dB, dBm, dBw

Decibel (dB), is a specific function that operates on a **unitless** parameter:

$$dB \doteq 10 \log_{10}(x)$$

where x is unitless!

Q: *A unitless parameter! What good is that !?*

A: **Many** values are unitless, such as **ratios** and **coefficients**.

For example, amplifier **gain** is a unitless value!

E.G., amplifier gain is the ratio of the output power to the input power:

$$\frac{P_{out}}{P_{in}} = G$$

$$\therefore \text{ Gain in dB } = 10 \log_{10} G \doteq G(dB)$$

Q: *Wait a minute! I've seen statements such as:*

.... the output power is 5 dBw ....
or
.... the input power is 17 dBm ....

*Of course, Power is **not** a unitless parameter!?!*

A: True! But look at how power is expressed; not in dB, but in **dBm** or **dBw**.

Q: *What the heck does dB**m** or dB**w** refer to ??*

A: It's sort of a **trick** !

Say we have some power P. Now say we divide this value P by one 1 Watt. The result is a unitless value that expresses the value of P in relation to 1.0 Watt of power.

For example, if $P = 2500\, mW$, then $P/1W = 2.5$. This simply means that power P is 2.5 times larger than one Watt!

Since the value $P/1W$ is unitless, we can express this value in decibels!

Specifically, we define this operation as:

$$P(dBw) \doteq 10 \log_{10}\left(\frac{P}{1\,W}\right)$$

For example, $P = 100$ Watts can alternatively be expressed as $P(dBw) = +20\,dBw$. Likewise, $P = 1\,mW$ can be expressed as $P(dBw) = -30\,dBw$.

Q: *OK, so what does* ***dBm*** *mean?*

A: This notation simply means that we have normalized some power P to one **Milliwatt** (i.e., $P/1mW$)—as opposed to one Watt. Therefore:

$$P(dBm) \doteq 10 \log_{10}\left(\frac{P}{1\,mW}\right)$$

For example, $P = 100$ Watts can alternatively be expressed as $P(dBm) = +50\,dBm$. Likewise, $P = 1\,mW$ can be expressed as $P(dBm) = 0\,dBm$.

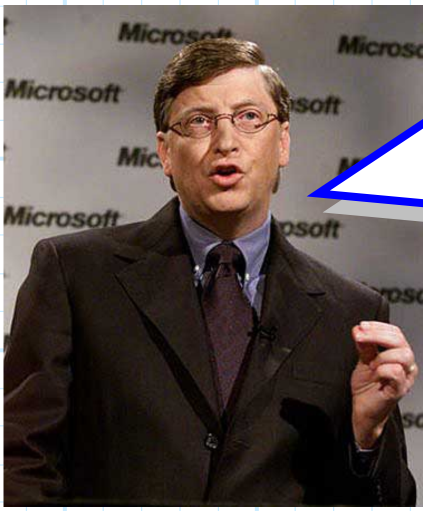
Make sure you are very **careful** when doing math with decibels!

Standard dB Values

Note that $10\,log_{10}(10) = 10dB$

Therefore an amplifier with a gain G = **10** is likewise said to have a gain of **10** dB.

Now consider an amplifier with a gain of **20** dB……



Q: *Yes, yes, I know. A* ***20 dB*** *amplifier has gain* ***G=20****, a* ***30 dB*** *amp has* ***G=30****, and so forth.*

Please speed this lecture up and quit wasting my valuable time making such ***obvious*** *statements!*

A: **NO!** Do **not** make this **mistake**!

Recall from **your** knowledge of logarithms that:

$$10\log_{10}\left[10^{n}\right] = n\,10\log_{10}\left[10\right] = 10n$$

Therefore, if we express gain as $G = 10^n$, we conclude:

$$G = 10^n \quad \leftrightarrow \quad G(dB) = 10n$$

In other words, G=100 = 10^2 (n=2) is expressed as 20 dB, while 30 dB (n=3) indicates G= 1000 = 10^3.

Likewise 100 mW is denoted as 20 dBm, and 1000 Watts is denoted as 30 *dBW*.

