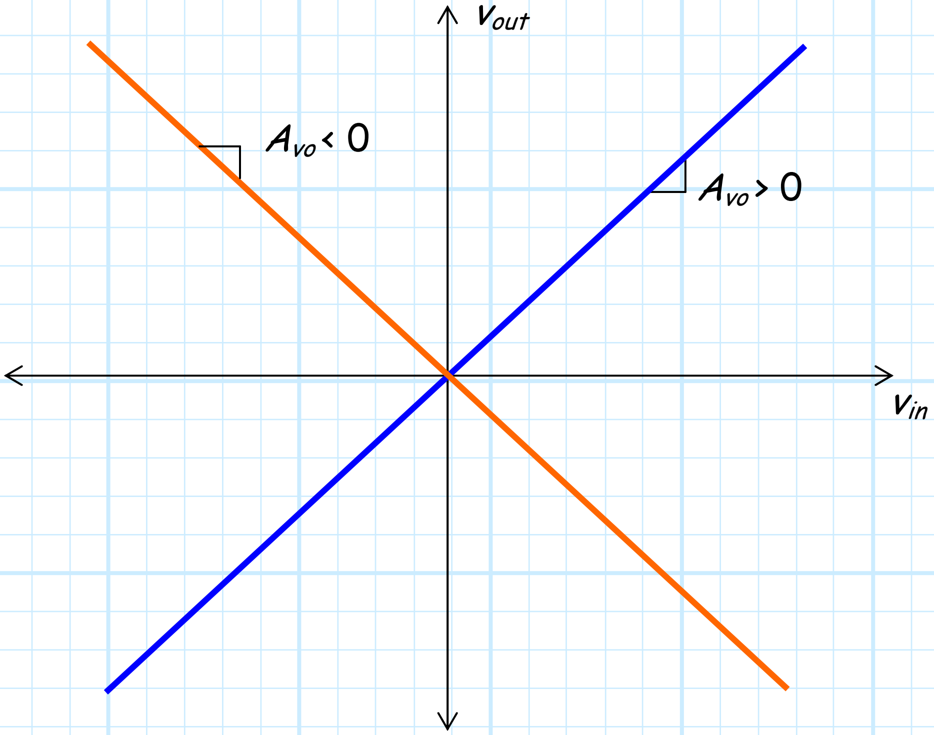
# Non-Linear Behavior of Amplifiers

Note that the **ideal** amplifier transfer function:

$$v_{out}^{oc}(t) = A_{vo}\, v_i(t)$$

is an equation of a **line** (with slope = $A_{vo}$ and $y$-intercept = 0).



# The output voltage is limited

This **ideal** transfer function implies that the **output voltage** can be **very** large, provided that the gain $A_{vo}$ and the input voltage $v_{in}$ are large.

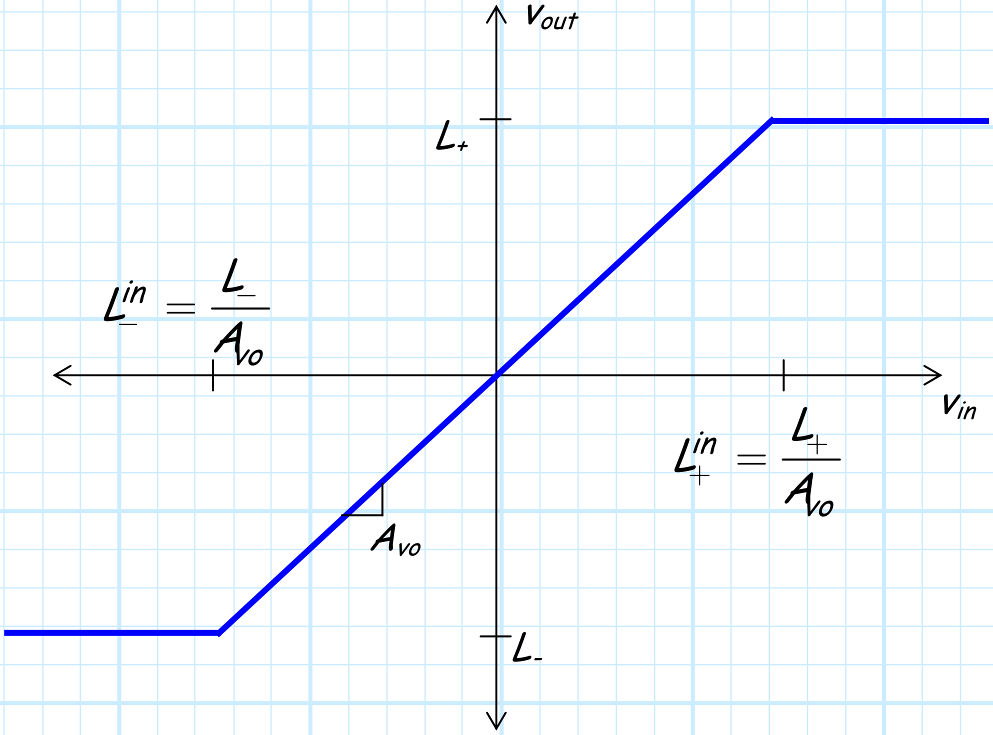
However, we find in a "real" amplifier that there are **limits** on how large the output voltage can become.

