The 4-Port Coupler

Guess what! I have determined that—unlike a 3-port device—a matched, lossless, reciprocal 4-port device is physically possible! In fact, I’ve found two general solutions!

The first solution is referred to as the symmetric solution:

\[
\mathbf{S}_\text{sym} = \begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0
\end{bmatrix}
\]

Note for this symmetric solutions, every row and every column of the scattering matrix has the same four values (i.e., \(\alpha\), \(j\beta\), and two zeros)!

The second solution is referred to as the anti-symmetric solution:

\[
\mathbf{S}_\text{anti-sym} = \begin{bmatrix}
0 & \alpha & \beta & 0 \\
\alpha & 0 & 0 & -\beta \\
\beta & 0 & 0 & \alpha \\
0 & -\beta & \alpha & 0
\end{bmatrix}
\]
Note that for this anti-symmetric solution, two rows and two columns have the same four values (i.e., $\alpha$, $\beta$, and two zeros), while the other two row and columns have (slightly) different values ($\alpha$, $-\beta$, and two zeros).

It is quite evident that each of these solutions are matched and reciprocal. However, to ensure that the solutions are indeed lossless, we must place an additional constraint on the values of $\alpha, \beta$. Recall that a necessary condition for a lossless device is:

$$\sum_{m=1}^{N} |S_{mn}|^2 = 1 \quad \text{for all } n$$

Applying this to the symmetric case, we find:

$$|\alpha|^2 + |\beta|^2 = 1$$

Likewise, for the anti-symmetric case, we get

$$|\alpha|^2 + |\beta|^2 = 1$$

It is evident that if the scattering matrix is unitary (i.e., lossless), the values $\alpha$ and $\beta$ cannot be independent, but must related as:

$$|\alpha|^2 + |\beta|^2 = 1$$
Generally speaking, we will find that $|\alpha| \geq |\beta|$. Given the constraint on these two values, we can thus conclude that:

$$0 \leq |\beta| \leq \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{1}{\sqrt{2}} \leq |\alpha| \leq 1$$

We will study three different passive, reciprocal, lossless devices:

1) The Directional Coupler

2) The 90° Hybrid

3) The 180° Hybrid