Amplifier Gain

Note that an amplifier is a two-port device.

As a result, we can describe an amplifier with a 2 x 2 scattering matrix:

\[
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\]

Q: What is the scattering matrix of an ideal amplifier??

A: Let's start with \( S_{11} \) and \( S_{22} \).

To insure maximum power transfer, the input and output ports would ideally be matched:

\[ S_{11} = S_{22} = 0 \]

Now, let's look at scattering parameter \( S_{21} \). We know that:

\[ P_2^- = |S_{21}|^2 P_1^+ \]
or, stated another way:

\[ P_{out} = |S_{21}|^2 P_{in} \]

Therefore, we can define the amplifier power gain as:

\[ G = \frac{P_{out}}{P_{in}} = |S_{21}|^2 \]

As the purpose of an amplifier is to boost the signal power, we can conclude that ideally:

\[ |S_{21}| \gg 1 \]

Clearly, an amplifier must be an active device!

As discussed earlier, the gain of an amplifier will change with signal frequency:

\[ G(\omega) = |S_{21}(\omega)|^2 \]

When radio engineers speak of amplifier gain, they almost always are speaking of this power gain \( G \). However, they do not generally state it as a specific function of frequency!

Rather, amplifier gain is typically specified as a numeric value such as \( G = 20 \) or \( G = 13 \) dB. This value is a statement of the approximate amplifier gain within the amplifier bandwidth.
Thus, amplifier gain and bandwidth are the two most fundamental performance specifications of any microwave amplifier—together they (approximately) describe the amplifier transfer function!

Additionally, radio engineers almost always speak of amplifier gain in **decibels** (dB):

\[
G(dB) = 10 \log_{10} G
\]

Finally, let’s consider \( S_{12} \). This scattering parameter relates the wave into port 2 (the output) to the wave out of port 1 (the input).

**Q:** Are amplifiers **reciprocal** devices? In other words, is \( S_{12} = S_{21} \)??
A: No! An amplifier is strictly a directional device; there is a specific input, and a specific output—it does not work in reverse!

Ideally, $S_{12} = 0$. Any other value can just cause problems!

Typically though, $S_{12}$ is small, but not zero. Generally speaking, radio engineers express $S_{12}$ as a value called reverse isolation:

$$\text{reverse isolation} = -10 \log_{10} |S_{12}|^2$$

Note when $S_{12} = 0$, reverse isolation will be infinite. Thus, the larger the reverse isolation, the better!

Summarizing, we find that the scattering matrix of the ideal amplifier is:

$$\overline{S}_{\text{ideal}} = \begin{bmatrix} 0 & 0 \\ S_{21} & 0 \end{bmatrix} \quad \text{where } |S_{21}| \gg 1$$

Sort of like an isolator with gain!

The non-ideal reality is that the zero valued terms will be small, but not precisely zero. Moreover, each scattering parameter will change with signal frequency—although they remain approximately constant within the amplifier bandwidth.