4. Receiver Dynamic Range

Q: *So can we apply our new knowledge about noise power to a **super-het** receiver?*

A: HO: Receiver Gain and Noise Figure

Q: *What about the input signal power? You said it can be very large or very small. Are there any **limits** to the signal power?*

A:

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HO: The Minimum Detectable Signal

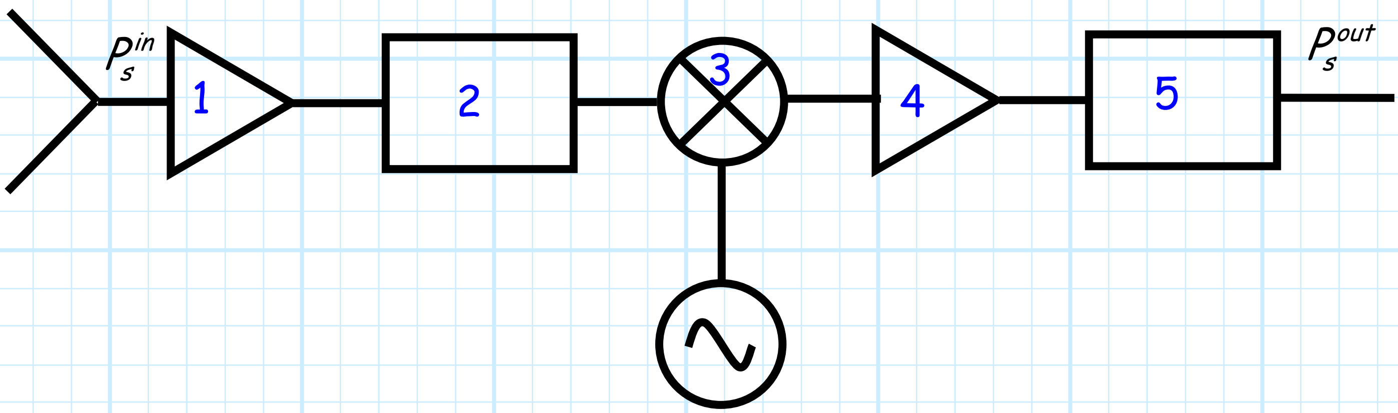
HO: Receiver 1dB Compression Point

HO: Receiver Dynamic Range

Example: Receiver Gain and Noise Figure

We cam now determine the overall gain and noise figure of a super-het **receiver**!

Consider the following **example**:



Let's look at each device individually:

Antenna - We assume that the antenna noise temperature is $T_A = T_o = 290\ K^\circ$, therefore $N_A = -174\ dBm/Hz$. Also, the antenna couples in a desired signal with power P_s^{in}.

1. LNA - This device has a gain $G_1 = 10.0$ (10 dB) and a noise figure $F_1 = 1.5$ (1.76 dB).

2. Preselector – This device has an **insertion loss** of 1 dB. Therefore:

$$G_2(dB) = -1.0 \quad \Rightarrow \quad G_2 = 0.8$$

$$F_2(dB) = 1.0 \quad \Rightarrow \quad F_2 = 1.26$$

3. Mixer - This device has an **conversion loss** of 6 dB. Therefore:

$$G_3(dB) = -6.0 \quad \Rightarrow \quad G_2 = 0.25$$

$$F_3(dB) = 6.0 \quad \Rightarrow \quad F_2 = 4.0$$

4. IF Amp - This device has a gain $G_4 = 10^3$ (30 dB) and a noise figure $F_4 = 4.0$ (6 dB).

5. IF Filter - This device has an **insertion loss** of 2 dB. Therefore:

$$G_5(dB) = -2.0 \quad \Rightarrow \quad G_2 = 0.63$$

$$F_5(dB) = 2.0 \quad \Rightarrow \quad F_5 = 1.58$$

The total **gain** of the receiver is easy to determine, its simply the product of the gains of all devices:

$$\begin{aligned} G_{R_x} &= G_1\, G_2\, G_3\, G_4\, G_5 \\ &= (10)(0.8)(0.25)(1000)(0.63) \\ &= 1260 \end{aligned}$$

or,

$$\begin{aligned} G_{Rx}(dB) &= G_1(dB) + G_2(dB) + G_3(dB) + G_4(dB) + G_5(dB) \\ &= 10 - 1 - 6 + 30 - 2 \\ &= 31 \end{aligned}$$

Note that:

$$10\, log_{10}[1260] = 31.0$$

Now, determining the **noise figure** of the receiver is a bit more challenging.

$$\begin{aligned} F_{R_x} &= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \frac{F_5 - 1}{G_1 G_2 G_3 G_4} \\ &= 1.5 + \frac{1.26 - 1}{(10)} + \frac{4 - 1}{(10)(0.8)} + \frac{4 - 1}{(10)(0.8)(0.25)} \\ &\quad + \frac{1.58 - 1}{(10)(0.8)(0.25)(1000)} \\ &= 1.5 + 0.026 + 0.375 + 0.094 + 0.0003 \\ &= 2.0 \end{aligned}$$

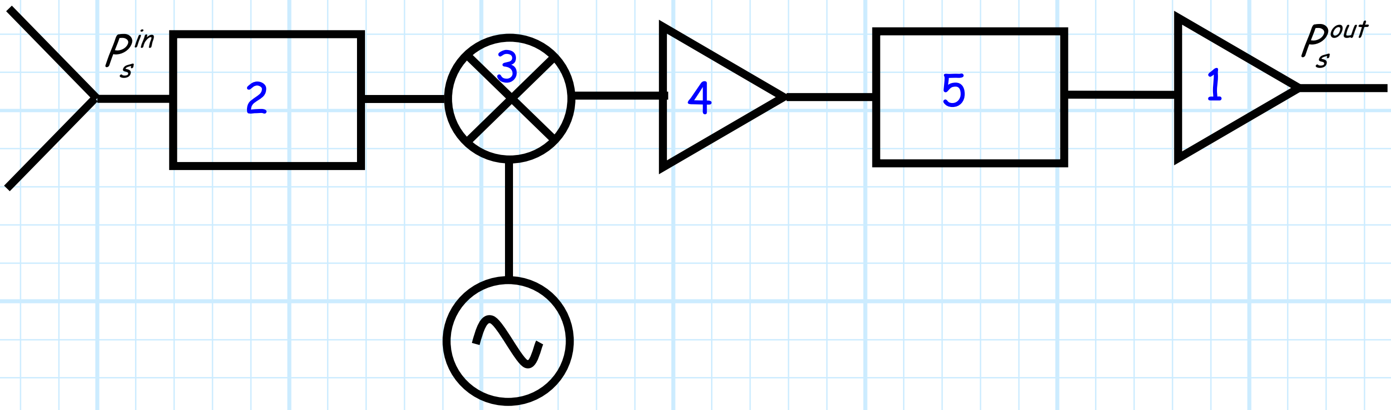
Therefore:

$$F_{R_x}(dB) = 3.0$$

Note then that the **noise power** at the receiver **output** is:

$$\begin{aligned} P_n^{out} &= F_{Rx} G_{Rx} k T_A \, B_{IF} \\ &= (2)(1260) k T_A \, B_{IF} \\ &= 10^{-17} \, B_{IF} \, W \end{aligned}$$

Now, let's se what happens if we **move the LNA** to the **end** of the receiver!



We find the **gain** is **unchanged**!

$$\begin{aligned} G_{R_x}(dB) &= G_2(dB) + G_3(dB) + G_4(dB) + G_5(dB) + G_1(dB) \\ &= -1 - 6 + 30 - 2 + 10 \\ &= 31 \end{aligned}$$

But **not** the noise figure:

$$\begin{aligned} F_{R_x} &= F_2 + \frac{F_3 - 1}{G_2} + \frac{F_4 - 1}{G_2 G_3} + \frac{F_5 - 1}{G_2 G_3 G_4} + \frac{F_1 - 1}{G_2 G_3 G_4 G_5} \\ &= 1.26 + \frac{4 - 1}{(0.8)} + \frac{4 - 1}{(0.8)(0.25)} \\ &\quad + \frac{1.58 - 1}{(0.8)(0.25)(1000)} + \frac{1.5 - 1}{(0.8)(0.25)(1000)(0.63)} \\ &= 1.26 + 3.75 + 15.0 + 0.003 + 0.004 \\ &= 20.0 \end{aligned}$$

