Power Flow and Return Loss

We have discovered that two waves propagate along a transmission line, one in each direction \((V^+(z)\) and \(V^-(z)\)).

\[
I(z) = \frac{V_0^+}{Z_0} \left[ e^{-j\beta \ell} - \Gamma_L e^{+j\beta \ell} \right]
\]

\[
V(z) = V_0^+ \left[ e^{-j\beta \ell} + \Gamma_L e^{+j\beta \ell} \right]
\]

The result is that electromagnetic energy flows along the transmission line at a given rate (i.e., power).

Q: How much power flows along a transmission line, and where does that power go?

A: We can answer that question by determining the power absorbed by the load!
The time average power absorbed by an impedance $Z_L$ is:

$$\rho_{abs} = \frac{1}{2} \text{Re}\{V_L I_L^*\}$$

$$= \frac{1}{2} \text{Re}\{V(z = 0) I(z = 0)^*\}$$

$$= \frac{1}{2} Z_0 \text{Re}\left\{(V_0^* \left[ e^{-j\beta_0} + \Gamma_L e^{+j\beta_0} \right]) \left(V_0^* \left[ e^{-j\beta_0} - \Gamma_L e^{+j\beta_0} \right]\right)^\ast\right\}$$

$$= \frac{|V_0|^2}{2 Z_0} \text{Re}\left\{1 - \left(\Gamma_L^* - \Gamma_L\right) - |\Gamma_L|^2\right\}$$

$$= \frac{|V_0|^2}{2 Z_0} \left(1 - |\Gamma_L|^2\right)$$

The significance of this result can be seen by rewriting the expression as:

$$\rho_{abs} = \frac{|V_0|^2}{2 Z_0} \left(1 - |\Gamma_L|^2\right)$$

$$= \frac{|V_0|^2}{2 Z_0} - \frac{|V_0^*\Gamma_L|^2}{2 Z_0}$$

$$= \frac{|V_0|^2}{2 Z_0} - \frac{|V_0|^2}{2 Z_0}$$

The two terms in above expression have a very definite physical meaning. The first term is the time-averaged power of the wave propagating along the transmission line toward the load.
We say that this wave is incident on the load:

\[ P_{\text{inc}} = P_+ = \frac{|V_0^+|^2}{2Z_0} \]

Likewise, the second term of the \( P_{\text{abs}} \) equation describes the power of the wave moving in the other direction (away from the load). We refer to this as the wave reflected from the load:

\[ P_{\text{ref}} = P_- = \frac{|V_0^-|^2}{2Z_0} = \frac{|\Gamma_L|^2|V_0^+|^2}{2Z_0} = |\Gamma_L|^2 P_{\text{inc}} \]

Thus, the power absorbed by the load is simply:

\[ P_{\text{abs}} = P_{\text{inc}} - P_{\text{ref}} \]

or, rearranging, we find:

\[ P_{\text{inc}} = P_{\text{abs}} + P_{\text{ref}} \]

This equation is simply an expression of the conservation of energy!

It says that power flowing toward the load (\( P_{\text{inc}} \)) is either absorbed by the load (\( P_{\text{abs}} \)) or reflected back from the load (\( P_{\text{ref}} \)).
Note that if $|\Gamma_L|^2 = 1$, then $P_{inc} = P_{ref}$, and therefore no power is absorbed by the load.

This of course makes sense!

The magnitude of the reflection coefficient ($|\Gamma_L|$) is equal to one only when the load impedance is purely reactive (i.e., purely imaginary).

Of course, a purely reactive element (e.g., capacitor or inductor) cannot absorb any power—all the power must be reflected!

**Return Loss**

The ratio of the reflected power to the incident power is known as return loss. Typically, return loss is expressed in dB:

$$R.L. = -10 \log_{10} \left[ \frac{P_{ref}}{P_{inc}} \right] = -10 \log_{10} |\Gamma_L|^2$$
For example, if the return loss is 10dB, then 10% of the incident power is reflected at the load, with the remaining 90% being absorbed by the load—we “lose” 10% of the incident power.

Likewise, if the return loss is 30dB, then 0.1% of the incident power is reflected at the load, with the remaining 99.9% being absorbed by the load—we “lose” 0.1% of the incident power.

Thus, a larger numeric value for return loss actually indicates less lost power! An ideal return loss would be $\infty$ dB, whereas a return loss of 0 dB indicates that $|\Gamma_L| = 1$--the load is reactive!