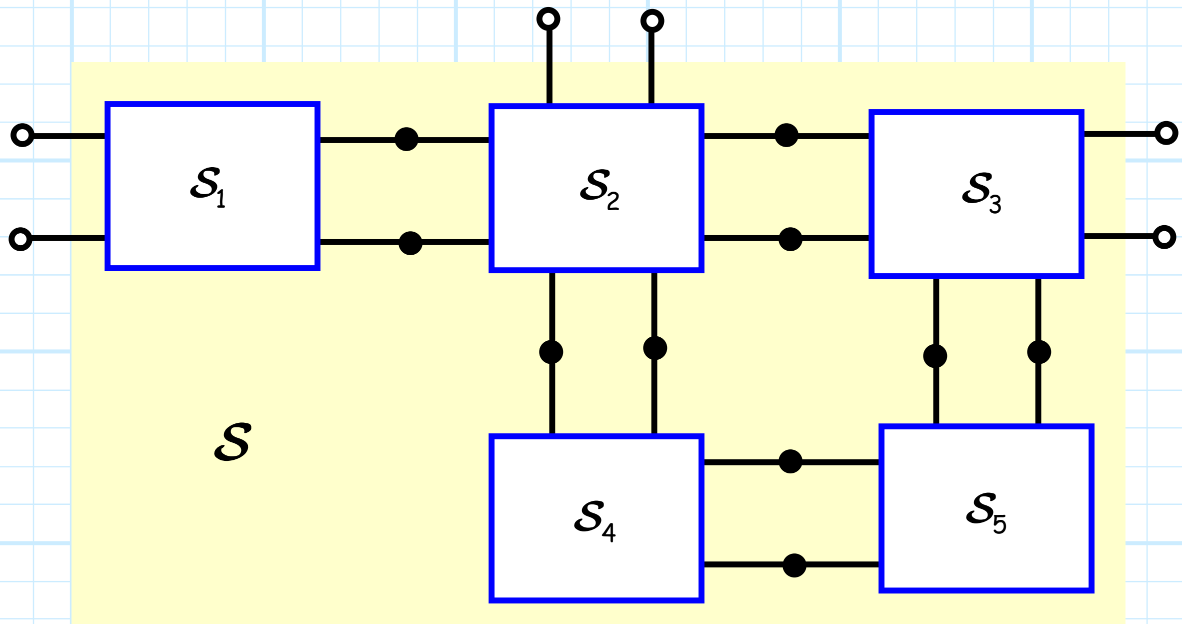
Signal Flow Graphs

Consider a complex **3-port** microwave network, constructed of **5** simpler microwave devices:



where $\mathcal{S}_n$ is the **scattering matrix** of each device (I've **changed** notations—$\mathcal{S}_1 = \bar{\bar{\mathbf{S}}}_1$!), and $\mathcal{S}$ is the **overall** scattering matrix of the **entire** 3-port network.

Q: *Is there any way to determine this **overall** network scattering matrix $\mathcal{S}$ from the **individual** device scattering matrices $\mathcal{S}_n$?*

A: **Definitely**! Note the wave **exiting** one port of a device is a wave **entering** (i.e., incident on) another (and vice versa). This is a **boundary condition** at the port connection between devices.

Add to this the scattering parameter equations from each individual device, and we have a **sufficient** amount of math to determine the relationship between the incident and exiting waves of the remaining three ports—in other words, the scattering matrix of the **3-port network**!

Q: *Yikes! Wouldn't that require a lot of **tedious** algebra!*

A: It sure would! We might use a **computer** to assist us, or we might use a tool employed since the early days of microwave engineering—the **signal flow graph**.

A signal flow graph of a microwave network is useful for **two** reasons:

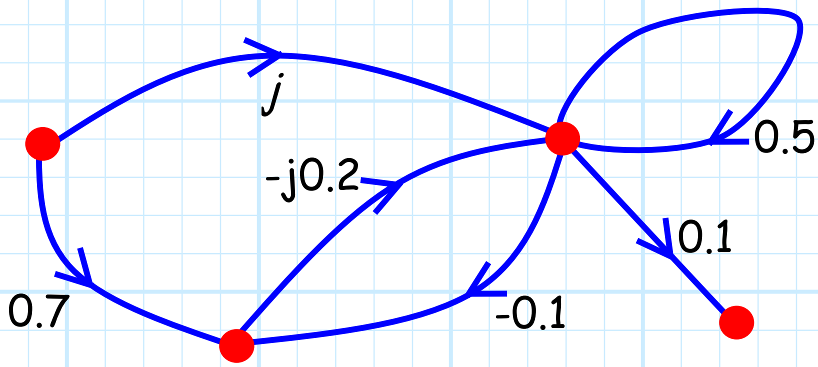
1) It provides a **graphical** representation of a microwave network, allowing for a **qualitative** analysis and interpretation of the network.

2) It provides a method for **simplifying** (i.e., reducing) a microwave network to a more **fundamental** form, such an easier **quantitative** analysis can be achieved.

Included in this quantitative analysis can be the determination of the network's **scattering matrix**; but more on that **later**.