The transfer function of an amplifier is more **accurately** expressed as:

$$v_{out}(t) = \begin{cases} L_+ & v_{in}(t) > L_+^{in} \\ A_{vo}\, v_{in}(t) & L_-^{in} < v_{in}(t) < L_+^{in} \\ L_- & v_{in}(t) < L_-^{in} \end{cases}$$

# Amplifier saturation

This expression is shown **graphically** as:



This expression (and graph) shows that electronic amplifiers have a **maximum** and **minimum** output voltage ($L_+$ and $L_-$).

If the **input** voltage is either too large or too small (too negative), then the amplifier **output** voltage will be equal to either $L_+$ or $L_-$ .

If $v_{out} = L_+$ or $v_{out}=L_-$, we say the amplifier is in **saturation** (or compression).

# Make sure the input isn't too large!

Amplifier saturation occurs when the **input** voltage is **greater** than:

$$v_{in} > \frac{L_+}{A_{vo}} \doteq L_+^{in}$$

or when the **input** voltage is **less** than:

$$v_{in} < \frac{L_-}{A_{vo}} \doteq L_-^{in}$$

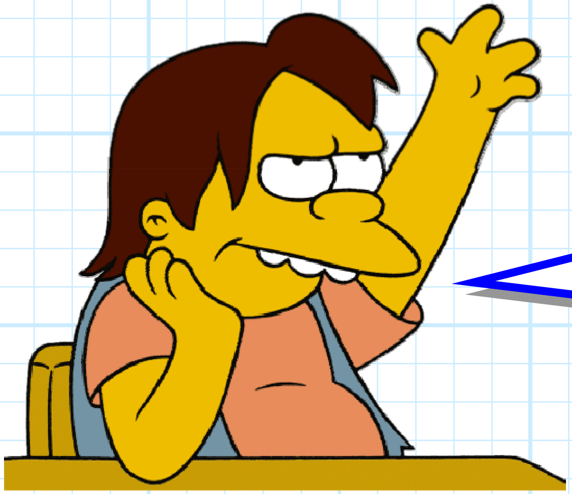
Often, we find that these voltage limits are **symmetric**, i.e.:

$$L_- = -L_+ \quad \text{and} \quad L_-^{in} = -L_+^{in}$$

For example, the output limits of an amplifier might be $L_+$ = 15 V and $L_-$ = -15 V.

However, we find that these limits are also often **asymmetric** (e.g., $L_+$ = +15 V and $L_-$ = +5 V).

