Section 5.4 – *BJT Circuits at DC*

Reading Assignment: *pp. 421-436*

To analyze a BJT circuit, we follow the **same** boring procedure as always: ASSUME, ENFORCE, ANALYZE and CHECK.

HO: Steps for D.C. Analysis of BJT Circuits

HO: Hints for BJT Circuit Analysis

For **example**:

Example: D.C. Analysis of a BJT Circuit

Example: An Analysis of a *pnp* BJT Circuit

Example: Another DC Analysis of a BJT Circuit

Example: A BJT Circuit in Saturation

Steps for D.C. Analysis of BJT Circuits

To analyze BJT circuit with D.C. sources, we **must** follow these **five steps**:

1. ASSUME an operating mode

2. ENFORCE the equality conditions of that mode.

3. ANALYZE the circuit with the enforced conditions.

4. CHECK the inequality conditions of the mode for consistency with original assumption. If consistent, the analysis is complete; if inconsistent, go to step 5.

5. MODIFY your original assumption and repeat all steps.

Let's look at each step in **detail**.

1. *ASSUME*

We can ASSUME Active, Saturation, or Cutoff!

2. ENFORCE

Active

For **active** region, we must ENFORCE **two equalities**.

a) Since the base-emitter junction is **forward** biased in the active region, we ENFORCE these equalities:

$$V_{BE} = 0.7 \text{ V} \quad \text{(npn)}$$

$$V_{EB} = 0.7 \text{ V} \quad \text{(pnp)}$$

b) We likewise know that in the **active** region, the base and collector currents are directly proportional, and thus we ENFORCE the equality:

$$i_C = \beta\, i_B$$

Note we can **equivalently** ENFORCE this condition with either of the the equalities:

$$i_C = \alpha i_E \qquad \text{or} \qquad i_E = (\beta + 1)\, i_B$$

Saturation

For **saturation** region, we must likewise ENFORCE **two equalities**.

a) Since the base-emitter junction is **forward** biased, we again ENFORCE these equalities:

$$V_{BE} = 0.7 \text{ V} \quad \text{(npn)}$$

$$V_{EB} = 0.7 \text{ V} \quad \text{(pnp)}$$

b) Likewise, since the collector base junction is **reverse** biased, we ENFORCE these equalities:

$$V_{CB} = -0.5 \text{ V} \quad \text{(npn)}$$

$$V_{BC} \simeq -0.5 \text{ V} \quad \text{(pnp)}$$

Note that from **KVL**, the above two ENFORCED equalities will require that these equalities **likewise** be true:

$$V_{CE} = 0.2 \text{ V} \quad \text{(npn)}$$

$$V_{EC} = 0.2 \text{ V} \quad \text{(pnp)}$$

Note that for saturation, you need to explicitly ENFORCE any **two** of these **three** equalities—the third will be ENFORCED **automatically** (via KVL)!!

To avoid **negative** signs (e.g., V_{CB}=-0.5), **I** typically ENFORCE the **first** and **third** equalities (e.g., V_{BE}= 0.7 and V_{CE}=0.2).

Cutoff

For a BJT in cutoff, both *pn* junctions are **reverse** biased—**no** current flows! Therefore we ENFORCE these equalities:

$$i_B = 0$$
$$i_C = 0$$
$$i_E = 0$$

3. *ANALYZE*

Active

The task in D.C. analysis of a BJT in **active** mode is to find **one** unknown **current** and **one** additional unknown **voltage**!

a) In addition the relationship $i_C = \beta\, i_B$, we have a **second** useful relationship:

$$i_E = i_C + i_B$$

This of course is a consequence of KCL, and is true **regardless** of the BJT mode.

But think about what this means! We have **two** current equations and **three** currents (i.e., i_E, i_C, i_B)—we only need to determine **one** current and we can then immediately find the other two!

Q: *Which current do we need to find?*

A: Doesn't matter! For a BJT operating in the active region, if we know **one** current, we know them **all**!

b) In addition to $V_{BE} = 0.7$ ($V_{EB} = 0.7$), we have a **second** useful relationship:

$$V_{CE} = V_{CB} + V_{BE} \quad \text{(npn)}$$

$$V_{EC} = V_{EB} + V_{BC} \quad \text{(pnp)}$$

This of course is a consequence of KVL, and is true **regardless** of the BJT mode.

Combining these results, we find:

$$V_{CE} = V_{CB} + 0.7 \quad \text{(npn)}$$

$$V_{EC} = 0.7 + V_{BC} \quad \text{(pnp)}$$

But think about what **this** means! If we find **one** unknown voltage, we can immediately determine the **other**.

Therefore, a D.C. analysis problem for a BJT operating in the active region reduces to:

find one of these values

$$i_B,\ i_C,\ or\ i_E$$

and find one of these values

$$V_{CE} \text{ or } V_{CB} \quad (V_{EC} \text{ or } V_{BC})$$

Saturation

For the saturation mode, we know **all** the BJT **voltages**, but know nothing about BJT **currents**!

Thus, for an analysis of circuit with a BJT in saturation, we need to find any **two** of the **three** quantities:

$$i_B,\ i_C,\ i_E$$

We can then use **KCL** to find the third.

Cutoff

Cutoff is a bit of the **opposite** of saturation—we know **all** the BJT **currents** (they're all **zero**!), but we know **nothing** about BJT **voltages** !

Thus, for an analysis of circuit with a BJT in cutoff, we need to find any **two** of the **three** quantities:

$$V_{BE},\ V_{CB},\ V_{CE} \qquad (npn)$$

$$V_{EB},\ V_{BC},\ V_{EC} \qquad (pnp)$$

We can then use **KVL** to find the third.

4. *CHECK*

You do not know if your D.C. analysis is correct unless you CHECK to see if it is consistent with your original assumption!

WARNING!-Failure to CHECK the original assumption will result in a SIGNIFICANT REDUCTION in credit on exams, regardless of the accuracy of the analysis !!!

Q: *What exactly do we CHECK?*

A: We ENFORCED the mode **equalities**, we CHECK the mode **inequalities**.

Active

We must CHECK **two** separate inequalities after analyzing a circuit with a BJT that we ASSUMED to be operating in **active** mode. One inequality involves BJT **voltages**, the other BJT **currents**.

a) In the **active** region, the Collector-Base Junction is "off" (i.e., **reverse** biased). Therefore, we must CHECK our analysis results to see if they are **consistent** with:

$$V_{CB} > 0 \quad \text{(npn)}$$

$$V_{BC} > 0 \quad \text{(pnp)}$$

Since $V_{CE} = V_{CB} + 0.7$, we find that an **equivalent** inequality is:

$$V_{CE} > 0.7 \quad \text{(npn)}$$

$$V_{EC} > 0.7 \quad \text{(pnp)}$$

We need to check **only** one of these two inequalities (**not both**!).

b) In the active region, the Base-Emitter Junction is "on" (i.e., **forward** biased). Therefore, we must CHECK the results of our analysis to see if they are **consistent** with:

$$i_B > 0$$

Since the active mode constants α and β are **always** positive values, **equivalent** expressions to the one above are:

$$i_C > 0 \qquad \text{and} \qquad i_E > 0$$

In other words, we need to CHECK and see if **any** one of the currents is positive—if one is positive, they are **all** positive!

