VSWR

Consider again the **voltage** along a terminated transmission line, as a function of **position** z:

$$V(z) = V_0^+\left[e^{-j\beta z} + \Gamma_L e^{+j\beta z}\right]$$

Recall this is a **complex** function, the magnitude of which expresses the **magnitude** of the **sinusoidal signal** at position z, while the phase of the complex value represents the relative **phase** of the sinusoidal signal.

Let's look at the **magnitude** only:

$$\begin{aligned}|V(z)| &= |V_0^+|\ |e^{-j\beta z} + \Gamma_L e^{+j\beta z}| \\ &= |V_0^+||e^{-j\beta z}||1+\Gamma_L e^{+j2\beta z}| \\ &= |V_0^+||1+\Gamma_L e^{+j2\beta z}|\end{aligned}$$

ICBST the **largest** value of $|V(z)|$ occurs at the location z where:

$$\Gamma_L e^{+j2\beta z} = |\Gamma_L| + j0$$

while the **smallest** value of $|V(z)|$ occurs at the location z where:

$$\Gamma_L e^{+j2\beta z} = -|\Gamma_L| + j0$$

As a result we can conclude that:

$$|V(z)|_{max} = |V_0^+|\left(1 + |\Gamma_L|\right)$$

$$|V(z)|_{min} = |V_0^+|\left(1 - |\Gamma_L|\right)$$

The ratio of $|V(z)|_{max}$ to $|V(z)|_{min}$ is known as the **Voltage Standing Wave Ratio** (VSWR):

$$\boxed{\text{VSWR} \doteq \frac{|V(z)|_{max}}{|V(z)|_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \qquad \therefore \qquad 1 \leq VSWR \leq \infty}$$

Note if $|\Gamma_L| = 0$ (i.e., $Z_L = Z_0$), then VSWR = 1. We find for this case:

$$|V(z)|_{\text{max}} = |V(z)|_{\text{min}} = |V_0^+|$$

In other words, the voltage magnitude is a **constant** with respect to position *z*.

Conversely, if $|\Gamma_L| = 1$ (i.e., $Z_L = jX$), then VSWR = ∞. We find for **this** case:

$$|V(z)|_{\text{min}} = 0 \qquad \text{and} \qquad |V(z)|_{\text{max}} = 2|V_0^+|$$

In other words, the voltage magnitude varies **greatly** with respect to position *z*.

As with **return loss**, VSWR is dependent on the **magnitude** of Γ_L (i.e, $|\Gamma_L|$) **only** !