# Saturation: Who really cares?

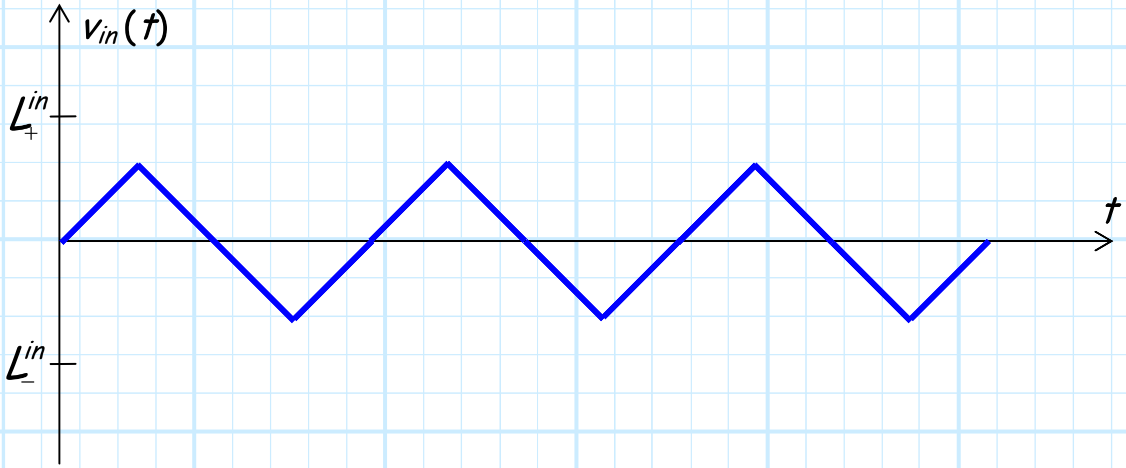


**Q:** *Why do we **care** if an amplifier saturates? Does it cause any **problems**, or otherwise result in performance **degradation**??*

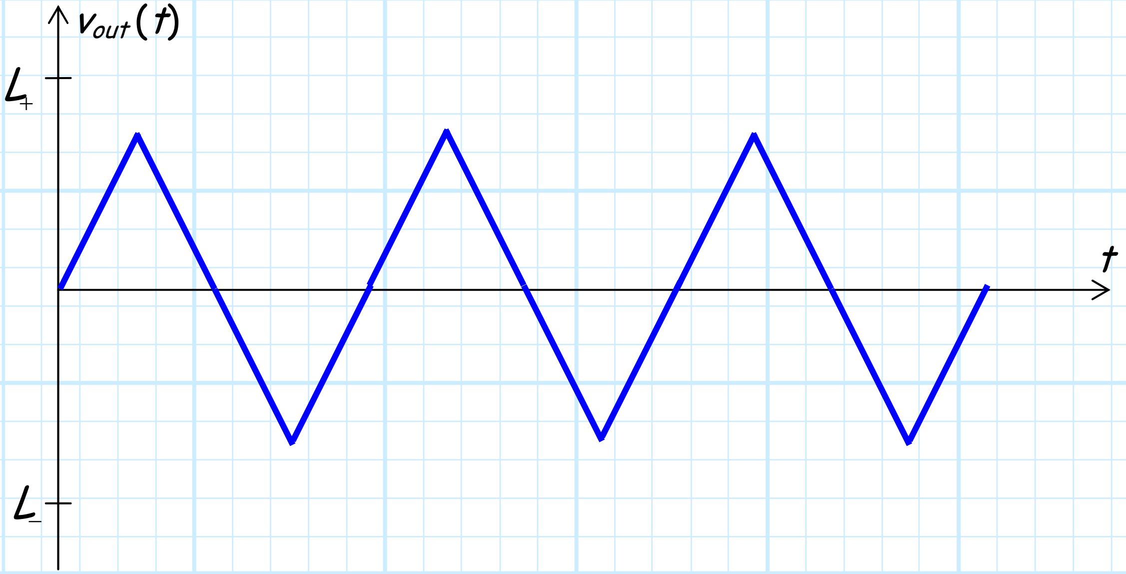
**A:** **Absolutely**! If an amplifier saturates—even momentarily—the unavoidable result will be a **distorted** output signal.

