligned} i_{in1} &= -i_{in2} \\ g_m v_{be1} &= -g_m v_{be2} \\ i_{c1} &= -i_{c2} \\ i_{e1} &= -i_{e2} \end{aligned}$$

This type of circuit symmetry is referred to as **odd symmetry**; the common-mode circuit, in contrast, possessed **even** symmetry.

Q: *Wait! You say that—because of "circuit symmetry"—that:*

$$v_{e1} = -v_{e2}.$$

*But, just **look** at the circuit; from **KVL** it is evident that:*

$$v_{e1} = v_{e2}$$

*How can **both** statements be correct?*

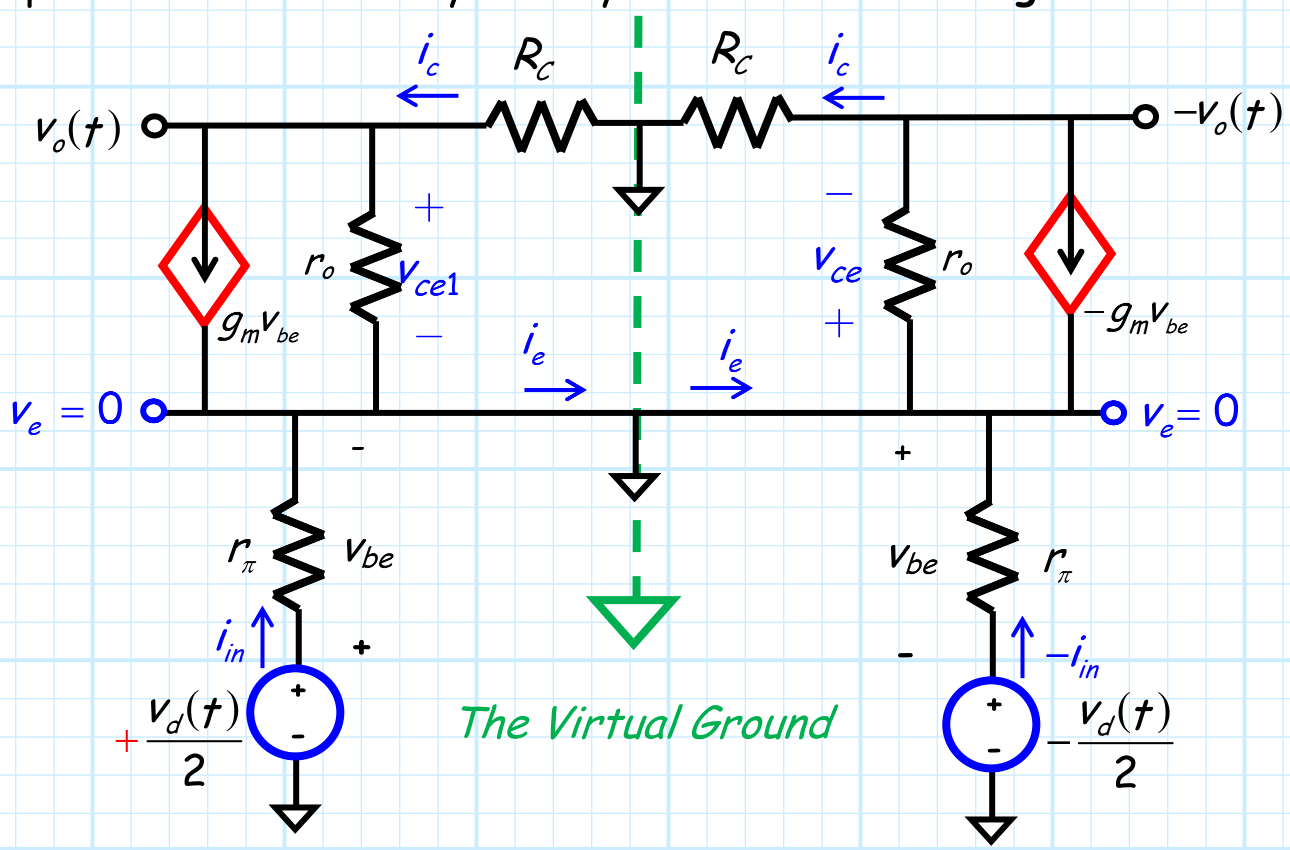
A: **Both** statements are correct!

In fact, the statements (taken together) tell us what the small-signal emitter voltages **must** be (for this differential-mode circuit).

