Phase Noise

There are also short-term instabilities (e.g., msec to \( \mu \text{sec} \)) in oscillator frequency!

We can model these as:

\[
\nu_c(t) = a \cos(\omega_0 t + \phi_n(t))
\]

where the relative phase \( \phi_n(t) \) is a random process called phase noise.

Q: It looks a lot like phase modulation!

A: Essentially, it is.

The random process \( \phi_n(t) \) has a small magnitude, i.e.:

\[
|\phi_n(t)| \ll 1
\]

Note since the phase changes as a function of time, the frequency will as well! Specifically:

\[
\omega(t) = \frac{d(\omega_0 t + \phi_n(t))}{dt} = \omega_0 + \frac{d\phi_n(t)}{dt} = \omega_0 + \omega_n(t)
\]
where:

$$\omega_n(t) = \frac{d\phi_n(t)}{dt}$$

As a result, the frequency of the oscillator is also a random process.

I.E., the oscillator frequency changes randomly as a function of time!

This random fluctuation spreads the oscillator signal spectrum.

In other words, instead of the spectrum of a perfect, "pure" tone:
we get a wider, **imperfect** spectrum:

\[ \frac{W}{Hz} \]

\[ f_0 \]

In this case, we say our oscillator has **spectral impurities**!

* Since the phenomenon of phase noise is a random process, we must describe the signal spectrum in terms of its **average** spectral power density.

* Spectral Power Density is expressed in units of **Watts/Hz**.

* For **white** noise, the spectral power density is a **constant** with respect to frequency:
* However, for phase noise, the resulting spectral power density changes as a function of frequency!

Specifically, the average spectral power density of an oscillator increases as frequency $f$ nears the nominal signal (i.e., carrier) frequency $f_0$. 

![Graph showing the change in average spectral power density as frequency changes.](image)

- Average SPD is relatively large near $f_0$.
- Average SPD is small away from $f_0$. 

![Graph showing the average spectral power density.](image)
Now, although we typically express average spectral power density in Watts/Hz or dBm/Hz, we generally express the spectral power density of an oscillator output in dBc!

In other words, we are only concerned about the magnitude of the phase noise spectral power density in comparison to the oscillator signal power $P_c$!

* Note we have a mathematical problem here! $P_c$ is in Watts, and SPD is in Watts/Hz. Therefore, the ratio of the two is not unitless!

* We get around this problem by specifying the noise as its power in a 1 Hz bandwidth.

→ Numerically, this is identical to the average spectral power density of the noise!

For example, if the noise power has an average spectral power density $2.0 \mu W/Hz$, then the noise power in a bandwidth of 1 Hz is:

$$2.0 \frac{\mu W}{Hz} (1 Hz) = 2.0 \mu W$$

Thus, phase noise is expressed as a rather cumbersome:

$dBc$ in a 1 Hz bandwidth
Q: But phase noise is a function of frequency $f$. Do we have to explicitly specify this function?

A: Generally speaking no. Phase noise is generally specified by stating the value of the noise power at one or two specific frequencies, with respect to the carrier frequency $f_0$.

Typically, the frequencies where the phase noise is specified ranges from 1 KHz to 100 KHz from the carrier.

For example, a typical oscillator spec might say:

-90 dBc in a 1 Hz bandwidth at 1 KHz from the carrier, and
-120 dBc in a 1 Hz bandwidth at 10 KHz from the carrier.

Make sure that you know how to properly specify the phase noise of an oscillator. It is often incorrectly done, and the source of many lost points on an exam or project!