# A distortion free example

For example, consider a case where the input to an amplifier is a **triangle** wave:

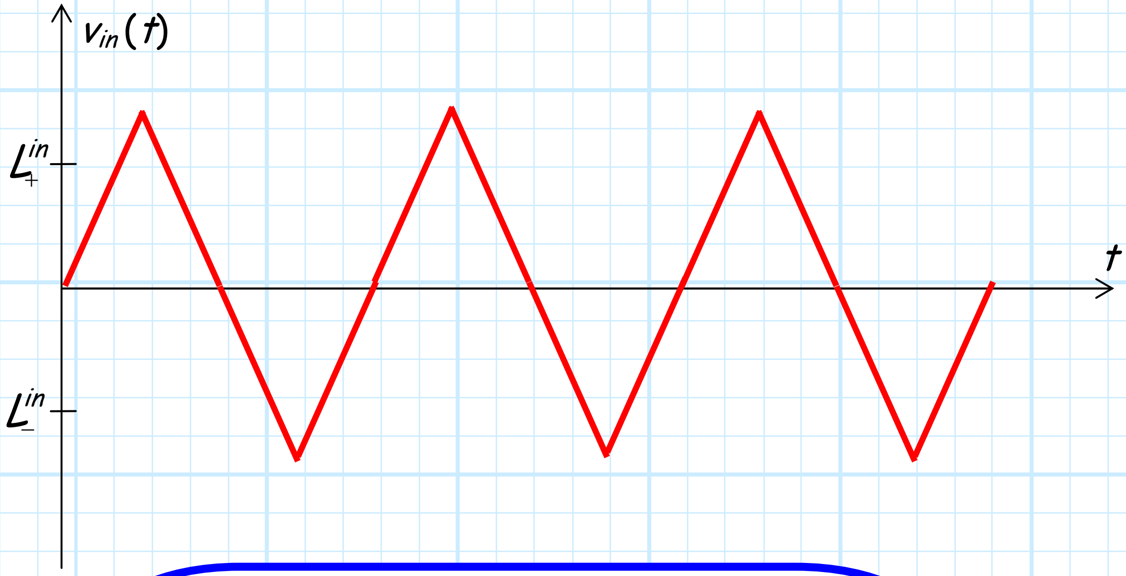


Since $L_-^{in} < v_{in}(t) < L_+^{in}$ for all time $t$, the **output** signal will be within the limits $L_+$ and $L_-$ for all time $t$, and thus the amplifier output will be $v_{out}(t) = A_{vo}\, v_{in}(t)$:



# The input is too darn big!

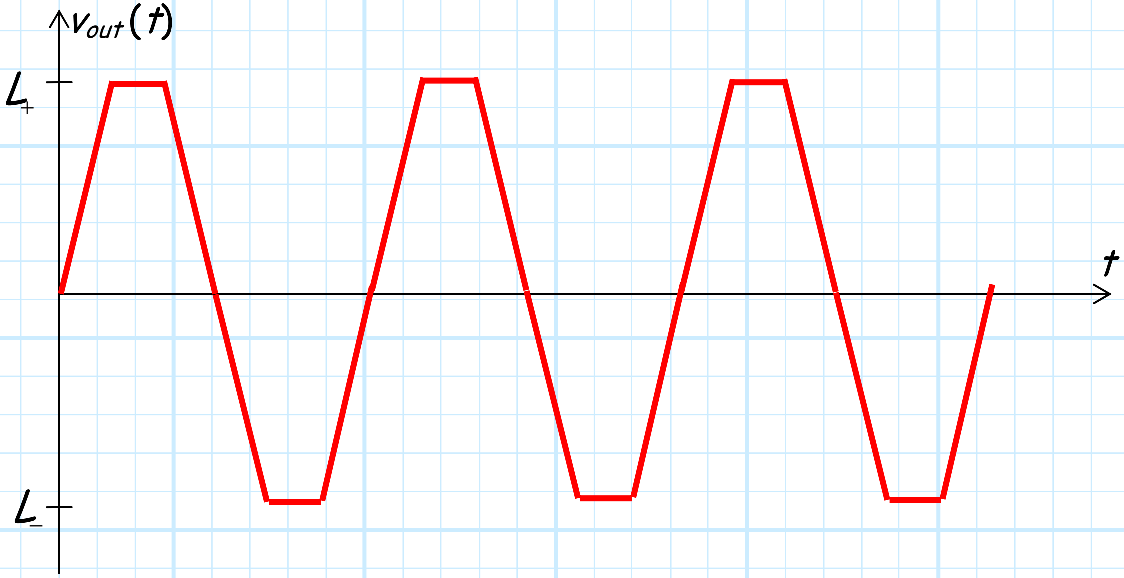
Consider now the case where the input signal is much **larger**, such that $v_{in}(t) > L_+^{in}$ and $v_{in}(t) < L_-^{in}$ for some time $t$ (e.g., the input triangle wave **exceeds** the voltage limits $L_+^{in}$ and $L_-^{in}$ some of the time):



*This is precisely the situation about which I earlier expressed **caution**.*

*We now must experience the palpable agony of **signal distortion**!*

# Palpable agony



Note that this output signal is **not** a triangle wave!

