System Equivalent Noise Temperature

Say we cascade three microwave devices, each with a different gain and equivalent noise temperature:

These three devices together can be thought of as one new microwave device.

Q: What is the equivalent noise temperature $T_e$ of this overall device?

A: First of all, we must define this temperature as the value $T_e$ such that:

$$T_{out} = G(T_{in} + T_e)$$

or specifically:
Q: Yikes! What is the value of $G$?

A: The value $G$ is the total system gain; in other words, the overall gain of the three cascaded devices. This gain is particularly easy to determine, as it simply the product of the three gains:

$$G = G_1 G_2 G_3$$

Now for the hard part! To determine the value of $T_{\text{out}}$, we must use our equivalent noise model that we studied earlier:
Thus, we cascade three models, one for each amplifier:

We can observe our model and note three things:

\[ T_{out1} = G_1 (T_{in} + T_{e1}) \]
\[ T_{out2} = G_1 (T_{out1} + T_{e1}) \]
\[ T_{out3} = G_1 (T_{out2} + T_{e1}) \]

Combining these three equations, we find:

\[ T_{out3} = G_1 G_2 G_3 (T_{in} + T_{e1}) + G_2 G_3 (T_{e2}) + G_3 (T_{e3}) \]

a result that is likewise evident from the model.
Now, since $T_{out} = T_{out3}$, we can determine the overall (i.e., system) equivalent noise temperature $T_e$:

$$T_e = \frac{T_{out} - T_{in}}{G} = \frac{G_1G_2G_3(T_{in} + T_{e1}) + G_2G_3(T_{e2}) + G_3(T_{e3})}{G_1G_2G_3} - T_{in}$$

$$= T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2}$$

Moreover, we will find if we cascade an $N$ number of devices, the overall noise equivalent temperature will be:

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \frac{T_{e4}}{G_1G_2G_3} + \cdots + \frac{T_{eN}}{G_1G_2G_3\cdots G_{N-1}}$$

I assume that you can use the above equation to get the correct answer—but I want to know if you understand why your answer is correct!

Make sure you understand where this expression comes from, and what it means.

Look closely at the above expression, for it tells us something very profound about the noise in a complex microwave system (like a receiver!).
Recall that we want the equivalent noise temperature to be as small as possible. Now, look at the equation above, which terms in this summation are likely to be the largest?

* Assuming this system has large gain $G$, we will find that the first few terms of this summation will typically dominate the answer.

* Thus, it is evident that to make $T_e$ as small as possible, we should start by making the first term as small as possible. Our only option is to simply make $T_{e1}$ as small as we can.

* To make the second term small, we could likewise make $T_{e2}$ small, but we have another option!

   $\Rightarrow$ We could likewise make gain $G_1$ large!

Note that making $G_1$ large has additional benefits, as it likewise helps minimize all the other terms in the series!

Thus, good receiver designers are particularly careful about placing the proper component at the beginning of a receiver. They covet a device that has high gain but low equivalent noise temperature (or noise figure).

$\Rightarrow$ The ideal first device for a receiver is a low-noise amplifier!
Q: Why don’t the devices at the end of the system make much of a difference when it comes to noise?

A: Recall that each microwave device adds more noise to the system. As a result, noise will generally steadily increase as it moves through the system.

* By the time it reaches the end, the noise power is typically so large that the additional noise generated by the devices there are insignificant and make little increase in the overall noise level.

* Conversely, the noise generated by the first device is amplified by every device in the overall system—this first device thus typically has the greatest impact on system noise temperature and system noise figure.