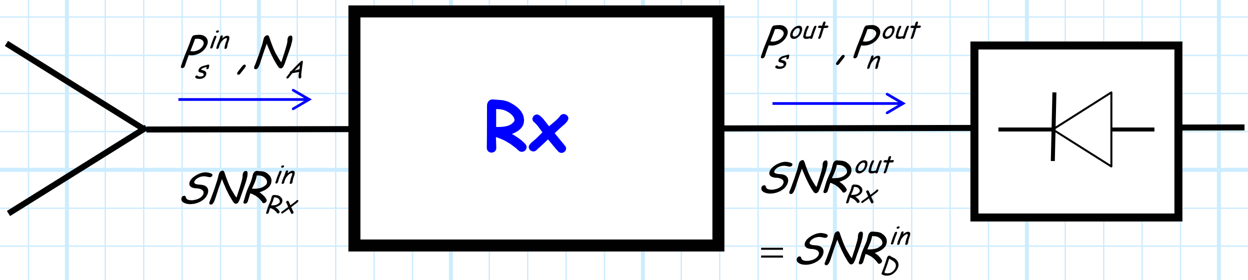
The receiver noise figure has increased to **13 dB**—10 dB larger than before!

As a result, the noise output power has likewise increased by a factor of **10 times**!

$$\begin{aligned} P_n^{out} &= F_{Rx} G_{Rx} k T_A \ B_{IF} \\ &= (2)(1260) k T_A \ B_{IF} \\ &= 10^{-16} \ B_{IF} \ W \end{aligned}$$

This example **demonstrates** how important the LNA is to effective receiver design.

Minimum Detectable Signal



Let's **review** what we have discovered! The **noise power** at the **output** of a receiver (i.e., the input of the demodulator) is:

$$P_n^{out} = F_{Rx}\, G_{Rx}\, kT_o\, B_{IF}$$

while the **signal power** at the receiver **output** is:

$$P_s^{out} = G_{Rx}\, P_s^{in}$$

Thus, the *SNR* at the receiver **output** (the detector input) is:

$$\begin{aligned} SNR_{Rx}^{out} &= \frac{P_s^{out}}{P_n^{out}} \\ &= \frac{G_{Rx}\, P_s^{in}}{F_{Rx}\, G_{Rx}\, kT_o\, B_{IF}} \\ &= \frac{P_s^{in}}{F_{Rx}\, kT_o\, B_{IF}} = SNR_D^{in} \end{aligned}$$

Q: *OK, so the expression above provides a method for determining the value of SNR_{Rx}^{out}; but what should this value* ***be?*** *What value is considered to be* ***sufficiently large*** *for accurate signal detection/demodulation??*

A: It **depends**! It depends on modulation type, demodulator design, and system accuracy requirements.

From all these considerations we can determine the **minimum required *SNR*** (i.e., SNR_D^{min})—a value that **must** be exceeded at the detector/demodulator input in order for an **sufficiently accurate** demodulation to occur. I.E.,

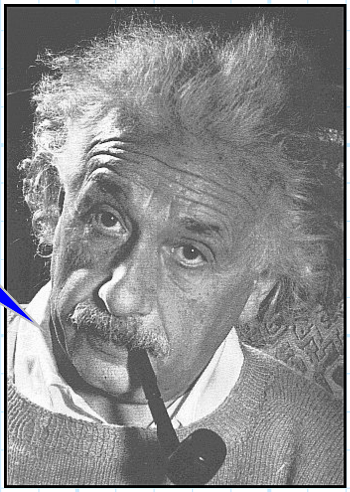
$$SNR_{Rx}^{out} > SNR_D^{min} \qquad \text{for accurate demodulation}$$

The value of this minimum SNR can be as small as -20 dB (or even lower), or as large as 40 dB (or even greater), depending on the application and its requirements.

Q: *How can we insure that $SNR_{Rx}^{out} > SNR_D^{min}$??*

A: Of course, we do need to make the noise figure of the receiver as small as possible. However, the value SNR_{Rx}^{out} **ultimately** depends on the **signal power** P_s^{in}—if this signal power drops toward **zero**, so **too** will SNR_{Rx}^{out}!

Thus, the requirement SNR_D^{min} ultimately translates into a **minimum signal power**—any signal above this minimum can be accurately detected, but signal power **below** this value **cannot**.



We call this minimum input signal power the **Minimum Detectable Signal** (MDS)—a.k.a the Minimum Discernable Signal. I.E.,

$$P_s^{in} > MDS \qquad \text{for accurate demodulation}$$

This Minimum Discernable Signal thus determines the **sensitivity** of the receiver.

