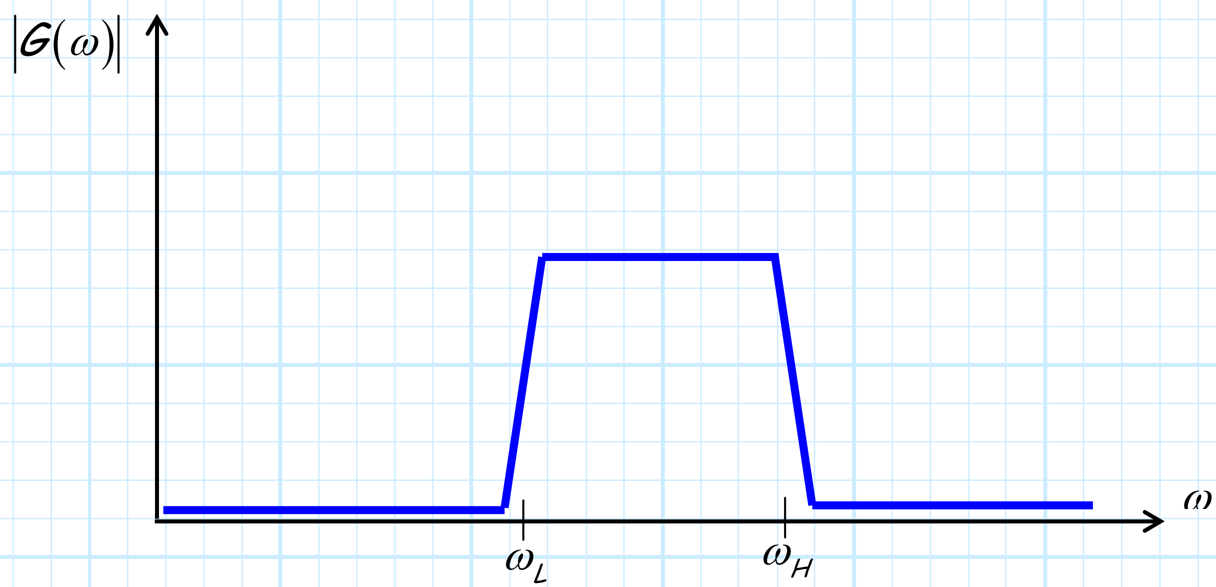
Frequency Bands

The Eigen value $G(\omega)$ of a linear operator is of course dependent on **frequency** ω—the numeric **value** of $G(\omega)$ depends on the frequency ω of the basis function $e^{j\omega t}$.



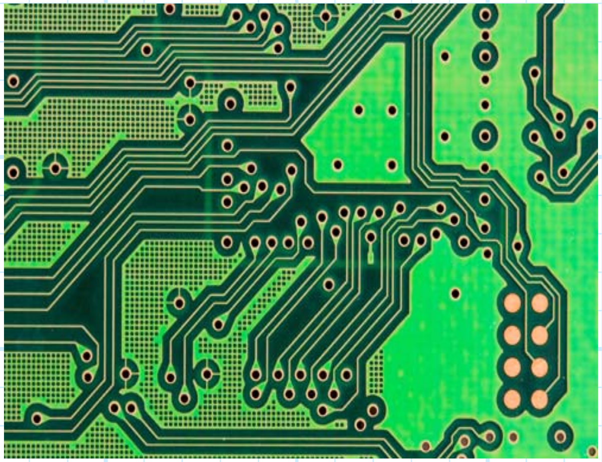
Frequency Response

The frequency ω has units of **radians/second**; it can likewise be expressed as:

$$\omega = 2\pi f$$

where f is the sinusoidal frequency in cycles/second (i.e., **Hertz**).

As a result, the function $G(\omega)$ is also known as the **frequency response** of a linear operator (e.g. a linear circuit).



