Filter Design Worksheet

Q: Given the order of a Butterworth or Chebychev bandpass filter, what will my stop band attenuation be? Or stated another way, what should the order of my filter be, in order to achieve a desired amount of stop-band attenuation ($-10 \log_{10} T(\omega)$)??

A: Consult the normalized attenuation charts (They’re in your book)!

For example, the normalized attenuation chart for a Butterworth filter is:
While the normalized attenuation chart for a **Chebychev** with **0.5 dB of passband ripple** is:

![Normalized attenuation chart for Chebychev with 0.5 dB of passband ripple](image1)

And the normalized attenuation chart for a **Chebychev** with **3.0 dB of passband ripple** is:

![Normalized attenuation chart for Chebychev with 3.0 dB of passband ripple](image2)
Q: Great, how the heck do I use these??

A: The variable $\alpha$ is a normalized frequency variable. The plots show attenuation versus frequency for a filter of order $n$.

Say we have a bandpass filter, whose (3 dB) passband extends from $f_1$ to $f_2$ ($f_2 > f_1$). The bandwidth of this filter would therefore be $f_2 - f_1$.

Using these values, we can define a normalized frequency $\alpha$ as:

$$\alpha = \left| \frac{1}{\Delta} \left( \frac{f - f_0}{f_0 - f} \right) \right| - 1$$

where:

$$f_0 = \sqrt{f_1 f_2}$$

$$\Delta = \frac{f_2 - f_1}{f_0}$$

Thus, given a frequency $f$, we can calculate a value $\alpha$.

* It turns out that all frequencies $f$ outside the pass band of the filter will have positive values of $\alpha$, while frequencies within the pass band will result in negative values of $\alpha$.

* Accordingly, if $f = f_1$ or $f = f_2$, the value of $\alpha$ will be zero (try it!).
* As a result, the attenuation charts give answers for positive values of \( \alpha \) only, corresponding to frequencies in the stop band.

* In other words, the attenuation charts provide information about the stop band attenuation only. Note as \( \alpha \) gets larger, the attenuation for all filter orders increases.

* This makes since, as an increasing \( \alpha \) corresponds to a frequency \( f \) either greater than \( f_2 \) and increasing, or a frequency \( f \) less than \( f_1 \) and decreasing.
For example, consider a Butterworth bandpass filter whose passband extends from 1 GHz to 4 GHz.

Therefore, \( f_1 = 1 \text{ GHz} \) and \( f_2 = 4 \text{ GHz} \), resulting in \( f_0 = 2 \text{ GHz} \) and \( \Delta = 1.5 \).

**Q1:** By how much is a 500 MHz signal attenuated if the filter has order \( n=6 \)?

For \( f = 0.5 \text{ GHz} \):

\[
\alpha = \left| \frac{1}{\Delta} \left( \frac{f}{f_0} - \frac{f_0}{f} \right) \right| - 1
\]

\[
= \left| \frac{1}{1.5} \left( \frac{0.5}{2.0} - \frac{2.0}{0.5} \right) \right| - 1
\]

\[
= 1.5
\]

It appears from the attenuation chart that this filter attenuates a 500 MHz signal approximately 50 dB.

**Q2:** What should the filter order \( n \) be, if we need to attenuate signals at 6.6 GHz by at least 40 dB?

For \( f = 8 \text{ GHz} \):

\[
\alpha = \left| \frac{1}{\Delta} \left( \frac{f}{f_0} - \frac{f_0}{f} \right) \right| - 1
\]

\[
= \left| \frac{1}{1.5} \left( \frac{6.6}{2.0} - \frac{2.0}{6.6} \right) \right| - 1
\]

\[
= 1.0
\]
Again from the chart, we find at $\alpha = 1.0$, a filter with order $n = 7$ (or higher) will attenuate a 6.6 GHz signal by more than 40 dB.

Now you too can determine filter attenuation and/or order. I hope you’ve been paying attention!!