An ideal amplifier takes an input signal and reproduces it exactly at its output, only with a larger magnitude!

Now, let’s express this result using our knowledge of linear circuit theory!

Recall, the output $v_{out}(t)$ of a linear device can be determined by convolving its input $v_{in}(t)$ with the device impulse response $g(t)$:

$$v_{out}(t) = \int_{-\infty}^{t} g(t - t')v_{in}(t')dt'$$

where $A_{vo}$ is the open-circuit voltage gain of the amplifier.
The impulse response for the ideal amplifier would therefore be:

\[ g(t) = A_v \delta(t) \]

so that:

\[ v_{\text{out}}(t) = \int_{-\infty}^{t} g(t - t')v_{\text{in}}(t')dt' \]

\[ = \int_{-\infty}^{t} A_v \delta(t - t')v_{\text{in}}(t')dt' \]

\[ = A_v v_{\text{in}}(t) \]

We can alternatively represent the ideal amplifier response in the frequency domain, by taking the Fourier Transform of the impulse response:

\[ T(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt \]

\[ = \int_{-\infty}^{\infty} A_v \delta(t)e^{-j\omega t}dt \]

\[ = A_v + j0 \]

This result, although simple, has an interesting interpretation. It means that the amplifier exhibits gain of \( A_v \) for sinusoidal signals of any and all frequencies!
Moreover, the ideal amplifier does not alter the relative phase of the sinusoidal signal (i.e., no phase shift).

In other words, if:

\[ v_{in}(t) = \cos(\omega t) \]

then at the output of the ideal amplifier we shall see:

\[ v_{out}(t) = |T(\omega)|\cos(\omega t + \angle T(\omega)) = A_v \cos(\omega t) \]

BUT, there is one big problem with an ideal amplifier:

They are **impossible** to build !!

**Q:** Why is that ??

**A:** Two reasons:

a) An ideal amplifier has **infinite** bandwidth.

b) An ideal amplifier has **zero** delay.

Not gonna happen !
Let's look at this second problem first. The ideal amplifier impulse response $g(t) = A_o \delta(t)$ means that the signal at the output occurs *instantaneously* with the signal at the input.

This of course *cannot* happen, as it takes some small, but non-zero amount of time for the signal to propagate through the amplifier. A more realizable amplifier impulse response is:

$$g(t) = A_o \delta(t - \tau)$$

resulting in an amplifier output of:

$$v_{out}(t) = \int_{-\infty}^{t} g(t - t') v_{in}(t')dt'$$

$$= \int_{-\infty}^{t} A_o \delta(t - \tau - t') v_{in}(t')dt'$$

$$= A_o v_{in}(t - \tau)$$

In other words, the output is both an amplified and *delayed* version of the input.

* Ideally, this delay does not *distort* the signal, as the output will have the same form as the input.

* Moreover, the delay for electronic devices such as amplifiers is *very small* in comparison to human time scales (i.e., $\tau \ll 1$ second).
* Therefore, propagation delay $\tau$ is generally not considered a problem for most amplifier applications.

Let's examine what this delay means in the frequency domain.

Evaluating the Fourier Transform of this modified impulse response gives:

\[
T(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt
\]

\[
= \int_{-\infty}^{\infty} A_v \delta(t - \tau) e^{-j\omega t} dt
\]

\[
= A_v \cos(\omega \tau) + j A_v \sin(\omega \tau)
\]

\[
= A_v e^{j\omega \tau}
\]

We see that, as with the ideal amplifier, the magnitude $|T(\omega)| = A_v$. However, the relative phase is now a linear function of frequency:

\[
\angle T(\omega) = \omega \tau
\]

As a result, if $v_{in}(t) = \cos(\omega t)$, the output signal will be:

\[
v_{out}(t) = |T(\omega)| \cos(\omega t - \angle T(\omega))
\]

\[
= A_v \cos(\omega t - \omega \tau)
\]

In other words, the output signal of a real amplifier is phase shifted with respect to the input.
In general, the amplifier phase shift $\angle T(\omega)$ will not be a perfectly linear function (i.e., $\angle T(\omega) \neq \omega \tau$), but instead will be a more general function of frequency $\omega$.

However, if the phase function $\angle T(\omega)$ becomes too “non-linear”, we find that signal dispersion can result—the output signal can be distorted!

Now, let’s examine the first problem with the ideal amplifier. This problem is best discussed in the frequency domain.

We discovered that the ideal amplifier has a frequency response of $|T(\omega)| = A_\omega$. Note this means that the amplifier gain is $A_\omega$ for all frequencies $0 < \omega < \infty$ (D.C. to daylight!).

The bandwidth of the ideal amplifier is therefore infinite!

* Since every electronic device will exhibit some amount of inductance, capacitance, and resistance, every device will have a finite bandwidth.

* In other words, there will be frequencies $\omega$ where the device does not work!

* From the standpoint of an amplifier, “not working” means $|T(\omega)| \ll A_\omega$ (i.e., low gain).

* Amplifiers will therefore have finite bandwidths.
There is a range of frequencies $\omega$ between $\omega_L$ and $\omega_H$ where the gain will (approximately) be $A_v$. For frequencies outside this range, the gain will typically be small (i.e. $|T(\omega)| \ll A_v$):

$$|T(\omega)| = \left\{ \begin{array}{ll}
A_v & \omega_L < \omega < \omega_H \\
\ll A_v & \omega < \omega_L \text{ or } \omega > \omega_H \end{array} \right.$$  

The width of this frequency range is called the amplifier bandwidth:

- **Bandwidth** $= \omega_H - \omega_L$ (radians/sec)
- $= f_H - f_L$ (cycles/sec)

**One result of having a finite bandwidth is that the amplifier impulse response is not an impulse function!**

$$g(t) = \int_{-\infty}^{\infty} T(\omega) e^{j\omega t} dt \neq A_v \delta(t - \tau)$$  

The **ideal** amplifier is not possible!