The numeric value of the signal frequency f has significant **practical** ramifications to us electrical engineers, beyond that of simply determining the numeric value $G(\omega)$.

These practical ramifications include the **packaging**, **manufacturing**, and **interconnection** of electrical and electronic devices.

The problem is that every real circuit is **awash** in inductance and capacitance!

Those darn parasitics!

Q: *If this is such a problem, shouldn't we just* ***avoid*** *using capacitors and inductors?*

A: Well, capacitors and inductors are **particular useful** to us EE's.

But, even **without** capacitors and inductors, we find that our circuits are **still** awash in capacitance and inductance!

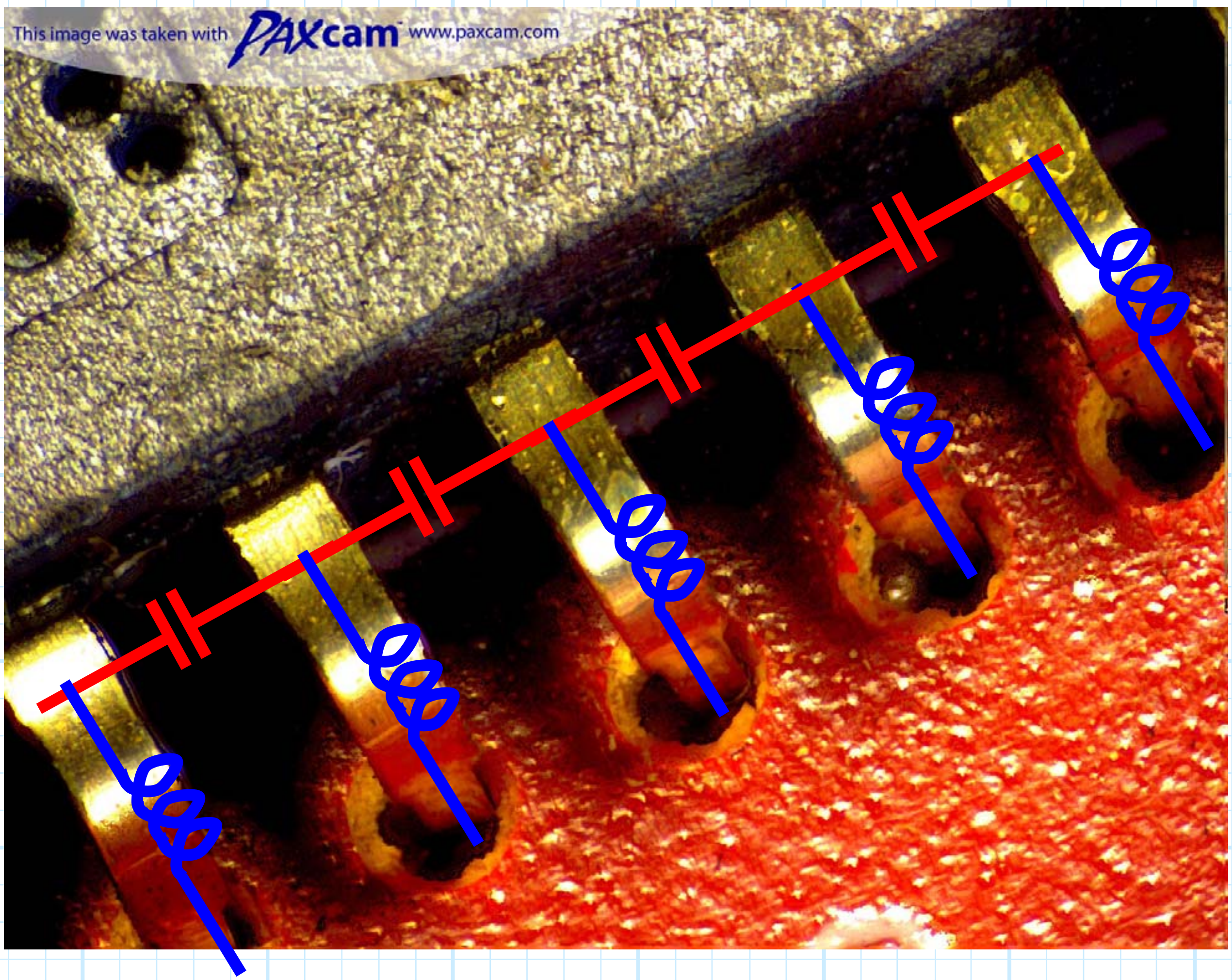
Q: *???*

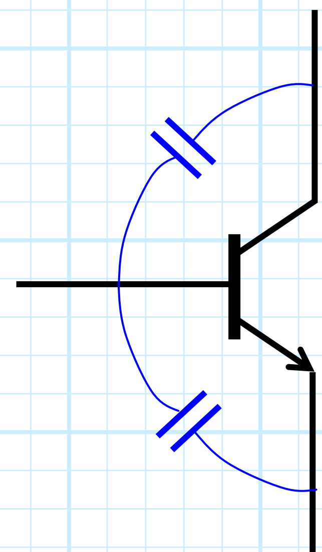
A: Every circuit that we construct will have a inherent set of **parasitic** inductance and capacitance.

Parasitic inductance and capacitance is associated with elements **other** than capacitors and inductors!

Every wire an inductor

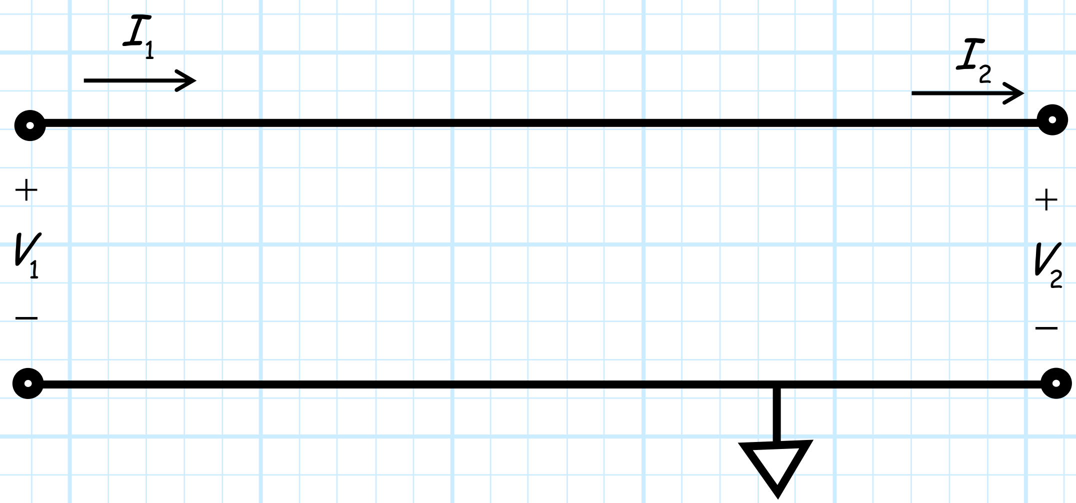
For example, every **wire** and lead has a small inductance associated with it:





Seems simple enough…

Consider then a "wire" above a ground plane:



From **KVL** and **KCL**, we "know" that:

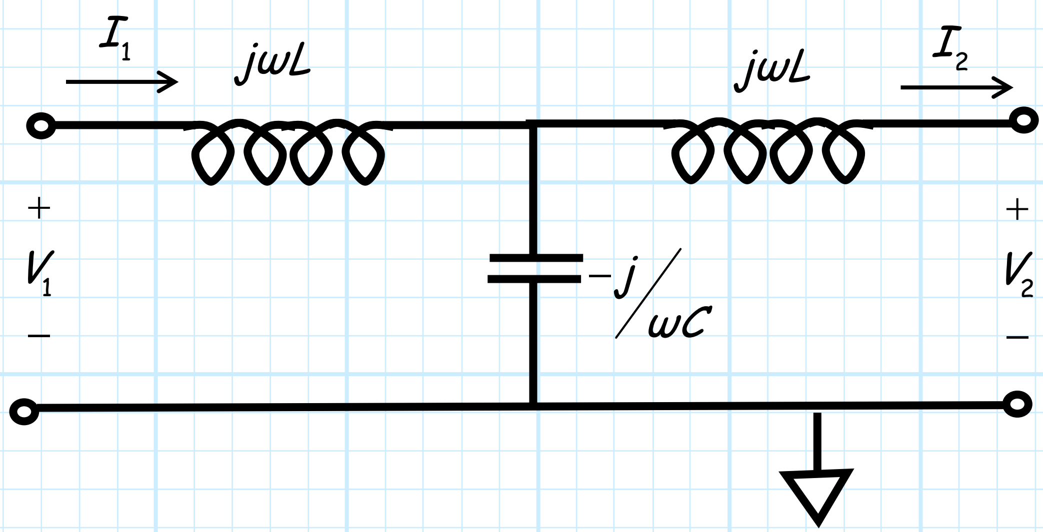
$$V_1 = V_2 \qquad\qquad I_1 = I_2$$

Thus, the linear operator (for example) relating voltage V_1 to voltage V_2 has an **Eigen value** equal to **1.0** for **all frequencies**:

$$\frac{V_2}{V_1} = G(\omega) = 1.0$$

...but its harder than you thought!

But, the unfortunate reality is that the "wire" exhibits **inductance**, and likewise a **capacitance** between it and the ground plane



We now see that the in fact the currents and voltage must be **dissimilar**:

$$V_1 \neq V_2 \qquad\qquad I_1 \neq I_2$$

And so the Eigen value of the linear operator is **not** equal to 1.0!

$$\frac{V_2}{V_1} = G(\omega) \neq 1.0$$

The parasitics are small

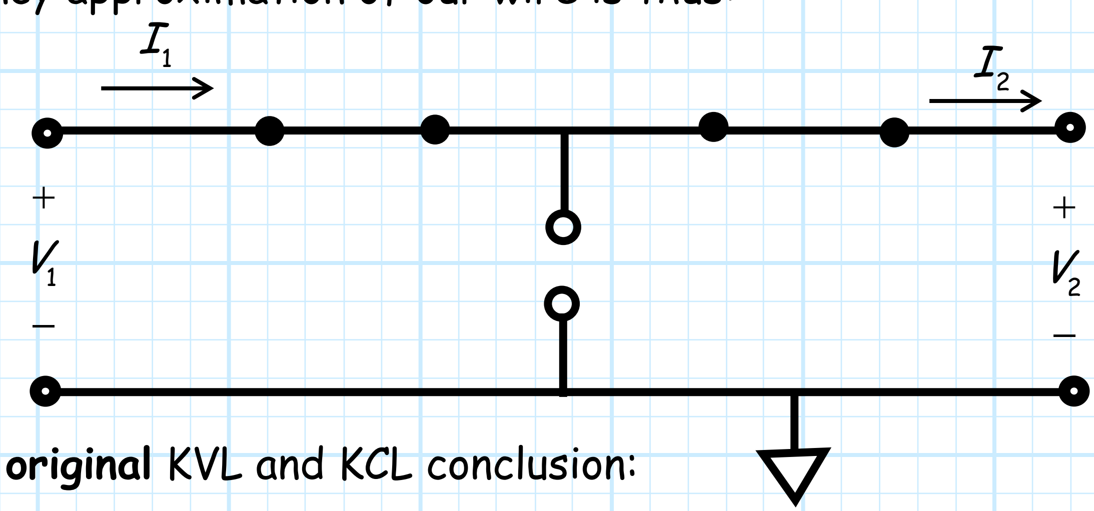
Now, these parasitic values of L and C are likely to be **very small**, so that if the frequency is "low" the **inductive impedance** is quite **small**:

$$|j\omega L| \ll 1 \quad \text{(almost a \textbf{short} circuit!)}$$

And, the **capacitive impedance** (if the frequency is low) is quite **large**:

$$\left|{-j}/{\omega C}\right| \ll 1 \qquad \text{(almost an \textbf{open} circuit!)}$$