Q: *What is the **value** of MDS? How can we determine it?*

A: We know that for **sufficiently** accurate demodulation:

$$SNR_{R_x}^{out} = \frac{P_s^{in}}{F_{R_x}\, kT_o\, B_{IF}} > SNR_D^{min}$$

Thus:

$$P_s^{in} > F_{R_x}\, kT_o\, B_{IF}\, SNR_D^{min}$$

And so it is evident that:

$$MDS = F_{R_x}\, kT_o\, B_{IF}\, SNR_D^{min}$$

Radio engineers often express MDS as **dBm**! The above expression can written **logarithmically** as:

$$MDS(dBm) = 10\log_{10}\left[\frac{F_{R_x}\, kT_o\, B_{IF}\, SNR_D^{min}}{1\,mW}\right]$$

$$= 10\log_{10}\left[\frac{F_{R_x}\, kT_o\, B_{IF}\, SNR_D^{min}}{1\,mW}\,\frac{1\,Hz}{1\,Hz}\right]$$

$$= 10\log_{10} F_{R_x} + 10\log_{10}\left[kT_o\frac{1\,Hz}{1\,mW}\right]$$

$$+10\log_{10}\left[\frac{B_{IF}}{1\,Hz}\right] + 10\log_{10} SNR_D^{min}$$

Recall that we earlier determined that :

$$10\log_{10}\left[kT_o\frac{1\,Hz}{1\,mW}\right] = -174$$

And so the **sensitivity** of a receiver can be determined as:

$$MDS(dBm) = -174 + F_{R_x}(dB) + SNR_D^{min}(dB) + 10\log_{10}\left[\frac{B_{IF}}{1\,Hz}\right]$$

Now, let's do an **example**!

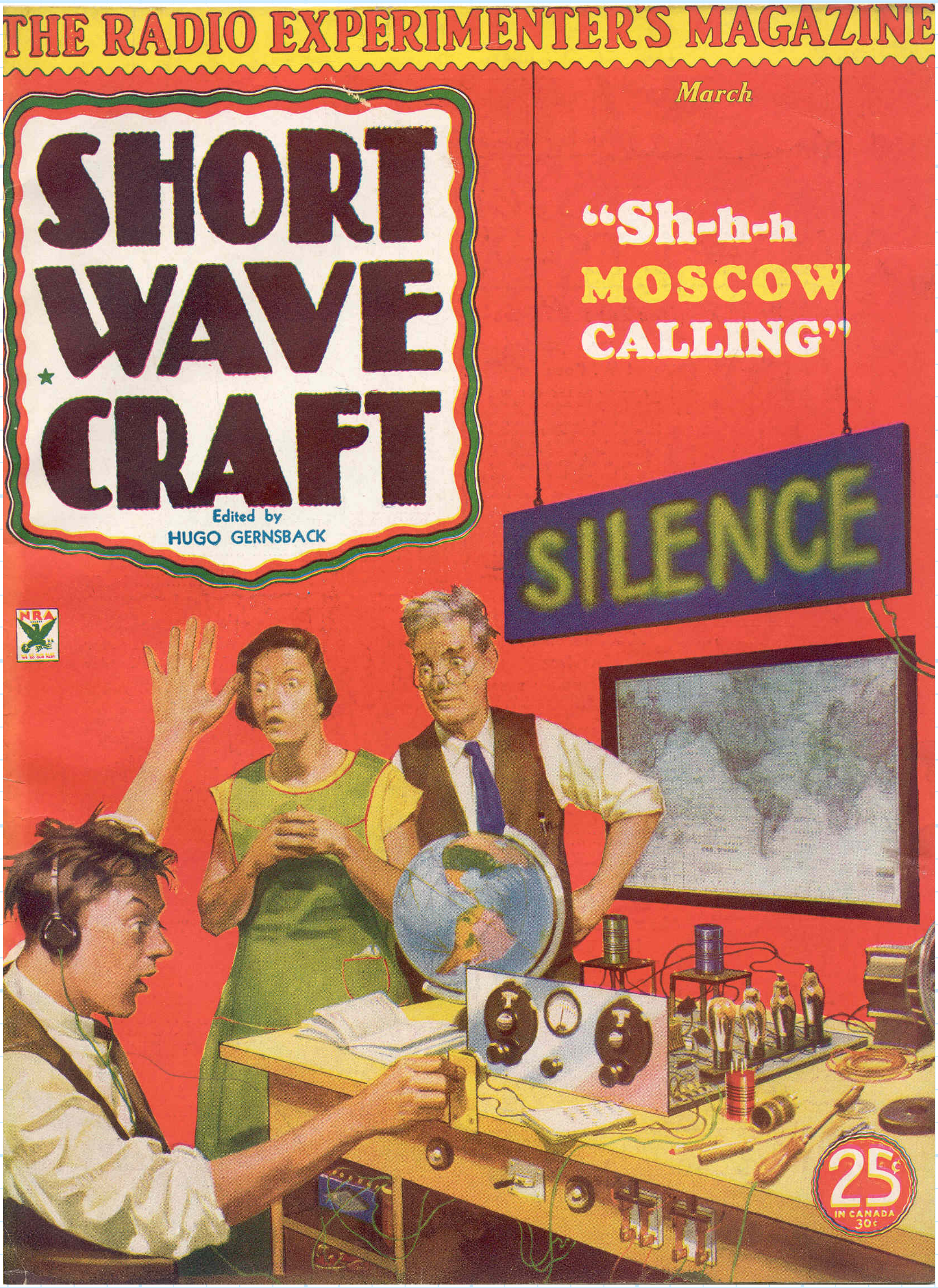
Say a receiver has a noise figure of **4.0 dB** and an IF bandwidth of **500 kHz**. The detector at the receiver output requires an SNR of **3.0 dB**. What is the **sensitivity** of this receiver?