Saturation

Here we must CHECK inequalities involving BJT **currents**.

a) We know that for saturation mode, the ratio of collector current to base current will be **less than beta**! Thus we CHECK:

$$i_C < \beta \, i_B$$

b) We know that **both** *pn* junctions are **forward** biased, hence we CHECK to see if all the **currents are positive**:

$$\begin{aligned} i_B &> 0 \\ i_C &> 0 \\ i_E &> 0 \end{aligned}$$

Cutoff

For **cutoff** we must CHECK two BJT **voltages**.

a) Since the EBJ is **reverse biased**, we CHECK:

$$V_{BE} < 0 \qquad (npn)$$

$$V_{EB} < 0 \qquad (pnp)$$

b) Likewise, since the CBJ is also **reverse biased**, we CHECK:

$$V_{CB} > 0 \qquad (npn)$$

$$V_{BC} > 0 \qquad (pnp)$$

If the results of our analysis are consistent with **each** of these inequalities, then we have made the **correct** assumption! The numeric results of our analysis are then likewise correct. We can stop working!

However, if **even one** of the results of our analysis is **inconsistent** with active mode (e.g., currents are negative, or $V_{CE} < 0.7$), then we have made the **wrong** assumption! Time to move to step 5.

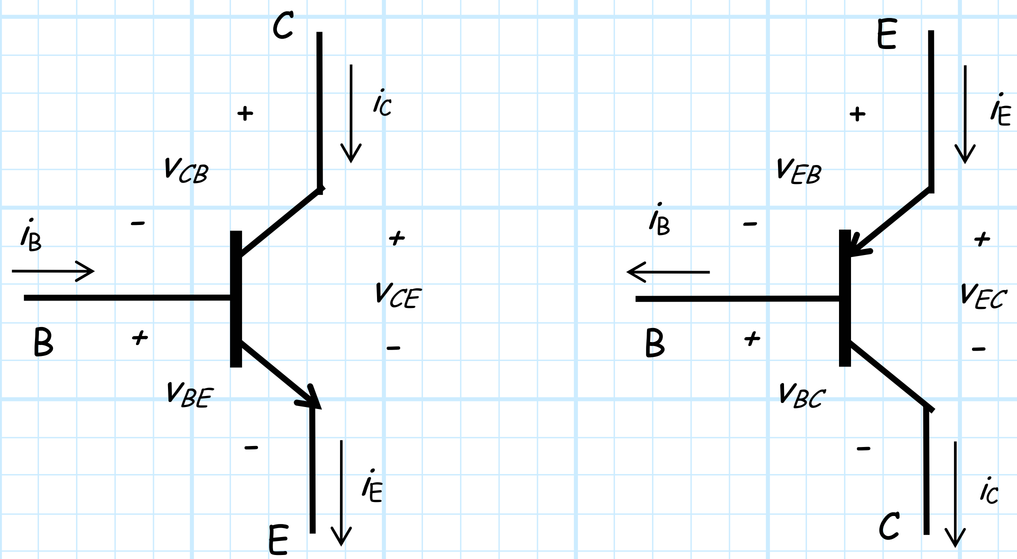
5. *MODIFY*

If one or more of the BJTs are **not** in the active mode, then it must be in either **cutoff** or **saturation**. We must change our assumption and start **completely** over!

In general, **all** of the results of our previous analysis are incorrect, and thus must be **completely** scraped!

Hints for BJT Circuit Analysis

1. Know the BJT **symbols** and **current/voltage** definitions!



2. Know what **quantities** must be determined for each **assumption** (e.g., for active mode, you must determine one BJT current and one BJT voltage).

3. Write **separate** equations for the BJT (device) and the remainder of the circuit (KVL, KCL, Ohm's Law).

4. Write the KVL equation for the circuit's "**Base-Emitter Leg**". In other words, write a KVL that includes v_{BE}.

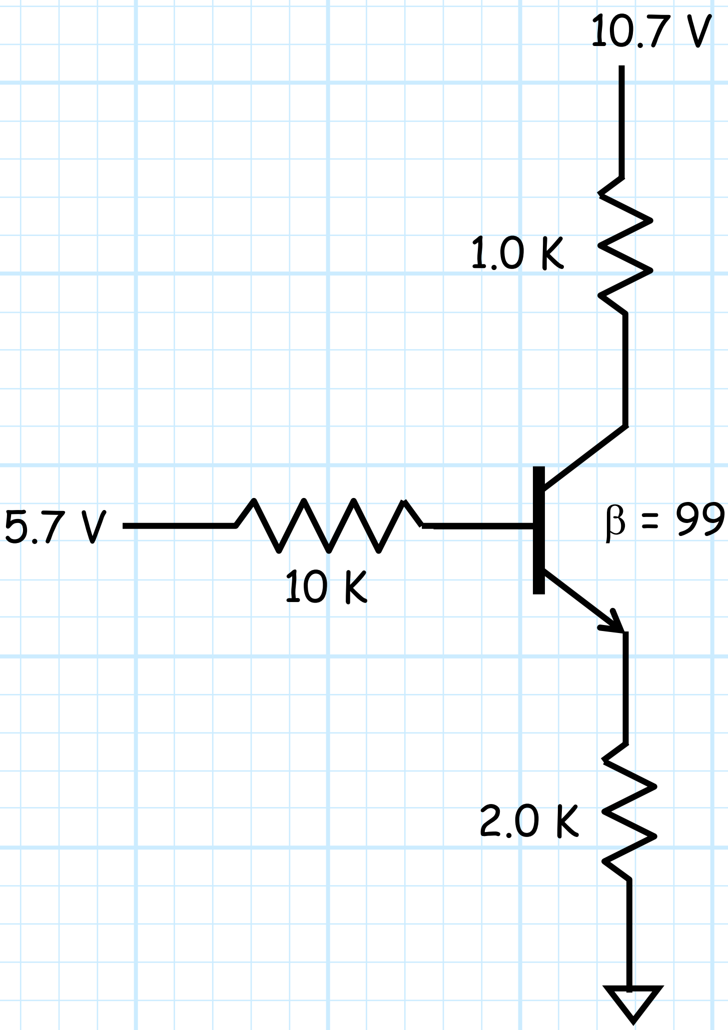
5. Forget about what the problem is asking for! Just start by determining **any** and **all** the circuit quantities that you can. If you end up solving the **entire** circuit, the answer will in there somewhere!