For time $t$ where $v_{in}(t) > L_+^{in}$ and $v_{in}(t) < L_-^{in}$, the value $A_{vo}\, v_{in}(t)$ is greater than $L_+$ and less than $L_-$, respectively.

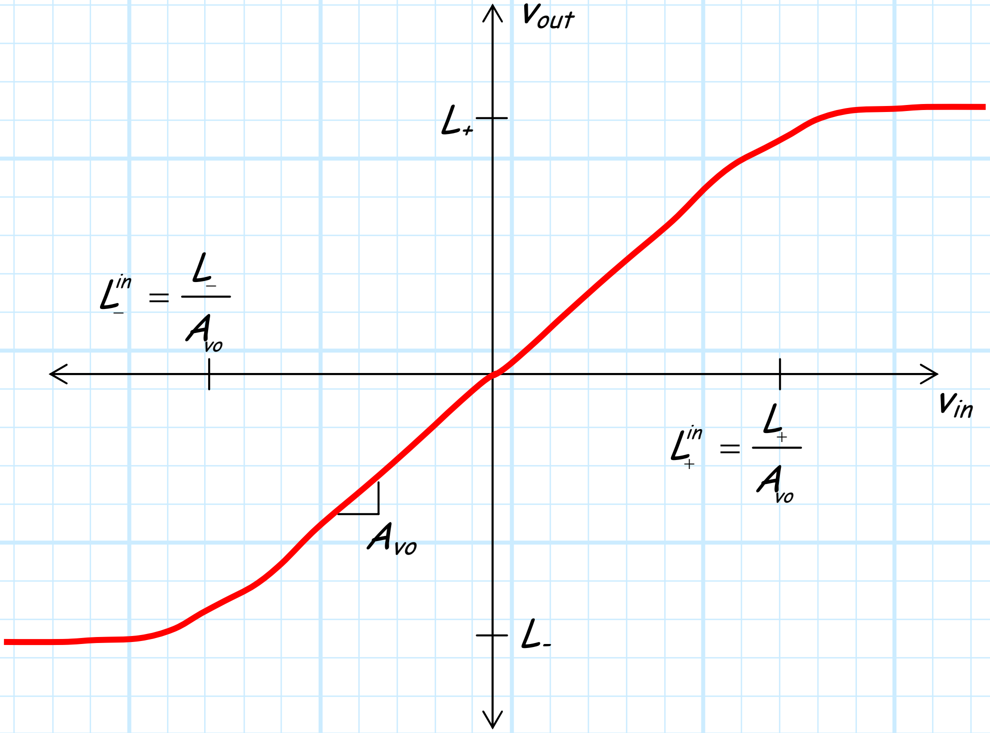
Thus, the output voltage is limited to $v_{out}(t) = L_+$ and $v_{out}(t) = L_-$ for these times.