$$
\begin{aligned}
MDS(dBm) &= -174 + F_{R_x}(dB) + SNR_D^{min}(dB) + 10\,log_{10}\left[\frac{B_{IF}}{1\,Hz}\right] \\
&= -174 + 4.0 + 3.0 + 10\,log_{10}\left[5 \times 10^3\right] \\
&= -174 + 4.0 + 3.0 + 57.0 \\
&= -110.0
\end{aligned}
$$

Q: *Yikes! The value -110 dBm is 10* ***femto-Watts****! Just* ***one*** *percent of* ***one*** *billionth of* ***one*** *milli-Watt! Could this receiver* ***actually*** *detect/demodulate a signal whose power is this* ***fantastically*** *small?*

A: You bet! The values used in this example are fairly **typical**, and thus an MDS of -110 dBm is **hardly unusual**.

It's a **good** thing too, as the signals delivered to the receiver by the antenna are **frequently** this tiny!



Receiver Saturation Point

Given the gain G_{Rx} of a **receiver**, we know that the **output** signal power (i.e., the signal power at the demodulator) is:

$$P_D^{in} = P_s^{out} = G_{Rx}\, P_s^{in}$$

Of course, P_s^{in} can theoretically be **any** value; but P_s^{out} is **limited**!

Q: *Limited by what?*

A: Many of the devices in a receiver have **compression points** (e.g., mixers and amplifiers)!

In other words, as P_s^{in} increases, **one** of the devices in the receiver will eventually compress (i.e., **saturate**). As we increase the signal power P_s^{in} beyond this point, we find that the receiver **output** power will be **less** than the value $G_{Rx}\, P_s^{in}$.

→ Precisely the **same** behavior as an amplifier or mixer!

Accordingly, we can define a **1dB compression point** for our **receiver**.

We can **approximately** determine the compression point of our receiver if we know **both** the gain (attenuation) and compression point of each and **every** one of its components.

Q: *This sounds **very** much like how we determined the overall **noise figure** of a receiver (i.e., with knowledge of G and F for **every** component). Give me the equivalent **equation** so I can get busy calculating the compression point of my receiver!*

A: Not so fast! The procedure for determining the compression point of a receiver is quite a bit **more complex** than finding its noise figure.

The problem is that the compression point of a receiver is not some function of the **all** the compression points of each device. Instead, it is dependent **only** on the compression point of the device that saturates **first** as P_s^{in} increases.