Note also that 0.001 mW = 10^{-3} mW is denoted as –30 *dBm*.

Another important relationship to keep in mind when using decibels is $10\log_{10}[2] \approx 3.0$. This means that:

$$10\log_{10}[2^n] = n\ 10\log_{10}[2] \simeq 3n$$

Therefore, if we express gain as $G = 2^n$, we conclude:

$$G = 2^n \quad \leftrightarrow \quad G(dB) \simeq 3n$$

As a result, a 15 dB (n=5) gain amplifier has G= 2^5 = 32. Similarly, 1/8 = 2^{-3} mW (n=-3) is denoted as –9 dBm.

Multiplicative Products and Decibels

Other logarithmic relationship that we will find useful are:

$$10\log_{10}[x\,y] = 10\log_{10}[x] + 10\log_{10}[y]$$

and its close cousin:

$$10\log_{10}\left[\frac{x}{y}\right] = 10\log_{10}[x] - 10\log_{10}[y]$$

Thus, the relationship $P_{out} = G\,P_{in}$ is written in **decibels** as:

$$P_{out} = G\,P_{in}$$

$$\frac{P_{out}}{1mW} = \frac{G\,P_{in}}{1mW}$$

$$10\log_{10}\left[\frac{P_{out}}{1mW}\right] = 10\log_{10}\left[\frac{G\,P_{in}}{1mW}\right]$$

$$10\log_{10}\left[\frac{P_{out}}{1mW}\right] = 10\log_{10}[G] + 10\log_{10}\left[\frac{P_{in}}{1mW}\right]$$

$$P_{out}(dBm) = G(dB) + P_{out}(dBm)$$

It is evident that "deebees" are **not** a unit! The units of the result can be found by **multiplying** the units of each term in a **summation** of decibel values.

For example, say some power $P_1 = 6\,dBm$ is combined with power $P_2 = 10\,dBm$. What is the resulting total power $P_T = P_1 + P_2$?

Q: *This result really **is** obvious—of **course** the total power is:*

$$\begin{aligned} P_T(dBm) &= P_1(dBm) + P_2(dBm) \\ &= 6\ dBm + 10\ dBm \\ &= 16\ dBm \end{aligned}$$

A: **NO!** Never do **this** either!

Logarithms are very helpful in expressing **products** or **ratios** of parameters, but they are **not** much help when our math involves sums and differences!

$$10\log_{10}[x + y] = ????$$

So, if you wish to add P_1 =6 dBm of power to P_2 =10 dBm of power, you must first **explicitly** express power in Watts:

$$P_1 = 10\ dBm = 10\ mW \quad \text{and} \quad P_2 = 6\ dBm = 4\ mW$$

Thus, the total power P_T is:

$$\begin{aligned} P_T &= P_1 + P_2 \\ &= 4.0\ mW + 10.0\ mW \\ &= 14.0\ mW \end{aligned}$$

Now, we can express this total power in *dBm*, where we find:

$$P_T\left(dBm\right) = 10\,log_{10}\left(\frac{14.0\ mW}{1.0\ mW}\right) = 11.46\ dBm$$

The result is **not** 16.0 *dBm* !.

We **can** mathematically add 6 *dBm* and 10 *dBm*, but we must understand what result means (nothing useful!).

$$\begin{aligned} 6\,dBm\ + 10\ dBm &= 10\log_{10}\left[\frac{4mW}{1mW}\right] + 10\log_{10}\left[\frac{10mW}{1mW}\right] \\ &= 10\log_{10}\left[\frac{40\ mW^2}{1mW^2}\right] \\ &= 16\ dB \text{ relative to } 1\ \text{mW}^2 \end{aligned}$$

Thus, mathematically speaking, 6 *dBm* + 10 *dBm* implies a multiplication of power, resulting in a value with units of **Watts squared** !

A few more **tidbits** about decibels:

1. $1.0 \quad \leftrightarrow \quad 0\ dB$

2. $0.0 \quad \leftrightarrow \quad -\infty\ dB$

3. $5^n \quad \leftrightarrow \quad \simeq 7n\ dB$ (can **you** show why?)