6. If you get stuck, try working the problem **backward**! For example, to find a resistor value, you must find the voltage across it and the current through it.

7. Make sure you are using **all** the information provided in the problem!

Example: D.C. Analysis of a BJT Circuit

Consider **again** this circuit from lecture:



Q: *What is I_B, I_C, I_E and also V_{CE}, V_{CB}, V_{BE} ??*

A: I don't know ! But, we can find out—IF we complete **each** of the five steps **required** for BJT DC analysis.

Step 1 - **ASSUME** an operating mode.

Let's ASSUME the BJT is in the ACTIVE region !

Remember, this is just a **guess**; we have no way of knowing for sure what mode the BJT is in at this point.

Step 2 - **ENFORCE** the conditions of the assumed mode.

For **active** region, these are:

$$V_{BE} = 0.7 \ V \quad \text{and} \quad I_C = \beta \, I_B = 99 \, I_B$$

Step 3 - **ANALYZE** the circuit.

This is the **BIG** step !

Q: *Where do we even start ?*

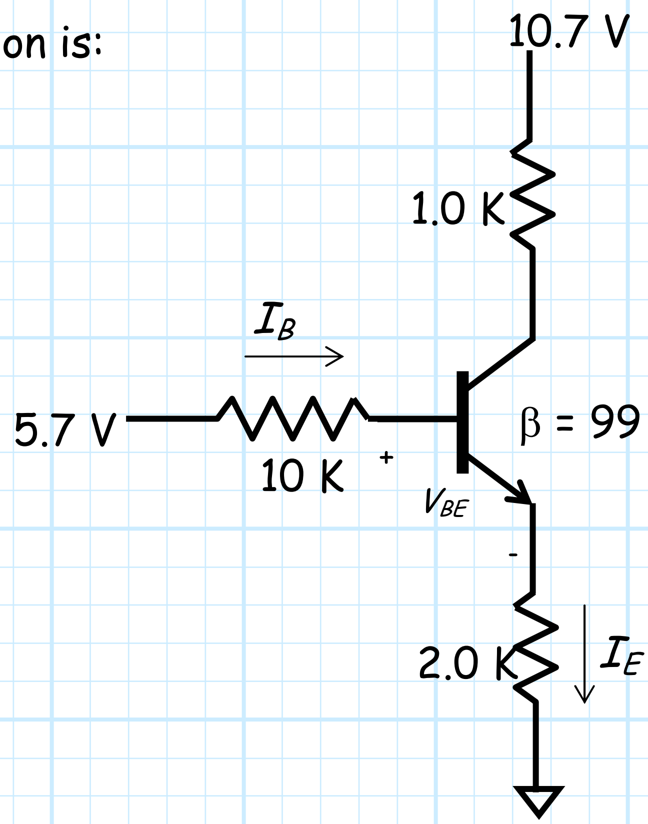
A: Recall what the hint sheet says:

"Write KVL equations for the base-emitter "leg"

I think we should try that !

The **base-emitter KVL** equation is:

$$5.7 - 10\,I_B - V_{BE} - 2\,I_E = 0$$



This is the **circuit** equation; note that it contains 3 unknowns (i_B, i_C, and V_{BE}).

Now let's add the relevant **device** equations:

$$V_{BE} = 0.7\ V$$

$$I_E = (\beta + 1)\,I_B = 100\,I_B$$

Look what we now have ! **3** equations and **3** unknowns (this is a **good** thing).

Inserting the device equations into the B-E KVL:

$$5.7 - 10\,I_B - 0.7 - 2(99+1)I_B = 0$$

Therefore:

$5.0 - 210\,I_B = 0$ → 1 equations and 1 unknown !

Solving, we get:

$$I_B = \frac{5.0}{210} = \underline{23.8\ \mu A}$$

Q: *Whew ! That was an **awful** lot of work for just one current, and we still have **two more** currents to find.*

A: No we don't ! Since we determined **one** current for a BJT in **active** mode, we've determined them **all** !

I.E.,

$$I_C = \beta\, I_B = \underline{2.356\ \text{mA}}$$

$$I_E = (\beta + 1)\, I_B = \underline{2.380\ \text{mA}}$$

(Note that $I_C + I_B = I_E$)

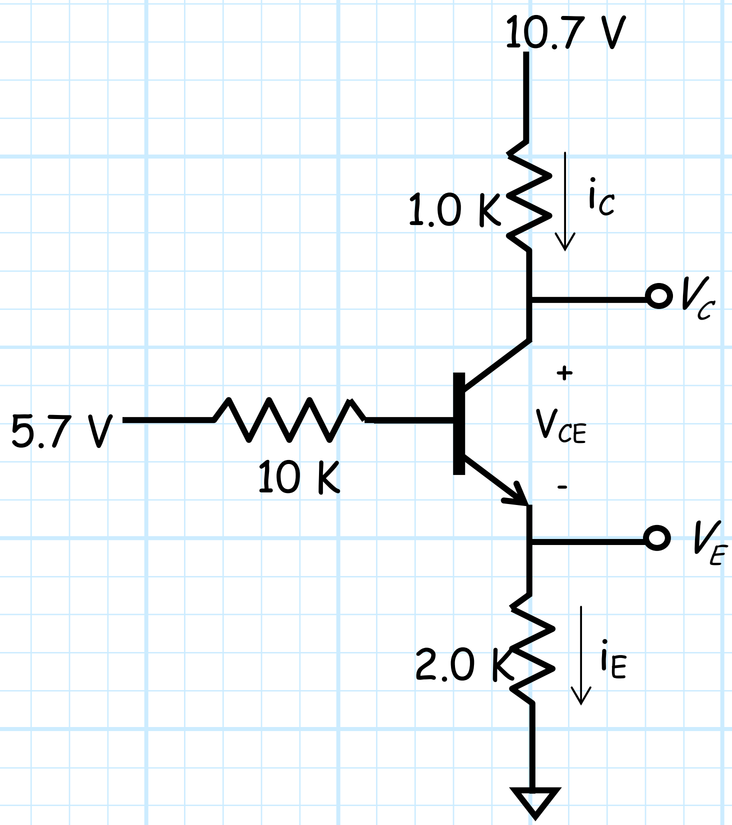
Now for the **voltages** !

Since we know the **currents**, we can find the voltages **using KVL**.

For example, let's **determine** V_{CE}. We can do this **either** by finding the voltage at the **collector** V_C (wrt ground) and voltage at the **emitter** V_E (wrt ground) and then subtracting ($V_{CE} = V_C - V_E$).