But first, some **definitions**!

Every signal flow graph consists of a set of **nodes**. These nodes are connected by **branches**, which are simply contours with a specified **direction**. Each branch likewise has a complex **value**.



Q: *What could this possibly have to do with* ***microwave engineering****?*

A: Each **port** of a microwave device is represented by **two nodes**—the "*a*" node and the "*b*" node. The "*a*" node simply represents the value of the **incident** wave on that port, evaluated **at** the plane of that port:

$$a_n \doteq V_n^+(z_n = z_{nP})$$

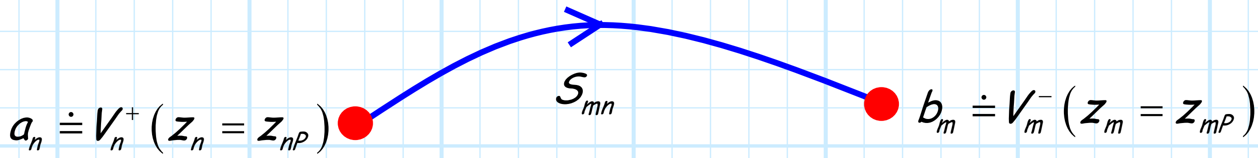
Likewise, the "*b*" node simply represents the value of the **exiting** wave from that port, evaluated **at** the plane of that port:

$$b_n \doteq V_n^-(z_n = z_{nP})$$

Note then that the **total voltage** at a port is simply:

$$V_n(z_n = z_{nP}) = V_n^+(z_n = z_{nP}) + V_n^-(z_n = z_{nP}) = a_n + b_n$$

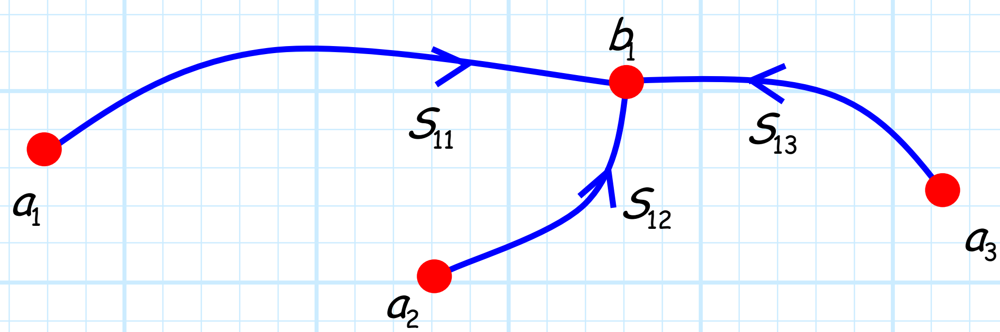
The value of the **branch** connecting two nodes is simply the value of the **scattering parameter** relating these two voltage values:



The signal flow graph above is simply a **graphical** representation of the equation:

$$V_m^-\left(z_m = z_{mP}\right) = S_{mn}\, V_n^+\left(z_n = z_{nP}\right)$$
$$b_m = S_{mn}\, a_n$$

Moreover, if **multiple** branches enter a node, then the voltage represented by that node is the **sum** of the values from each branch. For example, the signal flow graph:



is a **graphical** representation of the equation:

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2 + S_{13}\, a_3$$

Now, consider a **two-port device** with a scattering matrix $\mathcal{S}$:

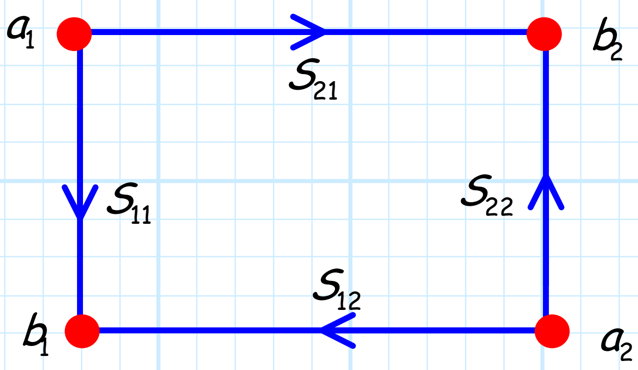
$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

So that:

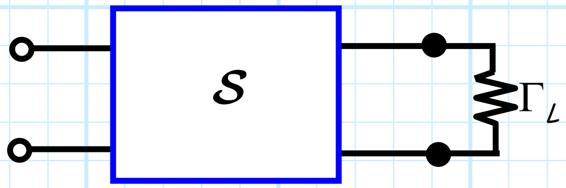
$$b_1 = S_{11}\, a_1 + S_{12}\, a_2$$

$$b_2 = S_{21}\, a_1 + S_{22}\, a_2$$

We can thus **graphically** represent a **two-port device** as:



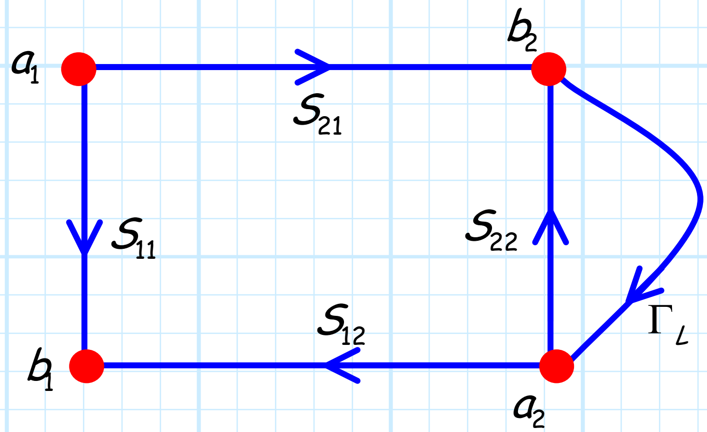
Now, consider a case where the second port is **terminated** by some load Γ_L:



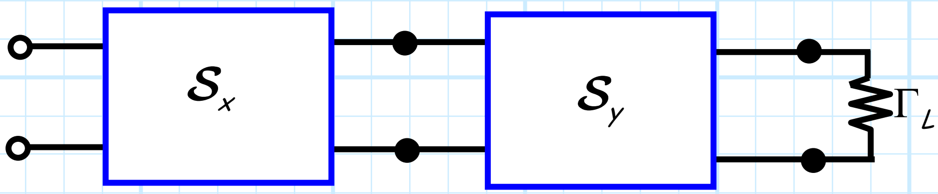
We now have yet **another** equation:

$$V_2^+\left(z_2 = z_{2P}\right) = \Gamma_L\, V_2^-\left(z_2 = z_{2P}\right)$$
$$a_2 = \Gamma_L\, b_2$$

Therefore, the signal flow graph of this **terminated** network is:



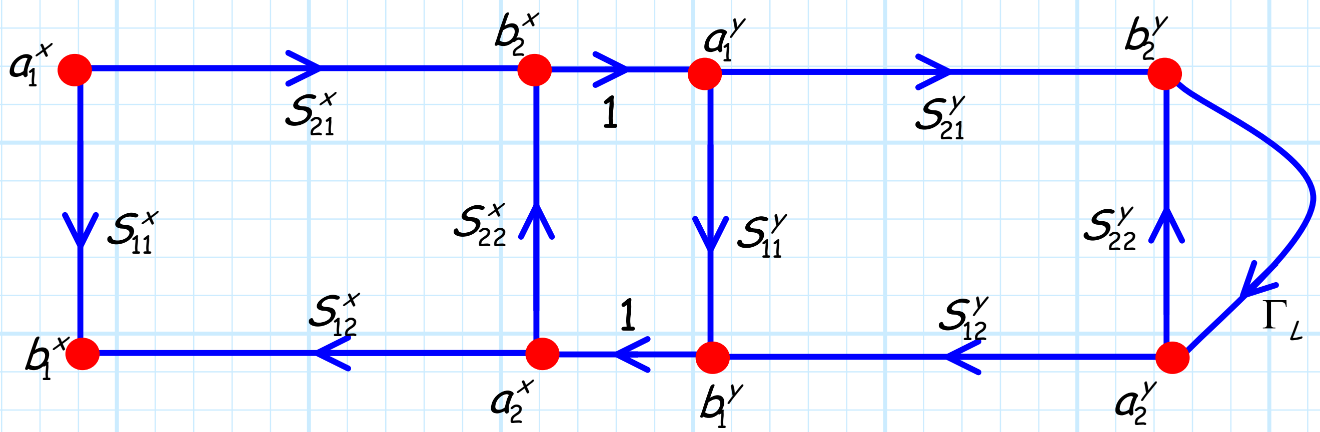
Now let's cascade **two different** two-port networks



Here, the output port of the first device is **directly** connected to the input port of the second device. We describe this mathematically as:

$$a_1^y = b_2^x \qquad \text{and} \qquad b_1^y = a_2^x$$

Thus, the signal flow graph of this network is:

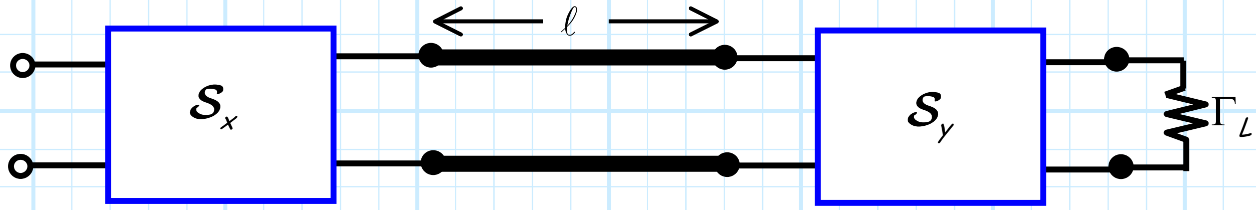


Q: *But what happens if the networks are connected with* ***transmission lines?***

A: Recall that a length ℓ of transmission line with characteristic impedance Z_0 is likewise a **two-port** device. Its scattering matrix is:

$$\mathcal{S} = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$

Thus, if the two devices are connected by a length of **transmission line**:



the signal flow graph is: