Antenna Gain

Recall that directivity pattern is:

\[ D(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{rad}} \]

Yet, because of ohmic and return losses, we find that:

\[ P_t \geq P_{ant} \geq P_{rad} \]

We find that \( P_{rad} \) is difficult to measure, whereas \( P_t \) and/or \( P_{ant} \) is not.
So, we define a new parameter, called antenna gain $G(\theta, \phi)$

$$G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_t}$$

or, assuming the antenna is matched:

$$G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{ant}}$$

Note this means that

$$\frac{G(\theta, \phi)}{D(\theta, \phi)} = \frac{P_{rad}}{P_{ant}}$$

But recall that $P_{rad} = eP_{ant}$, where $e$ is antenna efficiency.

In other words,

$$\frac{G(\theta, \phi)}{D(\theta, \phi)} = e$$
\[ G(\theta, \phi) = e \cdot D(\theta, \phi) \]

and since \( e \leq 1 \), we find

\[ G(\theta, \phi) \leq D(\theta, \phi). \]

Note that since \( e \) is a constant (with respect to \( \theta \) and \( \phi \)) we find that the patterns \( G(\theta, \phi) \) and \( D(\theta, \phi) \) are the same, only gain \( G(\theta, \phi) \) is slightly smaller than \( D(\theta, \phi) \) at every direction.

\( \text{e.g.,} \)
As a result, we find that the maximum value of the gain pattern $g(\theta, \phi)$ is:

$$g_0 = e \cdot D_0.$$