The 180° Hybrid Coupler

The 180° Hybrid Coupler (sometimes known as the “ring” or “rat-race” hybrid) is a lossless, matched and reciprocal 4-port device, with a scattering matrix of the anti-symmetric form:

\[
\mathbf{S} = \begin{bmatrix}
0 & \alpha & \beta & 0 \\
\alpha & 0 & 0 & -\beta \\
\beta & 0 & 0 & \alpha \\
0 & -\beta & \alpha & 0 \\
\end{bmatrix}
\]

Just like the quadrature coupler, however, we find that:

\[\alpha = \beta = \frac{1}{\sqrt{2}}\]

So that the scattering matrix for this device is:

\[
\mathbf{S} = \begin{bmatrix}
0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\
1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\
1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\
0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\
\end{bmatrix}
\]

Hence, this coupler is likewise a 3dB coupler—the power into a given port (with all other ports matched) is equally divided between two of the three output ports.
Note the relative phase between the outputs, however, is dependent on which port is the input.

For example, if the input is port 1 or port 3, the two signals will be in phase—no difference in their relative phase!

However, if the input is port 2 or port 4, the output signals will be $180^\circ$ out of phase ($e^{j\pi} = -1$)!

An interesting application of this coupler can be seen if we place two input signals into the device, at ports 2 and 3. Note the signal out of port 1 would therefore be:

$$V_1^- (z) = S_{12} V_2^+(z) + S_{13} V_3^+(z)$$

$$= \frac{1}{\sqrt{2}} (V_3^+(z) + V_2^+(z))$$

while the signal out of port 4 is:

$$V_4^- (z) = S_{42} V_2^+(z) + S_{43} V_3^+(z)$$

$$= \frac{1}{\sqrt{2}} (V_3^+(z) - V_2^+(z))$$

Note that the output of port 1 is proportional to the sum of the two inputs. Port 1 of a $180^\circ$ Hybrid Coupler is thus often referred to as the sum ($\Sigma$) port.
Likewise, port 4 is proportional to the difference between the two inputs. Port 4 a 180° Hybrid Coupler is thus often referred to as the delta (Δ) port.

There are many applications where we wish to take the sum and/or difference between two signals!

The 180° Hybrid Coupler can likewise be used in the opposite manner. If we have both the sum and difference of two signals available, we can use this device to separate the signals into their separate components!

http://paginas.fe.up.pt/~hmiranda/etele/microstrip/