OR, we can determine V_{CE} **directly** from **the C-E KVL equation**.

$$\begin{aligned} V_C &= 10.7 - I_C(1) \\ &= 10.7 - 2.36 \\ &= 8.34 \quad \text{V} \end{aligned}$$



and:

$$\begin{aligned} V_E &= 0 + I_E(2) \\ &= 0 + 4.76 \\ &= 4.76 \quad \text{V} \end{aligned}$$

Therefore,

$V_{CE} = V_C - V_E =$ <u>3.58 V</u>

Note we could have **likewise** written the C-E KVL:

$$10.7 - I_C(1) - V_{CE} - I_E(2) = 0$$

Therefore,

$$V_{CE} = 10.7 - I_C(1) - I_E(2) = 3.58 \qquad \text{V}$$

Q: *So, I guess we write the collector-base KVL to find V_{CB} ?*

A: You can, but a **wiser** choice would be to simply apply **KVL** to the **transistor** !

I.E., $V_{CE} = V_{CB} + V_{BE}$!!

Therefore $V_{CB} = V_{CE} - V_{BE} = \underline{2.88 \text{ V}}$

Q: *This has been **hard**. I'm glad we're finished !*

A: Finished ! We still have **2 more** steps to go!

Step 4 - CHECK to see if your results are **consistent** with your assumption.

For **active** mode:

$$V_{CE} = 3.58 \text{ V} > 0.7 \text{ V} \checkmark$$

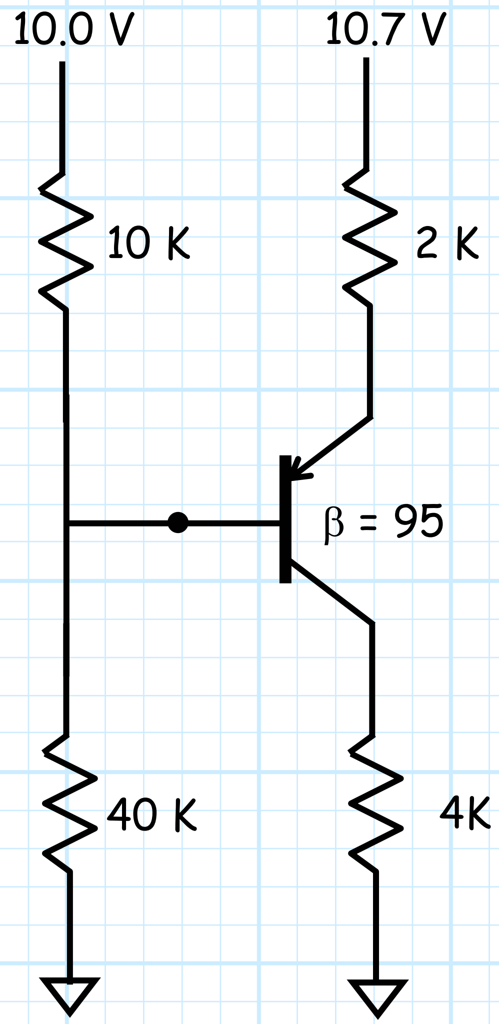
$$I_B = 23.8 \ \mu\text{A} > 0.0 \checkmark$$

Are assumption was **correct**, and therefore so are our **answers** !

No need to go on to Step 5 .

Example: An Analysis of a *pnp* BJT Circuit

Determine the **collector current** and **collector voltage** of the BJT in the circuit below.



1. ASSUME the BJT is in **active** mode.

2. ENFORCE the conditions:

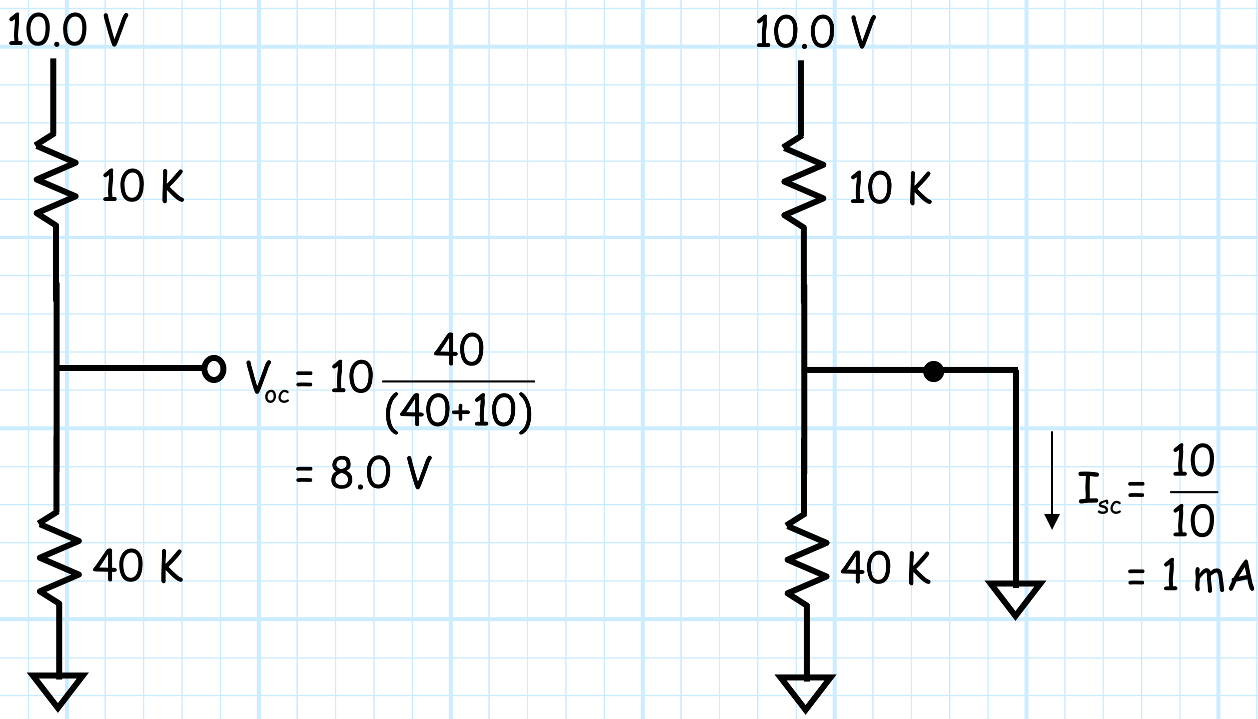
$$V_{EB} = 0.7 \text{ V} \quad \text{and} \quad i_C = \beta \, i_B$$

3. ANALYZE the circuit.

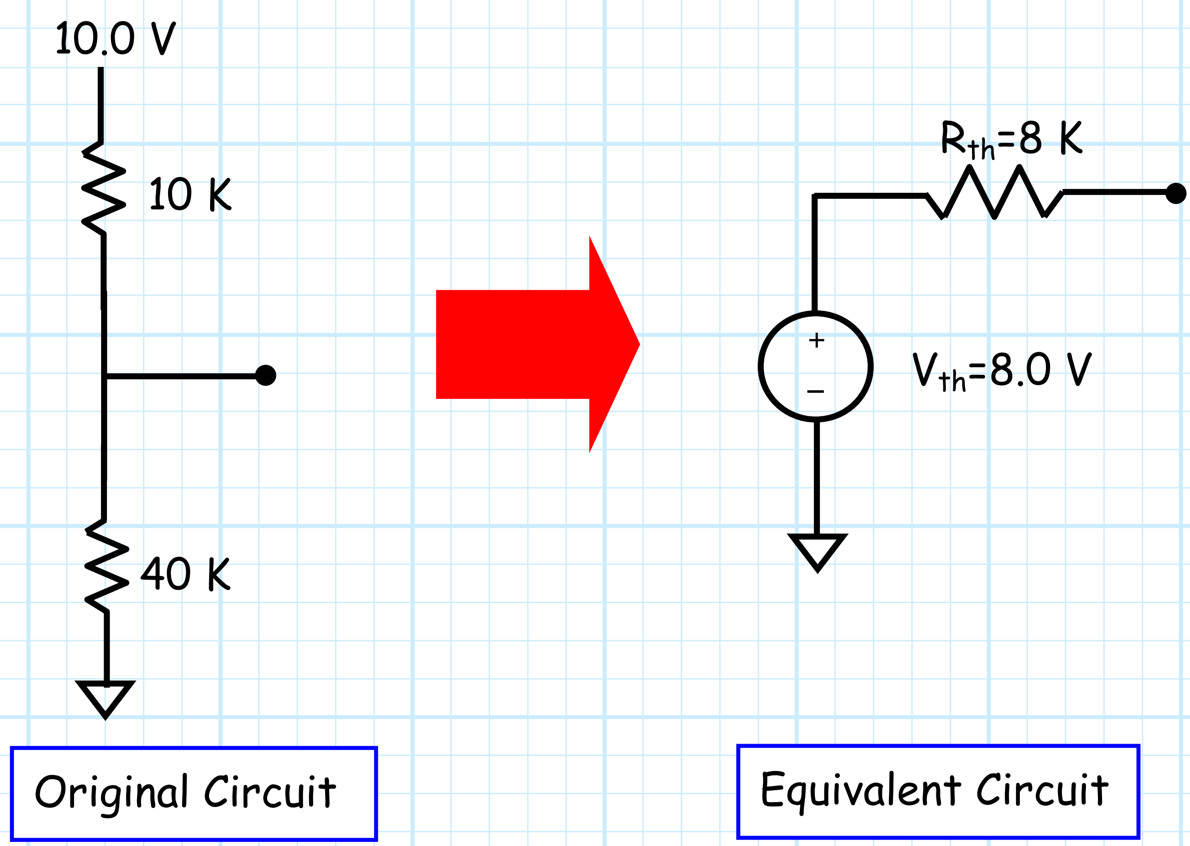
Q: *Yikes ! How do we write the base-emitter KVL ?*

A: This is a perfect opportunity to apply the **Thevenin's** equivalent circuit!

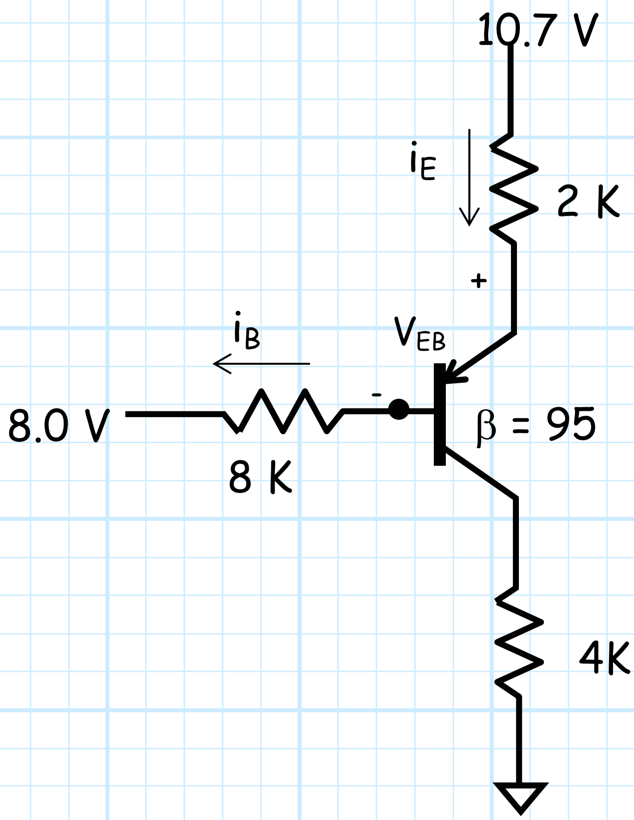
Thevenin's equivalent circuit:



Where $V_{th} = V_{oc} = 8.0$ V and $R_{th} = V_{oc}/I_{sc} = 8/1 = 8$ K



Therefore, we can write the BJT circuit as:



NOW we can easily write the emitter-base leg KVL:

$$10.7 - 2i_E - v_{EB} - 8i_B = 8.0$$

Along with our enforced conditions, we now have **three** equations and **three** unknowns !

Combining, we find:

$$10.7 - 2(96)i_B - 0.7 - 8\ i_B = 8.0$$

Therefore,

$$i_B = \frac{10.7 - 0.7 - 8.0}{2(96) + 8} = \frac{2}{200} = 0.01\text{ mA}$$

and **collector current** i_C is:

$$i_C = \beta\ i_B = 95(0.01) = \underline{0.95\text{ mA}}$$

Likewise, the **collector voltage** (wrt ground) V_C is:

$$V_C = 0.0 + 4\ i_C = \underline{3.8\text{ V}}$$

But wait ! We're **not** done yet ! We must **CHECK** our assumption.

First, i_B = 0.01 mA > 0 ✓

But, what is V_{EC} ??

Writing the emitter-collector KVL:

$$10.7 - 2\, i_E - V_{CE} - 4\, i_C = 0$$

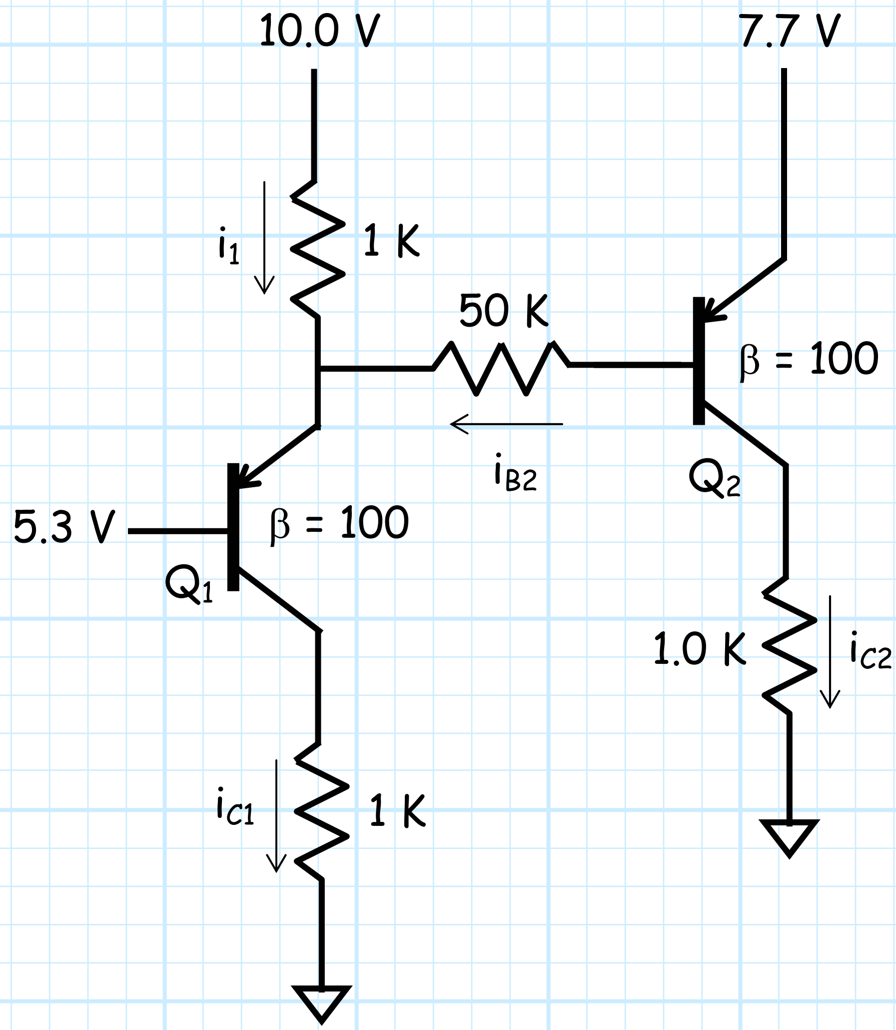
Therefore,

V_{EC} = 10.7 – 2(96) (0.01) – 4(0.95) = 4.98 V > 0.7 V ✓

Our assumption was **correct** !

Example: Another DC Analysis of a BJT Circuit

Find the **collector voltages** of the two BJTs in the circuit below.



ASSUME both BJTs are in **active** mode, therefore ENFORCE

$$V_{EB}^{1} = V_{EB}^{1} = 0.7 \text{ V}, \quad i_{C1} = 100\, i_{B1}, \quad \text{and} \quad i_{C2} = 100\, i_{B2}$$

Q: *Now, how do we ANALYZE the circuit ??*

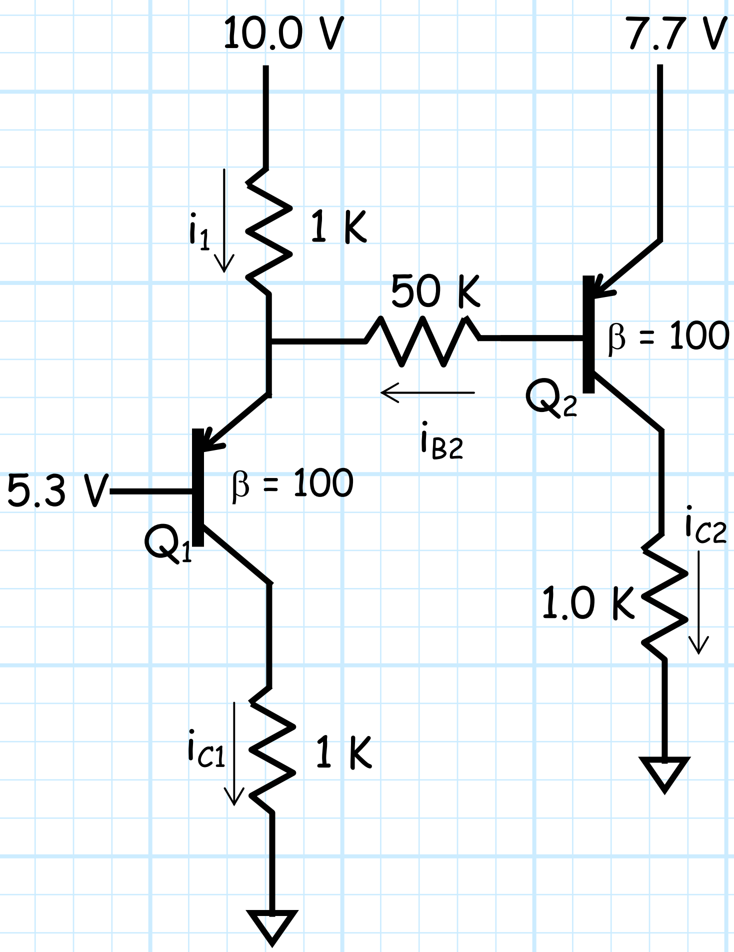
A: This seems to be a problem ! We cannot **easily** solve the emitter base KVL, as i_1 is NOT EQUAL to i_{E1} (make sure you understand this !). Instead, we find:

$$i_{E1} = i_1 + i_{B2}$$

So, what do we do ?

First, ask the question: **What do we know ??**

Look closely at the circuit, it is apparent that $\mathbf{V_{B1} = 5.3\ V}$ and $\mathbf{V_{E2} = 7.7\ V}$.



