Two-Tone Intermodulation

Q: It doesn’t seem to me that this dad-gum intermodulation distortion is really that much of a problem. I mean, the first and second harmonics will likely be well outside the amplifier bandwidth, right?

A: True, the harmonics produced by intermodulation distortion typically are not a problem in radio system design. There is a problem, however, that is much worse than harmonic distortion!

This problem is called two-tone intermodulation distortion.

Say the input to an amplifier consists of two signals at dissimilar frequencies:

\[ v_{in} = a \cos \omega_1 t + a \cos \omega_2 t \]

Here we will assume that both frequencies \( \omega_1 \) and \( \omega_2 \) are within the bandwidth of the amplifier, but are not equal to each other \((\omega_1 \neq \omega_2)\).
This of course is a much more realistic case, as typically there will be multiple signals at the input to an amplifier!

For example, the two signals considered here could represent two FM radio stations, operating at frequencies within the FM band (i.e., $88.1 \text{ MHz} \leq f_1 \leq 108.1 \text{ MHz}$ and $88.1 \text{ MHz} \leq f_2 \leq 108.1 \text{ MHz}$).

Q: My point exactly! Intermodulation distortion will produce those dog-gone second-order products:

$$\frac{a^2}{2} \cos 2\omega_1 t \quad \text{and} \quad \frac{a^2}{2} \cos 2\omega_2 t$$

and gul-durn third order products:

$$\frac{a^3}{4} \cos 3\omega_1 t \quad \text{and} \quad \frac{a^3}{4} \cos 3\omega_2 t$$

but these harmonic signals will lie well outside the FM band!

A: True! Again, the harmonic signals are not the problem. The problem occurs when the two input signals combine together to form additional second and third order products.
Recall an amplifier output is accurately described as:

\[ v_{out} = A_v v_{in} + B v_{in}^2 + C v_{in}^3 + \ldots \]

Consider first the second-order term if two signals are at the input to the amplifier:

\[ v_{2out}^2 = B v_{in}^2 \]

\[ = B (a \cos \omega_1 t + a \cos \omega_2 t)^2 \]

\[ = B (a^2 \cos^2 \omega_1 t + 2a^2 \cos \omega_1 t \cos \omega_2 t + a^2 \cos^2 \omega_2 t) \]

Note the first and third terms of the above expression are precisely the same as the terms we examined on the previous handout. They result in harmonic signals at frequencies \(2\omega_1\) and \(2\omega_2\), respectively.

The middle term, however, is something new. Note it involves the product of \(\cos \omega_1 t\) and \(\cos \omega_2 t\). Again using our knowledge of trigonometry, we find:

\[ 2a^2 \cos \omega_1 t \cos \omega_2 t = a^2 \cos (\omega_2 - \omega_1) t + a^2 \cos (\omega_2 + \omega_1) t \]

Note that since \(\cos (-x) = \cos x\), we can equivalently write this as:

\[ 2a^2 \cos \omega_1 t \cos \omega_2 t = a^2 \cos (\omega_1 - \omega_2) t + a^2 \cos (\omega_1 + \omega_2) t \]

Either way, the result is obvious—we produce two new signals!
These new **second-order** signals oscillate at frequencies \((\omega_1 + \omega_2)\) and \(|\omega_1 - \omega_2|\).

Thus, if we looked at the **frequency spectrum** (i.e., signal power as a function of frequency) of an amplifier **output** when two sinusoids are at the input, we would see something like this:

Note that the new terms have a frequency that is either much **higher** than both \(\omega_1\) and \(\omega_2\) (i.e., \((\omega_1 + \omega_2)\)), or much **lower** than both \(\omega_1\) and \(\omega_2\) (i.e., \(|\omega_1 - \omega_2|\)).

Either way, these new signals will typically be **outside** the amplifier bandwidth!
Q: I thought you said these "two-tone" intermodulation products were some "big problem". These sons of a gun appear to be no more a problem than the harmonic signals!

A: This observation is indeed correct for second-order, two-tone intermodulation products. But, we have yet to examine the third-order terms! I.E.,

\[ v_{out}^3 = C v_{in}^3 \]

\[ = C (a \cos \omega_1 t + a \cos \omega_2 t)^3 \]

If we multiply this all out, and again apply our trig knowledge, we find that a bunch of new third-order signals are created.

Among these signals, of course, are the second harmonics \( \cos 3\omega_1 t \) and \( \cos 3\omega_2 t \). Additionally, however, we get these new signals:

\[ \cos (2\omega_2 - \omega_1) t \quad \text{and} \quad \cos (2\omega_1 - \omega_2) t \]
Note since \( \cos(-x) = \cos x \), we can equivalently write these terms as:
\[
\cos(\omega_1 - 2\omega_2)t \quad \text{and} \quad \cos(\omega_2 - 2\omega_1)t
\]

Either way, it is apparent that the **third-order** products include signals at frequencies \( |\omega_1 - 2\omega_2| \) and \( |\omega_2 - 2\omega_1| \).

Now let's look at the output spectrum with these **new** third-order products included:

Now **you** should see the problem! These third-order products are very close in frequency to \( \omega_1 \) and \( \omega_2 \). They will likely lie **within** the bandwidth of the amplifier!

For example, if \( f_1 = 100 \) MHz and \( f_2 = 101 \) MHz, then \( 2f_2 - f_1 = 102 \) MHz and \( 2f_1 - f_2 = 99 \) MHz. All frequencies are well **within** the FM radio bandwidth!
Thus, these third-order, two-tone intermodulation products are the most significant distortion terms.

This is why we are most concerned with the third-order intercept point of an amplifier!

*I only use amplifiers with the highest possible 3rd-order intercept point!*