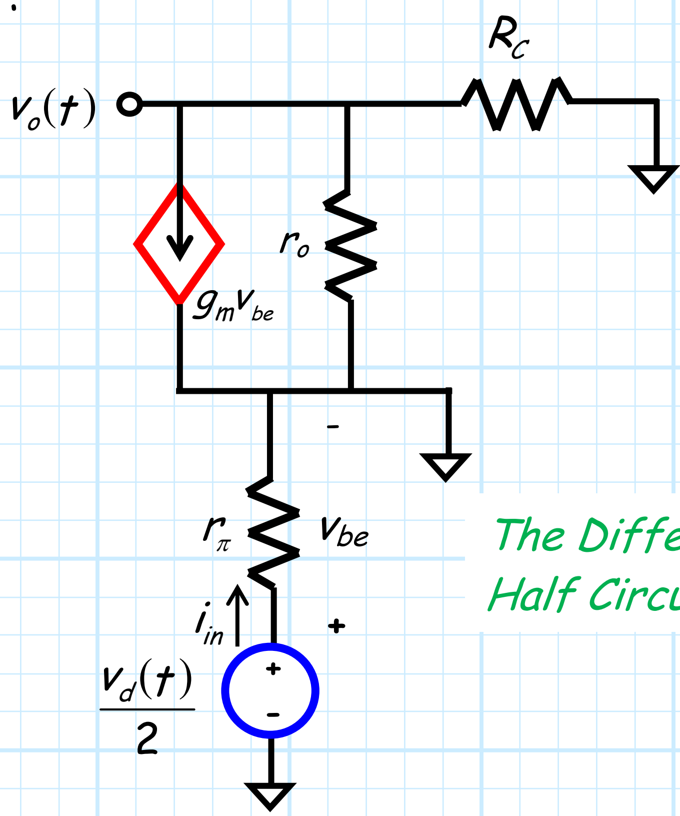
There is **only one possible solution** that satisfies the two equations—the differential-mode, small-signal emitter voltages must be equal to **zero**!

$$v_{e1} = -v_{e2} = v_{e2} = 0$$

More generally, the electric potential at every location along a plane of **odd** reflection symmetry is **zero volts**. Thus, the plane of odd circuit symmetry is known as **virtual ground**!



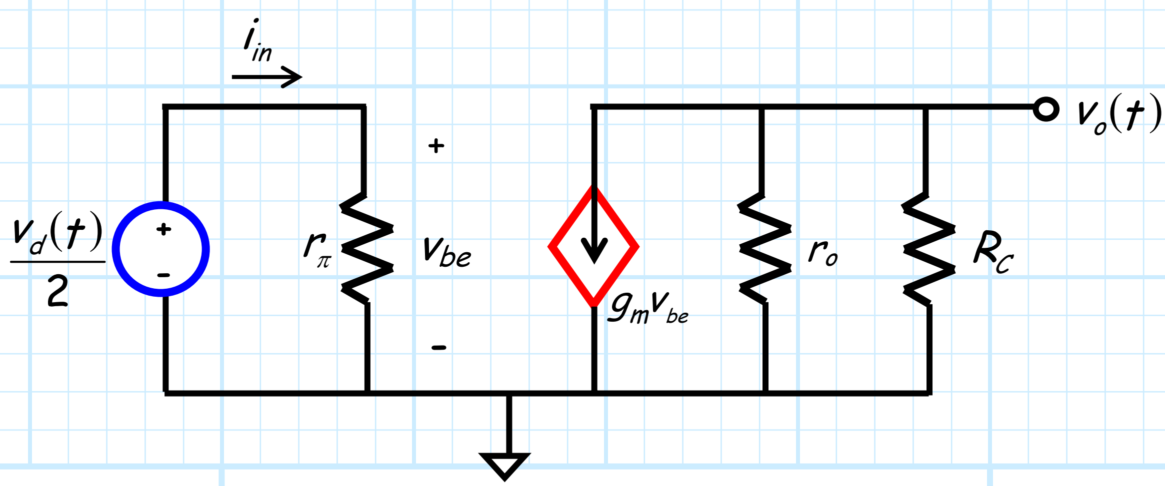
Again, the circuit has two **isolated and independent** halves. We can take our scissors and cut it into two separate "**half-circuits**":



The Differential-Mode Half Circuit

Note the only difference (aside from the small-signal source) between the differential half-circuit and its common-mode counterpart is that the emitter is connected to **ground** → it's a **common-emitter amplifier**!

Let's **redraw** this half-circuit and see if **you** recognize it:



Q: *Hey, we've seen this circuit (about a **million** times) before! We know that:*

$$v_o(t) = -g_m\left(r_o \| R_C\right)\frac{v_d(t)}{2} \cong -\frac{g_m R_C}{2} v_d(t)$$

And also:

$$i_{in}(t) = \frac{1}{r_\pi}\frac{v_d(t)}{2}$$

Right?

A: Exactly!

From this we can conclude that the **differential-mode small-signal gain** is:

$$A_d \doteq \frac{v_o(t)}{v_d(t)} = -\tfrac{1}{2}\, g_m R_C$$

And the **differential mode-input resistance** is:

$$R_{in}^d \doteq \frac{v_d(t)}{i_{in}(t)} = 2 r_\pi$$

In addition, it is evident (from past analysis) that the **output resistance** is:

$$R_{out}^d = r_o \| R_C \cong R_C$$

Now, putting the **two** pieces of our **superposition** together, we can conclude that, given small-signal inputs:

$$v_1(t) = v_{cm}(t) + \frac{v_d(t)}{2} \qquad\qquad v_2(t) = v_{cm}(t) - \frac{v_d(t)}{2}$$

The small-signal outputs are:

$$v_{o1}(t) = A_{cm}\, v_{cm}(t) + A_d\, v_d(t) \cong A_d\, v_d(t)$$

$$v_{o2}(t) = A_{cm}\, v_{cm}(t) - A_d\, v_d(t) \cong -A_d\, v_d(t)$$

Moreover, if we define a **differential output voltage**:

$$v_o^d(t) \doteq v_{o1}(t) - v_{o2}(t)$$

Then we find it is related to the differential input as:

$$v_o^d(t) = 2A_d\, v_d(t)$$

Thus, the **differential pair** makes a **very good difference amplifier**—the kind of gain stage that is required in every **operational-amplifier** circuit!