Radiation Intensity

We found that the power density of a wave produced by an antenna located at the origin has the form:

$$\vec{W}(\theta,\phi,r) = U(\theta,\phi)\,\frac{\hat{r}}{r^2}$$

And that the total radiated power from an antenna at the origin is:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

Q: So, what does radiation intensity $U(\theta,\phi)$ indicate? Does it have any physical meaning??

A: An antenna does not (in fact, CAN not) radiate power equally in all directions. $U(\theta,\phi)$ describes the <u>intensity</u> of the wave as a function of direction (θ,ϕ).

$$\boxed{\text{Radiation Intensity} \doteq U(\theta,\phi) \quad \frac{\text{Watts}}{\text{Steradian}}}$$

For example, say an antenna <u>could</u> radiate uniformly in all directions, then $U(\theta,\phi)$ is a <u>constant</u> (i.e., $U(\theta,\phi)=U_o$).

$$\therefore \quad P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta, d\phi$$

$$= U_o \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$= 4\pi \, U_o$$

∴ the radiation intensity of a uniformly radiating (i.e., isotropic) antenna is:

isotropic radiator

$$\boxed{U_0 = \frac{P_{rad}}{4\pi} \quad \left(\frac{\text{Watts}}{\text{steradian}}\right)}$$

Makes sense! If we radiate P_{rad} Watts uniformly over 4π steradians, the radiation intensity will be $\frac{P_{rad}}{4\pi}\ \frac{\text{Watts}}{\text{steradian}}$.

Note intensity $U(\theta,\phi)$ is a function of direction (θ,ϕ), whereas power density $\overline{W}(r,\theta,\phi)$ is a function of position.

∴ $U(\theta,\phi)$ is a description of how an antenna behaves, and $\overline{W}(r,\theta,\phi)$ is a description of the E.W. wave it (the antenna) creates!

Note the power density created by an isotropic radiator would be:

$$\boxed{\overline{W}(r,\theta,\phi) = U_0 \frac{\hat{r}}{r^2} = \frac{P_{rad}}{4\pi r^2}\hat{r}} \left(\frac{W}{m^2}\right)$$

Makes sense! The surface area of a sphere radius r is $4\pi r^2$. Thus, if energy is passing through this surface at a rate of P_{rad} J/s = Watts, the power density on the surface has a magnitude of $\frac{P_{rad}}{4\pi r^2}\left(\frac{\text{Watts}}{m^2}\right)$.