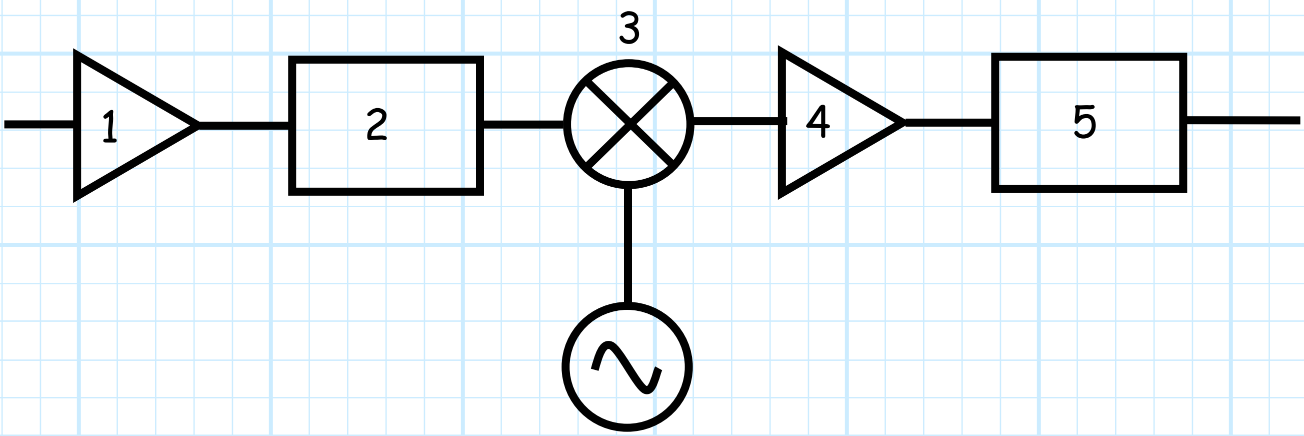
Big problem → we do **not** know what device will saturate first!

Q: *Won't it simply be the device with the **lowest** 1 dB compression point?*

A: **Nope!** The **gain** (or attenuation) of all of the devices that **precede** a component will likewise determine the value of receiver input power P_s^{in} at which that component saturates.

Thus, we must individually determine the **value** of receiver input power P_s^{in} that will cause **each** of the components in our receiver to saturate. The **smallest** of these values will be the compression point of the receiver!

Perhaps this is best explained by an **example**. Consider a **simple** super-het receiver with the following components:



Device	G_m	P_m^{1dB}	P_{in}^{sat}
m=1	10 dB	+10 dBm	
m=2	-1.0 dB	∞	
m=3	-6.0 dB	3 dBm	
m=4	15 dB	+14 dBm	
m=5	-2 dB	∞	

Here the value G_m represents the **gain** of the m-th component, P_m^{1dB} the 1dB **compression point** of the m-th component, and P_{in}^{sat} is the amount of **receiver input power** required to cause that particular component to **saturate**.

Now, let's look at **each** component, and determine its **particular** value for P_{in}^{sat}.

m=1: LNA

Recall the 1 dB compression point of an amplifier is specified in terms of **output** power. Thus, when this amplifier saturates, the **input** power will be:

$$P_{in1}^{1dB} = \frac{P_1^{1dB}}{G_1}$$

or equivalently:

$$\begin{aligned} P_{in1}^{1dB}(dBm) &= P_1^{1dB}(dBm) - G_1(dB) \\ &= 10 - 10 \\ &= 0\ dBm \end{aligned}$$

Q: *Wait! Shouldn't we add **1 dBm** to this answer??*

A: **Theoretically** yes, as when the device has compressed by 1 dB, the gain is effectively 1 dB less (i.e., $G_1 = 9dB$). However, we typically do **not** consider this fact when computing compression point, as:

1. 1 dBm is generally not large enough to be **numerically significant**, particularly when considering all the other approximations and **uncertainties** in our design!

2. By not adding the 1 dBm to the solution, we have a bit more **conservative** estimate of receiver performance. After all, our goal is to **avoid** receiver saturation!

Now, since the input to the **LNA** is likewise the input to the **receiver**, we can conclude that the LNA will saturate when the **receiver** input power is 0 dBm. Thus, "according to" our first component:

$$P_{in}^{sat} = 0\, dBm$$

However, this very well may **not** be the input value at which the receiver saturates, as some **other** component may compress at an even **lower** receiver input power.

→ Let's find out **if** there is such a component!

m=2: Preselector Filter

Q: *Wait a second! I don't recall ever hearing about a **filter** compression point!?*

A: True enough! A filter, since it is a **passive** and **linear** device, has **no** compression point. Of course, if we put too much power into the device, it will damage (e.g. **melt**) it, but this power is typically **very large** compared to most amplifier or mixer compression points.

Thus, we can conclude that the compression point of a filter is **effectively infinity**, as is the input receiver power required to "saturate" it (i.e., $P_{in}^{sat} = \infty$).