Hey! We therefore also know $\mathbf{V_{E1}}$ and $\mathbf{V_{B2}}$:

$$V_{E1} = V_{B1} + V_{EB}^{1} = 5.3 + 0.7 = 6.0\ V$$

$$V_{B2} = V_{E2} - V_{EB}^{2} = 7.7 - 0.7 = 7.0\ V$$

Wow ! From these values we get:

$$i_1 = \frac{10 - V_{E1}}{1} = \frac{10-6}{1} = 4\ mA$$

and

$$i_{B2} = \frac{V_{B2} - V_{E1}}{50} = \frac{7-6}{50} = 0.02\ mA$$

This is easy! Since we know i_1 and i_{B2}, we can **find** $\mathbf{i_{E1}}$:

$$i_{E1} = i_1 + i_{B2} = 4.0 + 0.02 = 4.02 \text{ mA}$$

Since we know **one** current for each BJT, we know **all** currents for each BJT:

$$i_{C1} = \alpha\, i_{E1} = \frac{\beta}{\beta+1}\, i_{E1} = \frac{100}{101}\, 4.02 = 3.98 \text{ mA}$$

$$i_{C2} = \beta\, i_{B2} = 100(0.02) = 2 \text{ mA}$$

Finally, we can determine the voltages $\mathbf{V_{C1}}$ and $\mathbf{V_{C2}}$.

$$V_{C1} = 0.0 + 1\, i_{C1} = 0.0 + 1(3.98) = \underline{3.98 \text{ V}}$$

$$V_{C2} = 0.0 + 1\, i_{C2} = 0.0 + 1(2.0) = \underline{2.0 \text{ V}}$$

Now, let's CHECK to see if our assumptions were correct:

$i_{C2} = 2\text{mA} > 0$ ✓ $\qquad$ $i_{C1} = 3.98 \text{ mA} > 0$ ✓

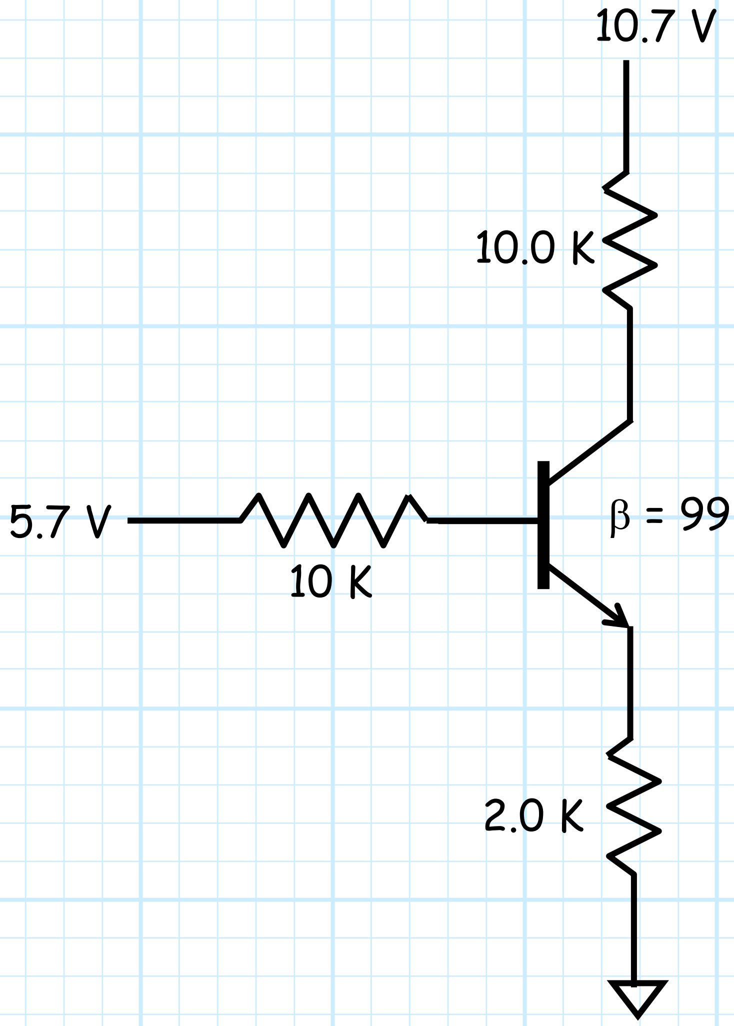
$$V_{EC}^{1} = V_{E1} - V_{C1} = 6.0 - 3.98 = 2.02 \text{ V} > 0.7 \text{ V} \checkmark$$

$$V_{BC}^{2} = V_{B1} - V_{C1} = 7.0 - 2.0 = 5.0 \text{ V} > 0 \checkmark$$

Assumptions are **correct** !

Example: A BJT Circuit in Saturation

Determine all **currents** for the BJT in the circuit below.



*Hey! I remember this circuit, its just like a **previous example**. The BJT is in **active** mode!*

Let's see if you are correct! ASSUME it is in active mode and ENFORCE $V_{CE} = 0.7$ V and $i_C = \beta\ i_B$.

The B-E KVL is therefore:

$$5.7 - 10\, i_B - 0.7 - 2\,(99+1)\, i_B = 0$$

Therefore $i_B = 23.8\ \mu A$

See! Base current i_B = 23.8 μA, just like before. Therefore collector current and emitter current are again $i_C = 99 i_B$ = 2.356 mA and i_E = 100 i_B = 2.380 mA. Right ?!

Well **maybe**, but we still need to CHECK to see if our assumption is correct!

We know that $i_B = 23.8\ \mu A > 0$ ✓ , but what about V_{CE} ?

From collector-emitter KVL we get:

$$10.7 - 10\, i_C - V_{CE} - 2\, i_E = 0$$

Therefore,

$$V_{CE} = 10.7 - 10(2.36) - 2(2.38) = -17.66 \text{ V} < 0.7 \text{ V} \quad \times$$

Our assumption is **wrong** ! The BJT is **not** in active mode.

In the previous example, the collector resistor was **1K** , whereas in this example the collector resistor is **10K**. Thus, there is 10X the **voltage drop** across the collector resistor, which **lowers** the collector voltage so much that the BJT cannot remain in the active mode.

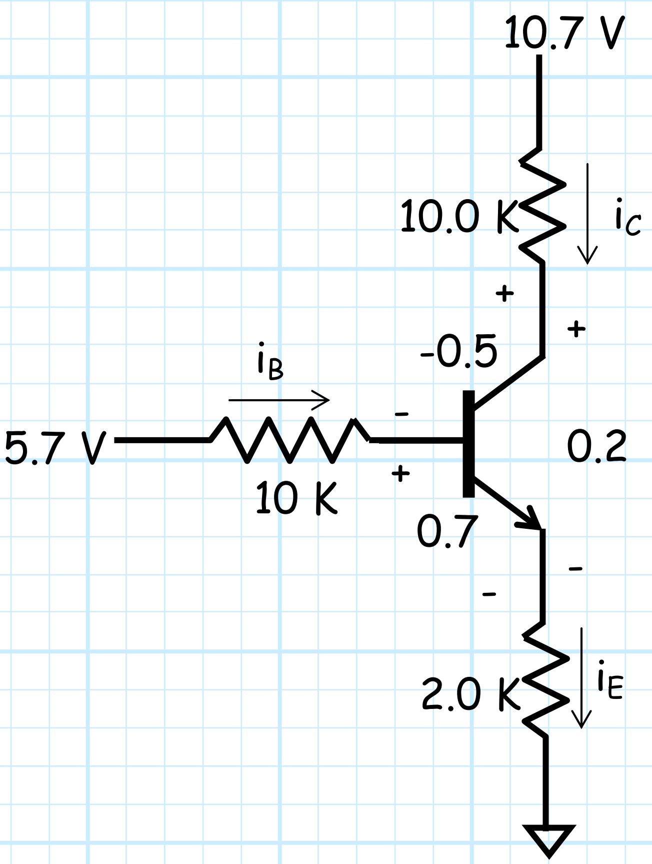
Q: *So what do we do **now** ?*

A: Go to **Step 5**; change the assumption and try it again!

