Line Impedance

Now let’s define line impedance \( Z(z) \), which is simply the ratio of the complex line voltage and complex line current:

\[
Z(z) = \frac{V(z)}{I(z)}
\]

**Q:** Hey! I know what this is! The ratio of the voltage to current is simply the characteristic impedance \( Z_0 \), right???

**A:** NO! The line impedance \( Z(z) \) is (generally speaking) NOT the transmission line characteristic impedance \( Z_0 \)!!!

⇒ It is unfathomably important that you understand this!!!!

To see why, recall that:

\[
V(z) = V^+(z) + V^-(z)
\]
And that:

\[ I(z) = \frac{V^+(z) - V^-(z)}{Z_0} \]

Therefore:

\[ Z(z) = \frac{V(z)}{I(z)} = Z_0 \left( \frac{V^+(z) + V^-(z)}{V^+(z) - V^-(z)} \right) \neq Z_0 \]

Or, more specifically, we can write:

\[ Z(z) = Z_0 \left( \frac{V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}}{V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}} \right) \]

Q: I'm confused! Isn't:

\[ \frac{V^+(z)}{I^+(z)} = Z_0 \]???

A: Yes! That is true! The ratio of the voltage to current for each of the two propagating waves is \( \pm Z_0 \). However, the ratio of the sum of the two voltages to the sum of the two currents is not equal to \( Z_0 \) (generally speaking)!

This is actually confirmed by the equation above. Say that \( V^-(z) = 0 \), so that only one wave \( (V^+(z)) \) is propagating on the line.
In this case, the ratio of the total voltage to the total current is simply the ratio of the voltage and current of the one remaining wave—the characteristic impedance $Z_0$!

$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \left( \frac{V^+(z)}{V^+(z)} \right) = \frac{V^+(z)}{I^+(z)} = Z_0 \quad \text{(when } V^+(z) \text{)}$$

**Q:** So, it appears to me that characteristic impedance $Z_0$ is a **transmission line parameter**, depending only on the transmission line values $R$, $G$, $L$, and $C$.

**A:** Exactly! Moreover, note that characteristic impedance $Z_0$ is simply a **number**, whereas line impedance $Z(z)$ is a **function** of position $(z)$ on the transmission line.

**Q:** Therefore, it appears to me that characteristic impedance $Z_0$ is a **transmission line parameter**, depending only on the transmission line values $R$, $G$, $L$, and $C$.

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**A:** **Exactly!** Moreover, note that characteristic impedance $Z_0$ is simply a **number**, whereas line impedance $Z(z)$ is a **function** of position $(z)$ on the transmission line.