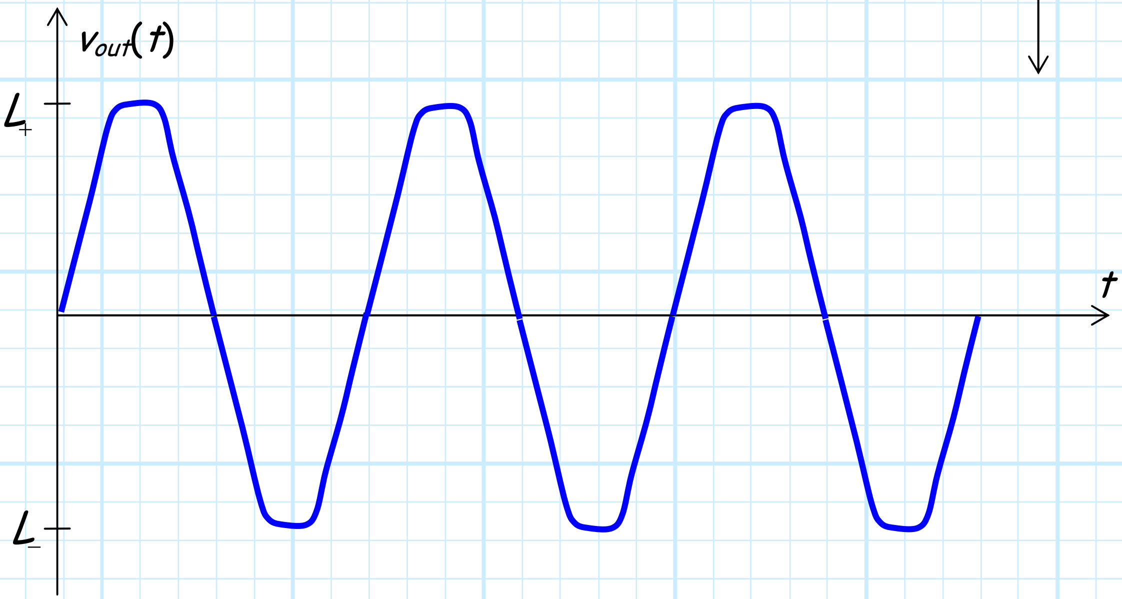
As a result, we find that output $v_{out}(t)$ does **not** equal $A_{vo}\, v_{in}(t)$ —the output signal is **distorted**!

# "Soft" Saturation

In reality, the **saturation** voltages $L_+$, $L_-$, $L_+^{in}$, and $L_-^{in}$ are not so **precisely** defined.

The transition from the linear amplifier region to the saturation region is **gradual**, and cannot be unambiguously defined at a precise point.

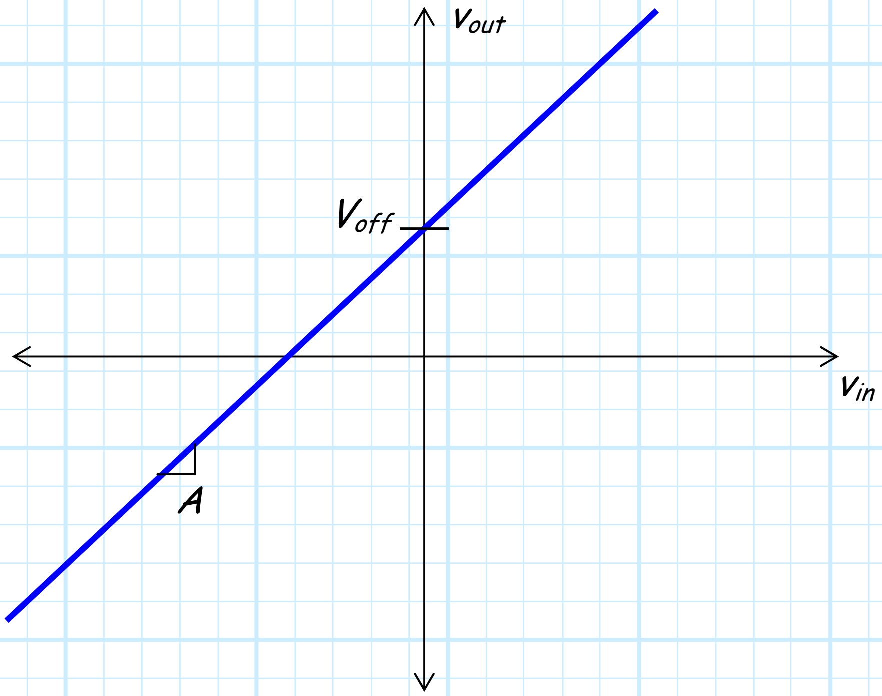




# Yet another problem: DC offset

Now for another **non-linear** problem!

We will find that many amplifiers exhibit a **DC offset** (i.e., a DC **bias**) at their output.



# How do we define gain?

The output of these amplifiers can be expressed as:

$$v_{out}(t) = A\, v_{in}(t) + V_{off}$$

where $A$ and $V_{off}$ are constants.