Thus, a low-frequency approximation of our wire is thus:



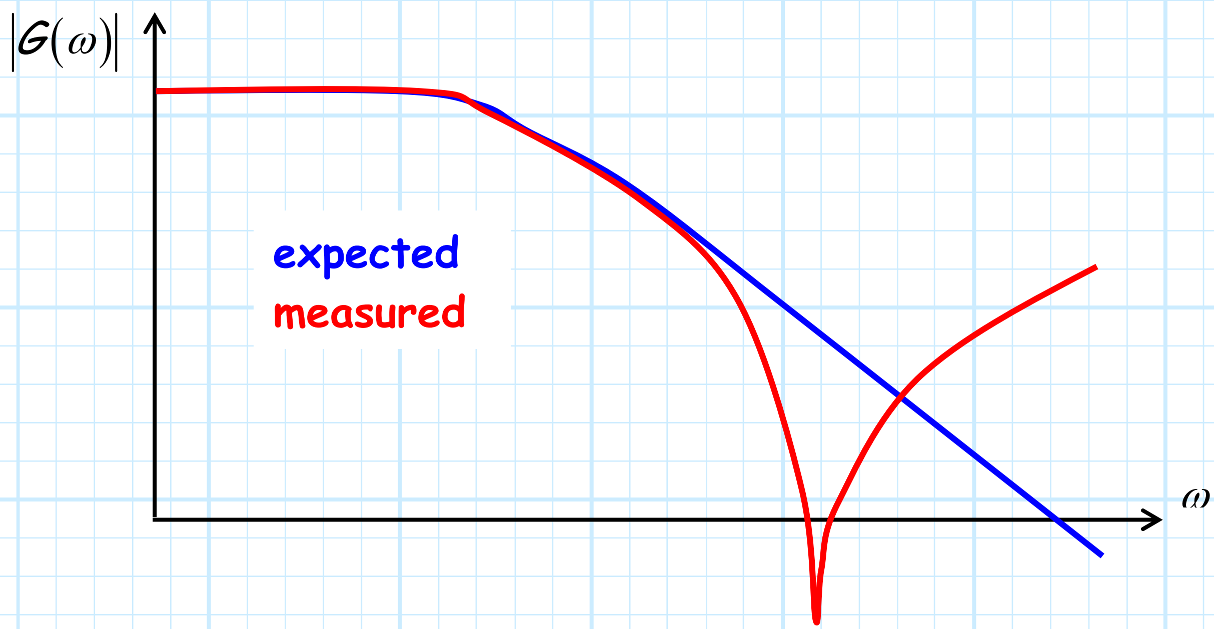
Which leads to our **original** KVL and KCL conclusion:

$$V_1 = V_2 \qquad\qquad I_1 = I_2$$

Parasitics are a problem at "high" frequencies

Thus, as our signal frequency increases, the we often find that the "frequency response" $G(\omega)$ will in reality be **different** from that **predicted** by our circuit model—**unless** explicit parasitics are considered in that model.

As a result, the response $G(\omega)$ may **vary** from our **expectations** as the signal frequency increases!



Frequency Bands

For frequencies in the kilohertz (**audio** band) of megahertz (**video** band), parasitics are generally not a problem.

However, as we move into the 100's of megahertz, or gigahertz (**RF** and **microwave** bands), the effects of parasitic inductance and capacitance are not only significant—they're **unavoidable**!