Lets ASSUME instead that the BJT is in **saturation**. Thus, we ENFORCE the conditions:

$$V_{CE} = 0.2 \text{ V} \quad V_{BE} = 0.7 \text{ V} \quad V_{CB} = -0.5 \text{ V}$$

Now lets ANALYZE the circuit !



Note that we **cannot** directly determine the currents, as we **do not** know the base voltage, emitter voltage, or collector voltage.

But, we **do** know the **differences** in these voltages!

For example, we know that the collector voltage is 0.2 V **higher** than the emitter voltage, but we **do not** know what the collector or emitter voltages are!

Q: *So, how the heck do we ANALYZE this circuit !?*

A: Often, circuits with BJTs in **saturation** are somewhat more **difficult** to ANALYZE than circuits with **active** BJTs. There are often **many approaches**, but all result from a logical, systematic application of **Kirchoff's Laws**!

ANALYSIS EXAMPLE 1 - Start with **KCL**

We **know** that $i_B + i_C = i_E$ (KCL)

But, what **are** i_B, i_C, and i_E ??

Well, from **Ohm's Law**:

$$i_B = \frac{5.7 - V_B}{10} \qquad i_C = \frac{10.7 - V_C}{10} \qquad i_E = \frac{V_E - 0}{10}$$

Therefore, combining with KCL:

$$\frac{5.7 - V_B}{10} + \frac{10.7 - V_C}{10} = \frac{V_E}{10}$$

Look what we have, **1 equation** and **3 unknowns**.

→ We need **2** more **independent** equations involving V_B, V_C, and V_E!

Q: ***Two more*** *independent equations !? It looks to me as if we have written all that we can about the circuit using Kirchoff's Laws.*

A: True! There are no more **independent** circuit equations that we can write using KVL or KCL ! But, recall the hint sheet:

"Make sure you are using ***all*** *available information".*

There is more **information** available to us – the ENFORCED conditions!

$$V_{CE} = V_C - V_E = 0.2 \quad \Rightarrow \quad V_C = V_E + 0.2$$

$$V_{BE} = V_B - V_E = 0.7 \quad \Rightarrow \quad V_B = V_E + 0.7$$

Two more **independent** equations! Combining with the earlier equation:

$$\frac{5.7 - (0.7 + V_E)}{10} + \frac{10.7 - (0.2 + V_E)}{10} = \frac{V_E}{10}$$

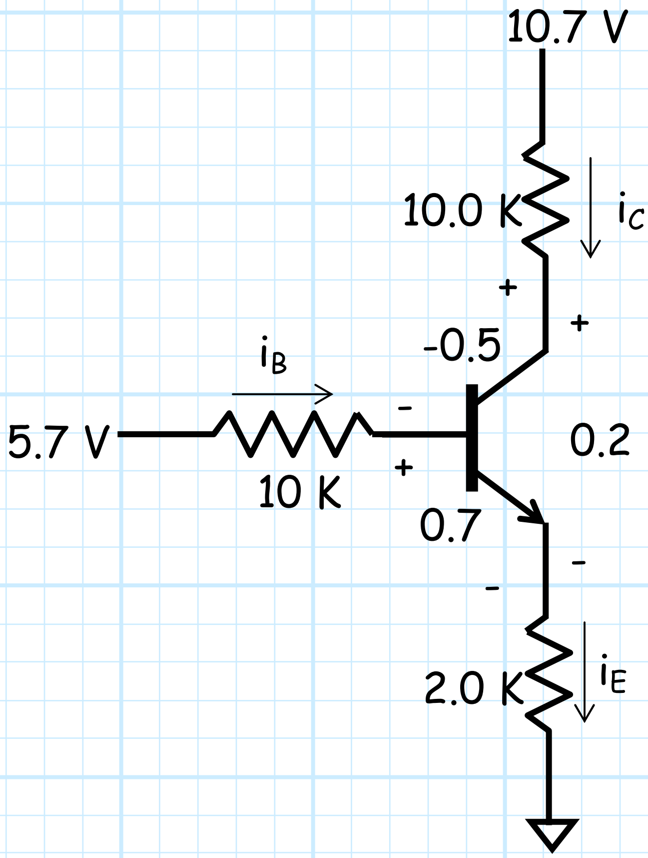
One equation and **one** unknown ! Solving, we get $\mathbf{V_E = 2.2\ V}$.

Inserting this answer into the above equations, we get:

$$V_B = 2.9\text{ V} \qquad V_C = 2.4\text{ V}$$

$$i_C = 0.83\text{ mA} \qquad i_B = 0.28\text{ mA} \qquad i_E = 1.11\text{ mA}$$

ANALYSIS EXAMPLE 2 - Start with **KVL**



We can write the KVL equation for any **two** circuit legs:

B-E KVL:

$$5.7 - 10\, i_B - 0.7 - 2\, i_E = 0.0$$

C-E KVL:

$$10.7 - 10\, i_C - 0.2 - 2\, i_E = 0.0$$

Note the ENFORCED conditions are **included** in these KVL equations.

Simplifying, we get these **2** equations with **3** unknowns:

$$5.0 = 10\, i_B + 2\, i_E$$

$$10.5 = 10\, i_C + 2\, i_E$$

We need **one more** independent equation involving i_B, i_C, and i_E.

Try KCL ! $i_B + i_C = i_E$

Inserting the KCL equation into the 2 KVL equations, we get:

$$5.0 = 12\, i_B + 2\, i_C$$

$$10.5 = 2\, i_B + 12\, i_C$$

Solving, we get the **same answers** as in analysis example 1.

> **Lesson:** There are **multiple** strategies for analyzing these circuits; use the ones that you feel most **comfortable** with !

However you ANALYZE the circuit, you **must** in the end also CHECK your results.

First CHECK to see that **all** currents are **positive**:

$i_C = 0.83$ mA > 0 ✓ $i_B = 0.28$ mA > 0 ✓ $i_E = 1.11$ mA > 0 ✓

Also CHECK **collector current**:

$$i_C = 0.83 \text{ mA} < \beta\, i_B = 27.7 \text{ mA}$$ ✓

Our solution is **correct** !!!