Microwave Filter Design

Recall that a *lossless* filter can be described in terms of either its power transmission coefficient $T(\omega)$ or its power reflection coefficient $\Gamma(\omega)$, as the two values are completely dependent:

$$\Gamma(\omega) = 1 - T(\omega)$$

**Ideally**, these functions would be quite simple:

1. $T(\omega) = 1$ and $\Gamma(\omega) = 0$ for all frequencies within the pass-band.

2. $T(\omega) = 0$ and $\Gamma(\omega) = 1$ for all frequencies within the stop-band.

For example, the **ideal** low-pass filter would be:

![Diagram of ideal low-pass filter](image-url)
Add to this a **linear phase** response, and you have the **perfect** microwave filter!

There’s just one small problem with this **perfect** filter → It’s **impossible** to build!

Now, if we consider only possible (i.e., **realizable**) filters, we must limit ourselves to filter functions that can be expressed as **finite polynomials** of the form:

\[
T(\omega) = \frac{a_0 + a_1 \omega + a_2 \omega^2 + \cdots}{b_0 + b_1 \omega + b_2 \omega^2 + \cdots + b_N \omega^N}
\]

The **order** \( N \) of the (denominator) polynomial is likewise the **order** of the filter.

There are many different **types** of polynomials that result in good filter responses, and each type has its own set of **characteristics**.

The type of **polynomial** likewise describes the type of microwave **filter**. Let’s consider **three** of the most popular types:

1. **Elliptical**

**Elliptical** filters have three primary characteristics:

a) They exhibit very **steep “roll-off”**, meaning that the transition from pass-band to stop-band is very rapid.
b) They exhibit ripple in the pass-band, meaning that the value of $T$ will vary slightly within the pass-band.

c) They exhibit ripple in the stop-band, meaning that the value of $T$ will vary slightly within the stop-band.

![Graph showing pass-band and stop-band ripple](image)

We find that we can make the roll-off steeper by accepting more ripple.

2. Chebychev

Chebychev filters are also known as equal-ripple filters, and have two primary characteristics

a) Steep roll-off (but not as steep as Elliptical).

b) Pass-band ripple (but not stop-band ripple).
We likewise find that the roll-off can be made steeper by accepting more ripple.

3. Butterworth

Also known as maximally flat filters, they have two primary characteristics

a) **Gradual** roll-off.

b) **No ripple**—not anywhere.
Q: So we always chose elliptical filters; since they have the steepest roll-off, they are closest to ideal—right?

A: Ooops! I forgot to talk about the phase response $\angle S_{21}(\omega)$ of these filters. Let's examine $\angle S_{21}(\omega)$ for each filter type before we pass judgment.

Butterworth $\angle S_{21}(\omega)$ $\rightarrow$ Close to linear phase.

Chebychev $\angle S_{21}(\omega)$ $\rightarrow$ Not very linear.

Elliptical $\angle S_{21}(\omega)$ $\rightarrow$ A big non-linear mess!

Thus, it is apparent that as a filter roll-off improves, the phase response gets worse (watch out for dispersion!).

$\rightarrow$ There is no such thing as the “best” filter type!

Q: So, a filter with perfectly linear phase is impossible to construct?

A: No, it is possible to construct a filter with near perfect linear phase—but it will exhibit a horribly poor roll-off!
Now, for any type of filter, we can improve roll-off (i.e., increase stop-band attenuation) by increasing the filter order $N$. However, be aware that increasing the filter order likewise has these deleterious effects:

1. It makes phase response $\angle S_{21}(\omega)$ worse (i.e., more non-linear).

2. It increases filter cost, weight, and size.

3. It increases filter insertion loss (this is bad).

4. It makes filter performance more sensitive to temperature, aging, etc.

From a practical viewpoint, the order of a filter should typically be kept to $N < 10$.

**Q:** So exactly what are these filter polynomials $T(\omega)$? How do we determine them?

**A:** Fortunately, radio engineers do not need to determine specific filter polynomials in order to specify (to filter manufacturers) what they want built.

Instead, radio engineers simply can specify the type and order of a filter, saying things like:
"I need a 3rd-order Chebychev filter!"

or

"Get me a 5th-order Butterworth filter!"

or

"I wish I’d paid more attention in EECS 622!"

Thus, the most important filter specifications are:

1. Filter bandwidth and center frequency

2. Filter type and order.

However, there are many more important filter specifications!