It is evident that if the input is **zero**, the output voltage will **not** be (zero, that is)!

$$\text{i.e.,} \qquad v_{out} = V_{off} \qquad \text{if} \qquad v_{in} = 0$$

**Q:** ***Yikes!*** *How do we determine the* ***gain*** *of such an amplifier?*

*If:*

$$v_{out}(t) = A\, v_{in}(t) + V_{off}$$

*then what is:*

$$\frac{v_{out}(t)}{v_{in}(t)} = ?????$$

*The* ***ratio*** *of the output voltage to input voltage is* ***not a constant****!*

# Calculus: is there anything it can't do?

**A:** The gain of **any** amplifier can be defined more precisely using the **derivative** operator:

$$\boxed{A_{vo} \doteq \frac{d\,v_{out}}{d\,v_{in}}}$$

Thus, for an amplifier with an output DC offset, we find the voltage gain to be:

$$A_{vo} = \frac{d\,v_{out}}{d\,v_{in}} = \frac{d\left(A\,v_{in} + V_{off}\right)}{d\,v_{in}} = A$$

In other words, the gain of an amplifier is determined by the **slope** of the transfer function!

# This sort of makes sense!

For an amplifier with **no** DC offset (i.e., $v_o = A_{vo}\, v_i$), it is easy to see that the gain is **likewise** determined from this definition:

$$A_{vo} = \frac{d\, v_{out}}{d\, v_{in}} = \frac{d\, A_{vo}\, v_{in}}{d\, v_{in}} = A_{vo}$$

*Hey, hey! This definition makes sense if you think about it—gain is the* ***change*** *of the output voltage with respect to a* ***change*** *at the input.*

*For example, of small change* $\Delta v_{in}$ *at the* ***input*** *will result in a change of* $A_{vo}\, \Delta v_{in}$ *at the* ***output****.*

*If* $A_{vo}$ *is* ***large****, this change at the output will be* ***large****!*

# Both problems collide

OK, here's **another** problem.

The derivative of the transfer curve for **real** amplifiers will **not be a constant**.

We find that the gain of a amplifier will often be **dependent** on the input voltage!

The main reason for this is amplifier **saturation**.

Consider again the transfer function of an amplifier that **saturates**:

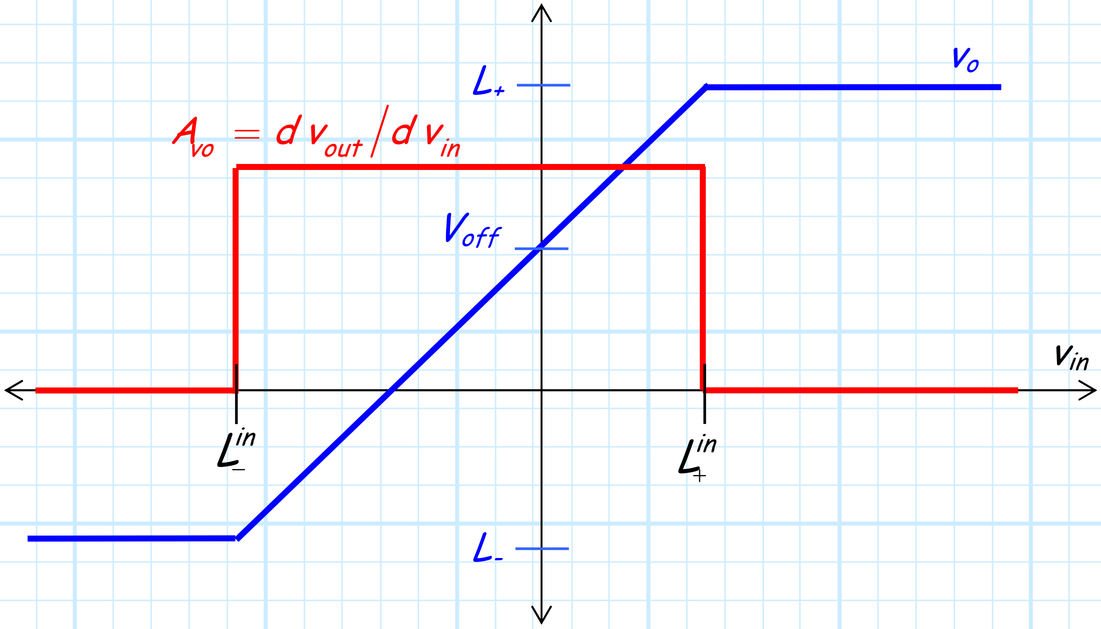
$$v_{out} = \begin{cases} L_+ & v_{in} > L_+^{in} \\ A v_{in} + V_{off} & L_-^{in} < v_i < L_+^{in} \\ L_- & v_{in} < L_-^{in} \end{cases}$$