Q: *I see! Filters make no difference in determining the satuaration point of a receiver. Can we **ignore** them altogether?*

A: Absolutely **not**! Although filters will not saturate, they **will** help determine the saturation point of a receiver.

The reason is that filters have **insertion loss**. Note the gain of this filter is –1.0 dB, which indicates an insertion loss of +1.0 dB. This loss **will** affect the input power of all the components further down the receiver "chain", and thus **may** affect the receive saturation point!

m=3: Mixer

Don't forget that the 1dB compression point of a mixer (unlike an amplifier) is specified in terms of its **input** power!

Thus, from the mixer compression point, we can **immediately** conclude that:

$$P_{in3}^{1dB}\left(dBm\right) = P_{3}^{1dB}\left(dBm\right) = 3\,dBm$$

In other words, we do **not** have to "remove" the **mixer** conversion loss to find the input power, the way we had to subtract the LNA gain from the LNA (output) compression point.

However, since the mixer is not directly connected to the input of the receiver, we **must** "remove" the gain of the **preceding components** in order to determine what **input receiver power** will cause the mixer to saturate.

Since the power into the **mixer** is simply the power into the **receiver** times the gain of the LNA and preselector filter:

$$P_{in3} = P_{in}\, G_1 G_2$$

we can conclude that:

$$P_{in}^{sat} = \frac{P_{in3}^{sat}}{G_1 G_2}$$

or equivalently:

$$\begin{aligned} P_{in}^{sat}\left(dBm\right) &= P_{in3}^{sat}\left(dBm\right) - G_1\left(dB\right) - G_2\left(dB\right) \\ &= 3 - 10 - \left(-1\right) \\ &= -6.0\ dBm \end{aligned}$$

Note here that the 1dB insertion loss (i.e., $G_2 = -1.0\ dB$) of the **filter** was involved in our computation, and thus affected the value of this saturation point!

m=4: IF Amp

This IF amplifier saturates when its **output** power is $P_4^{1dB} = +14\ dBm$. The **receiver** input power that would cause this much output power at our **IF amp** can be (approximately) determined by "removing" the gain of each preceding device, **including** the gain of the amplifier itself (do **you** see why?):

$$\begin{aligned} P_{in}^{sat}(dBm) &= P_4^{1dB}(dBm) - G_4(dB) - G_3(dB) - G_2(dB) - G_1(dB) \\ &= 14 - 15 - (-6) - (-1) - 10 \\ &= -4 dBm \end{aligned}$$

Note that the **gain** of the mixer is -6 dB; meaning that the mixer **conversion loss** is +6 dB.

It is now evident that we can write a **general equation** for determining the input receiver power that will cause an **amplifier** to saturate, when that amp is the *m*-th component of a receiver:

$$\begin{aligned} P_{in}^{sat}(dBm) &= P_m^{1dB}(dBm) - G_m(dB) - G_{m-1}(dB) - \cdots - G_1(dB) \\ &= P_m^{1dB}(dBm) - \sum_{n=1}^{m} G_n(dB) \qquad \text{(for amplifiers)} \end{aligned}$$

Note the expression for **mixers** will be slightly **different**, given their definition of 1dB compression point:

$$
\begin{aligned}
P_{in}^{sat}(dBm) &= P_m^{1dB}(dBm) - G_{m-1}(dB) - G_{m-2}(dB) - \cdots - G_1(dB) \\
&= P_m^{1dB}(dBm) - \sum_{n=1}^{m-1} G_n(dB) \qquad (\text{for mixers})
\end{aligned}
$$

m=5: IF Filter

As we determined earlier, the compression point of a **filter** is effectively **infinity**, and so for this device:

$$P_{in}^{sat} = \infty$$

So, let's **summarize** in our table what we have found:

Device	G_m	P_m^{1dB}	P_{in}^{sat}
m=1	10 dB	+10 dBm	0 dBm
m=2	-1.0 dB	∞	∞
m=3	-6.0 dB	3 dBm	-6 dBm
m=4	15 dB	+14 dBm	-4 dBm
m=5	-2 dB	∞	∞

Q: *Wait a second! We have determined **four different answers** for the receiver input power that will saturate our receiver. Can they **all** be correct?*

A: Absolutely **not**! There is **one**, and **only** one, answer for receiver saturation point P_{in}^{sat}.

→ The receiver saturation point is the **smallest** of all of our calculated values P_{in}^{sat}!

