Example: Input Impedance
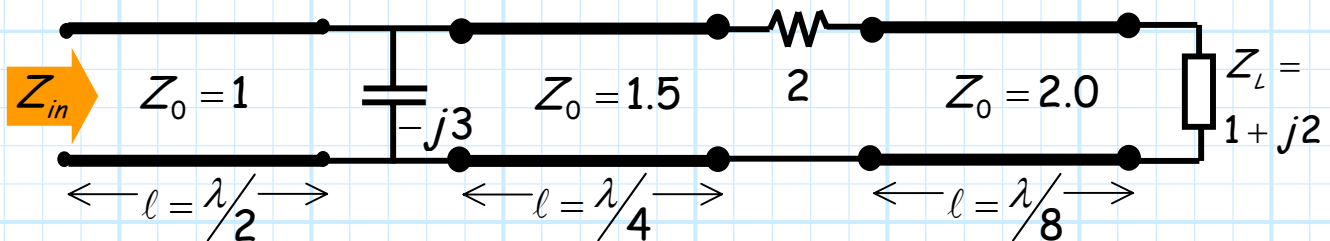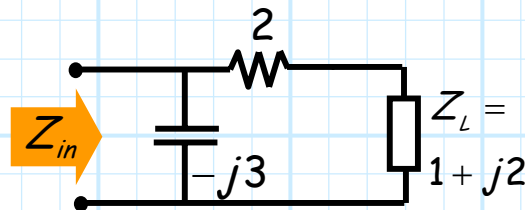
Consider the following circuit:

If we **ignored** our new μ-wave knowledge, we might **erroneously** conclude that the input impedance of this circuit is:
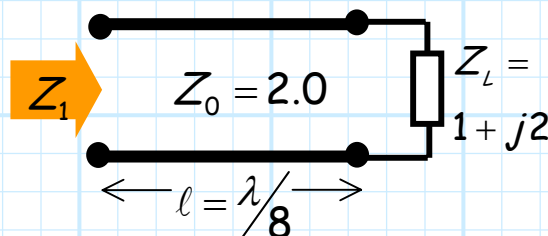

Therefore:

$$Z_{in} = \frac{-j3(2+1+j2)}{-j3+2+1+j2} = \frac{6-j9}{3-j} = 2.7 - j2.1$$
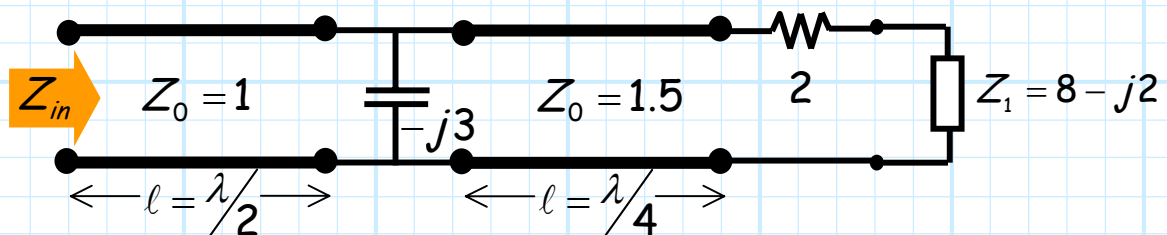
Of course, this is **not** the correct answer!

We must use our **transmission line theory** to determine an accurate value. Define Z_1 as the input impedance of the last section:
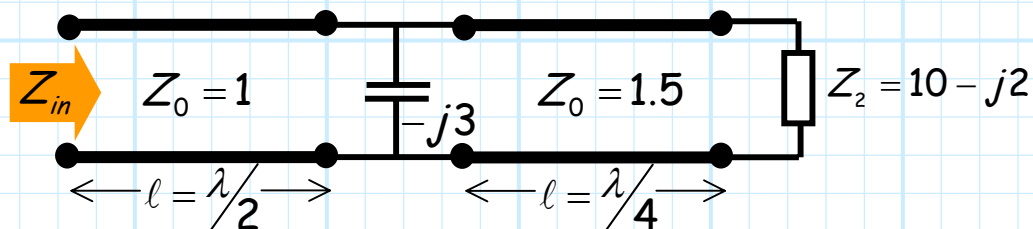
we find that Z_1 is :

$$Z_1 = Z_0 \left(\frac{Z_L \cos \beta \ell + j\, Z_0 \sin \beta \ell}{Z_0 \cos \beta \ell + j\, Z_L \sin \beta \ell} \right)$$

$$= 2 \left(\frac{(1+j2) \cos\left(\pi/4\right) + j\, 2 \sin\left(\pi/4\right)}{2 \cos\left(\pi/4\right) + j(1+j2)\ \sin\left(\pi/4\right)} \right)$$

$$= 2 \left(\frac{1+j4}{j} \right)$$

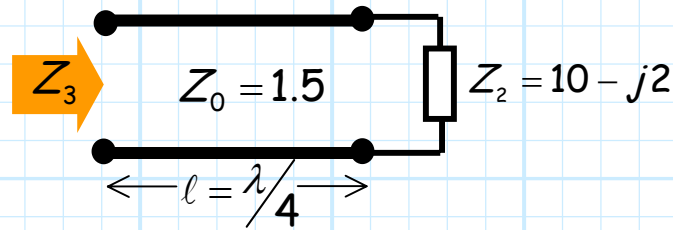$$= 8 - j2$$

Therefore, our circuit now becomes:

Note the resistor is in **series** with impedance Z_1. We can **combine** these two into one impedance defined as Z_2:

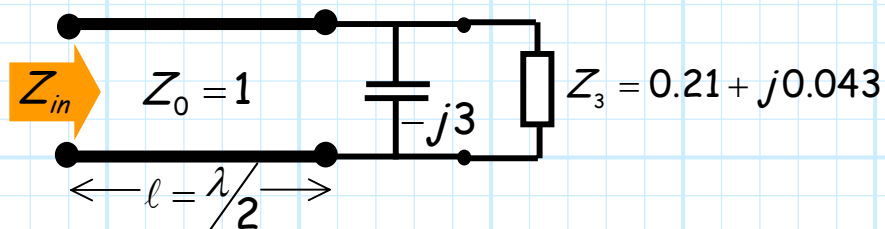$$Z_2 = 2 + Z_1 = 2 + (8 - j2) = 10 - j2$$

Now let's define the input impedance of the **middle** transmission line section as Z_3:

Note that this transmission line is a **quarter wavelength** ($\ell = \lambda/4$). This is one of the **special** cases we considered earlier! The input impedance Z_3 is:

$$Z_3 = \frac{Z_0^2}{Z_L}$$

$$= \frac{Z_0^2}{Z_2}$$
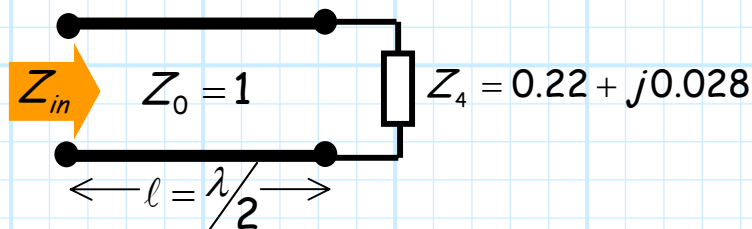
$$= \frac{1.5^2}{10 - j2}$$

$$= 0.21 + j0.043$$

Thus, we can further **simplify** the original circuit as:

Now we find that the impedance Z_3 is **parallel** to the capacitor. We can **combine** the two impedances and define the result as impedance Z_4:

$$Z_4 = -j3 \| (0.21 + j0.043)$$
$$= \frac{-j3(0.21 + j0.043)}{-j3 + 0.21 + j0.043}$$
$$= 0.22 + j0.028$$

Now we are left with **this** equivalent circuit:

Note that the remaining transmission line section is a **half wavelength**! This is one of the **special** situations we discussed in a previous handout. Recall that the **input** impedance in this case is simply equal to the **load** impedance:
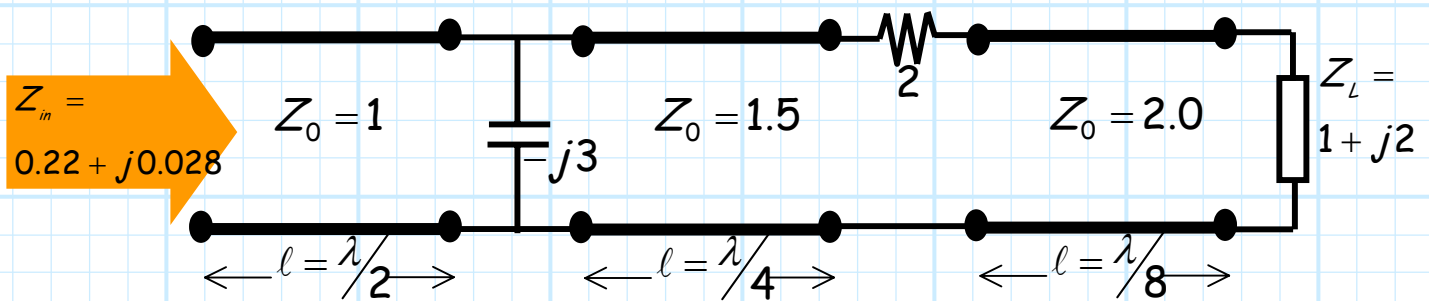
$$Z_{in} = Z_L = Z_4 = 0.22 + j0.028$$

Whew! We are **finally** done. The **input impedance** of the original circuit is:
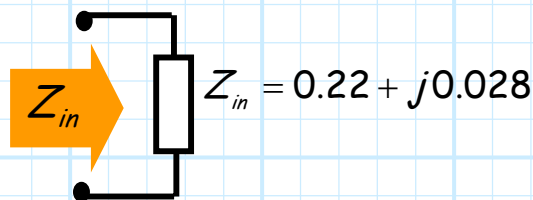
Note this means that **this** circuit:

$$Z_{in} = 0.22 + j0.028$$

and **this** circuit:

$$Z_{in} = 0.22 + j0.028$$
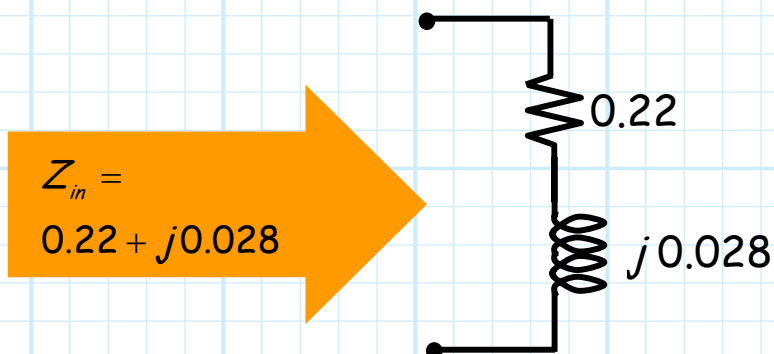
are precisely the **same**! They have **exactly** the same impedance, and thus they "behave" precisely the **same** way in any circuit (but **only** at frequency ω_0!).