# Gain is a function of $v_{in}$

We find the **gain** of this amplifier by taking the **derivative** with respect to $v_{in}$:

$$A_{vo} = \frac{d\,v_{out}}{d\,v_{in}} = \begin{cases} 0 & v_{in} > L_+^{in} \\ A & L_-^{in} < v_{in} < L_+^{in} \\ 0 & v_{in} < L_-^{in} \end{cases}$$

Graphically, this result is:

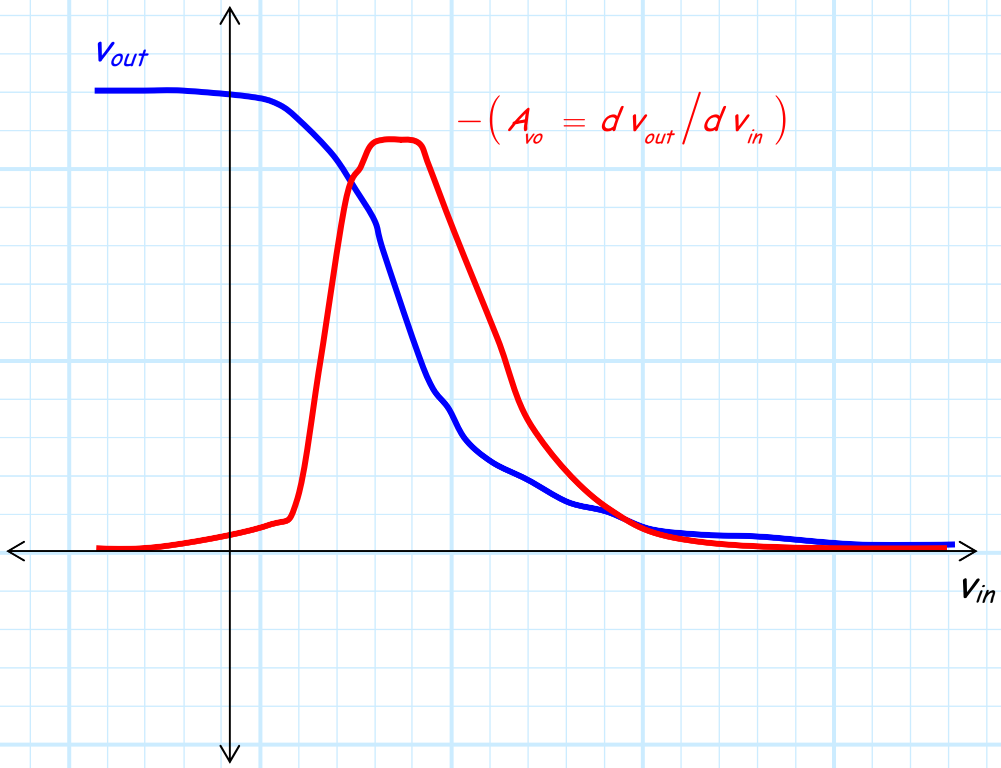


# You'll see this transfer function again!

Thus, the gain of this amplifier when in saturation is **zero**. A change in the input voltage will result in **no change** on the output—the output voltage will simply be $v_o = L_{\pm}$.

Again, the transition into saturation is **gradual** for real amplifiers.

In fact, we will find that many of the amplifiers studied in this class have a **transfer function** that looks something like this→



We will find that the voltage gain of many amplifiers is **dependent** on the input voltage.

Thus, a **DC bias** at the input of the amplifier is often required to **maximize** the amplifier gain.