Thus, for **this** example, the receiver compression point is:

$$P_{in}^{sat} = -6 \ dBm$$

Q: *Why do we consider the **smallest** of all the values P_{in}^{sat} as the receiver compression point? Why not the **largest**? Why not the **average**?*

A: A receiver is considered saturated when **any** of its components are in saturation. Remember, saturation causes our signal to **distort**, and thus it may **not** be accurately demodulated. As a result, an input signal power that causes the saturation of **even one** receiver component is **unacceptable**.

Thus, by choosing the **smallest** of the input saturation powers, we have selected a value that will **unambiguously** define the point where **even one** component is saturated—**if** the receiver input power is **less** than even the smallest of our calculated P_{in}^{sat}, then **none** of the receiver components will be saturated.

Q: *In this example, it is the **mixer** that saturates first. Is this **always** the case?*

A: It is indeed **often** the case that the mixer is the device that determines the receiver saturation point. However, the **LNA** can likewise be the component that saturates first.

Q: *What can we do to **improve** (i.e., increase) the saturation point of a receiver?*

A: Considering the discussion of this handout, it should be quite **evident** how to accomplish this.

We can do **two** different things:

1. *Find a **different** component part with a **higher** saturation point* P_m^{1dB}

This strategy at first seems **very simple**. However, a component designer of mixers or amplifiers is faced with the same **design conflicts** and trade-offs that typically face all other design engineers. If he or she **improves** the 1dB compression point, it will undoubtedly **mess-up** some other important parameter like gain, or bandwidth, or noise figure, or conversion loss.

Thus, a **receiver designer** that attempts to replace a mixer or amplifier with another exhibiting a **higher** 1dB compression point will almost certainly cause some **degradation** in some other receiver performance parameter, like gain, or noise figure, or bandwidth.

→ The selection of receiver component parts is typically a **compromise** between competing and conflicting component parameters. You will **never** find a "perfect" microwave component, only a component which **best suits** the receiver specifications and design goals.

2. *Decrease the **gain** (increase the attenuation) of the component parts **preceding** the component that saturates.*

Decreasing the gain and or increasing the loss of components in a receiver **will** generally improve (i.e, increase) the receiver saturation point. However, it will also **mess-up** the receiver **noise figure** and **MDS**!

→ Again, we are faced with a design trade-off!

Note however, that we only need to decrease the total gain of the components **preceding** the device that is saturating. **Another** way to accomplish this is simply to **rearrange** the order of the devices in the receiver chain.

For example, we might move an amplifier **from** a location **preceding** the saturating component, **to** a location **after** the saturating component—this of course **reduces** the gain of the components **preceding** the device, but does **not alter** the overall receiver gain!

Likewise, we might move a lossy component **from** a location **after** a saturating component, **to** a location **preceding** the saturating component—this again **reduces** the gain of the components preceding the device, but again does **not alter** the overall receiver gain!

Thus, we can conclude that the compression point of a receiver will typically i**mprove** if we move **lossy** components to the **front** and **amplifiers** to the **back**!

However, we must keep in mind **two** things:

* The **order** of some devices **cannot** be changed. For example, we cannot put the mixer before the preselector filter!

* Although rearranging the order of the components in a receiver will **not** change the receiver gain, it **can** play **havoc** with receiver **noise figure** and **MDS**!

In fact, it should be evident to you that the **receiver noise figure** typically improves if we move **lossy** components to the **back** and **amplifiers** to the **front**—exactly the **opposite** strategy for improving receiver saturation!

We find that very often, receiver **saturation point** and receiver **sensitivity** are in **direct conflict**—improve one and you degrade the other!

Receiver Dynamic Range

We now know there is a **minimum input** signal power that a receiver can accurately demodulate.

→ *The Minimum Detectable Signal (MDS) defines the* ***sensitivity*** *of the receiver*

We also know there is a **maximum input** signal power that a receiver can accurately demodulate.

→ *The receiver 1 dB compression point defines the* ***saturation*** *point of the receiver.*

The **ratio** of the input saturation point and the minimum detectable signal is defined as the **total dynamic range** of the receiver.

$$\text{total dynamic range} \doteq \frac{P_{in}^{sat}}{MDS}$$

Note dynamic range is a **unitless** value, therefore dynamic range is most often expressed in **dB**:

$$\text{total dynamic range}(\text{dB}) \doteq P_{in}^{sat}(\text{dBm}) - MDS(\text{dBm})$$